A- 0-)

Design Matrix

$$X = \begin{bmatrix} 1 & x_1 & \sin(2\pi x_1/12) \\ \vdots & \vdots & \vdots \\ 1 & x_n & \sin(2\pi x_n/12) \end{bmatrix}$$

Parameter Vector

$$\beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}$$

1-) By definition a matrix is symmetric when "a matrix A s.t. $A^T = A$"

Hence, since A is symmetric, then $A^T = A$.
Similarly, $(A^2)^T = A^T A^T = AA = A^2$ which means A^2 is also symmetric.

2-) ~~By definition a matrix A is orthogonally diagonalizable if~~ ~~$A = PDP^T = PDP^{-1}$~~

A matrix, A, is orthogonally diagonalizable if and only if A is a symmetric matrix.
Since we know A is orthogonally diagonalizable, then A is symmetric which means $A^T = A$. By using the proof above, we showed that A^2 is also symmetric, so A^2 is also orthogonally diagonalizable

B-

1-) First, let's write $Q(x)$ as $x^T A x$.

$$x^T A x = [x_1 \ x_2] \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$ Now let's find its eigenvalues:

$$A - \lambda I = \begin{bmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{bmatrix} \quad \det\begin{pmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{pmatrix} = \lambda^2 - 6\lambda + 8 = (\lambda-4)(\lambda-2)$$

so $\lambda = 4$ & $\lambda = 2$

Hence, we can make $D = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$ and usually we'd calculate P but since $x = Py$, then $Q(x) = Q(Py) = (Py)^T A (Py) = y^T (P^T A P) y = y^T D y = \lambda_1 y_1^2 + \lambda_2 y_2^2$

So the answer is $\boxed{4y_1^2 + 2y_2^2}$

2-) As the professor said during lecture, "if A is a symmetric matrix, then $M = \max\limits_{x^T x = 1} x^T A x$" which means that under these conditions M is the greatest eigenvalue of A which we calculated to be $\boxed{4}$