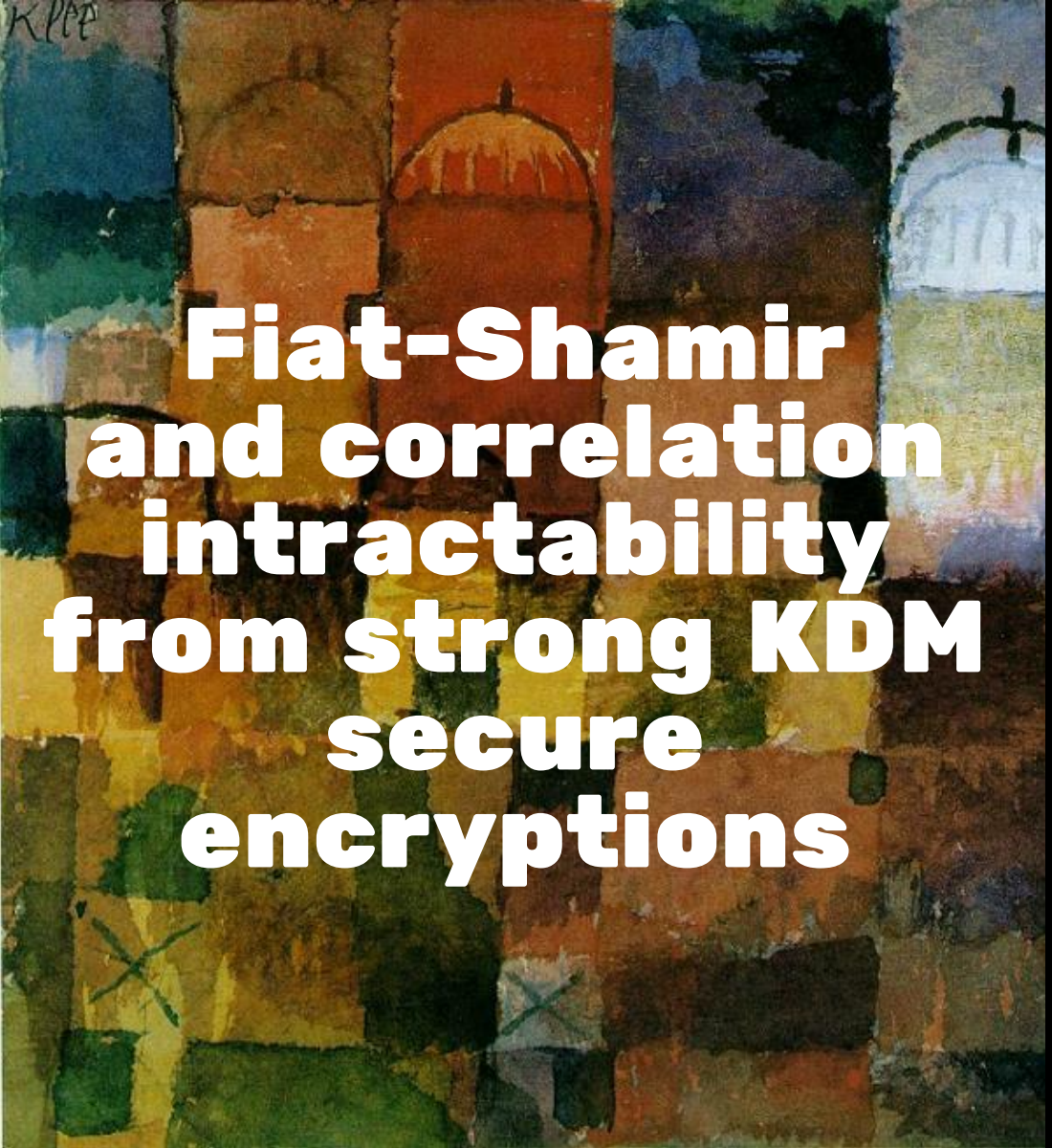
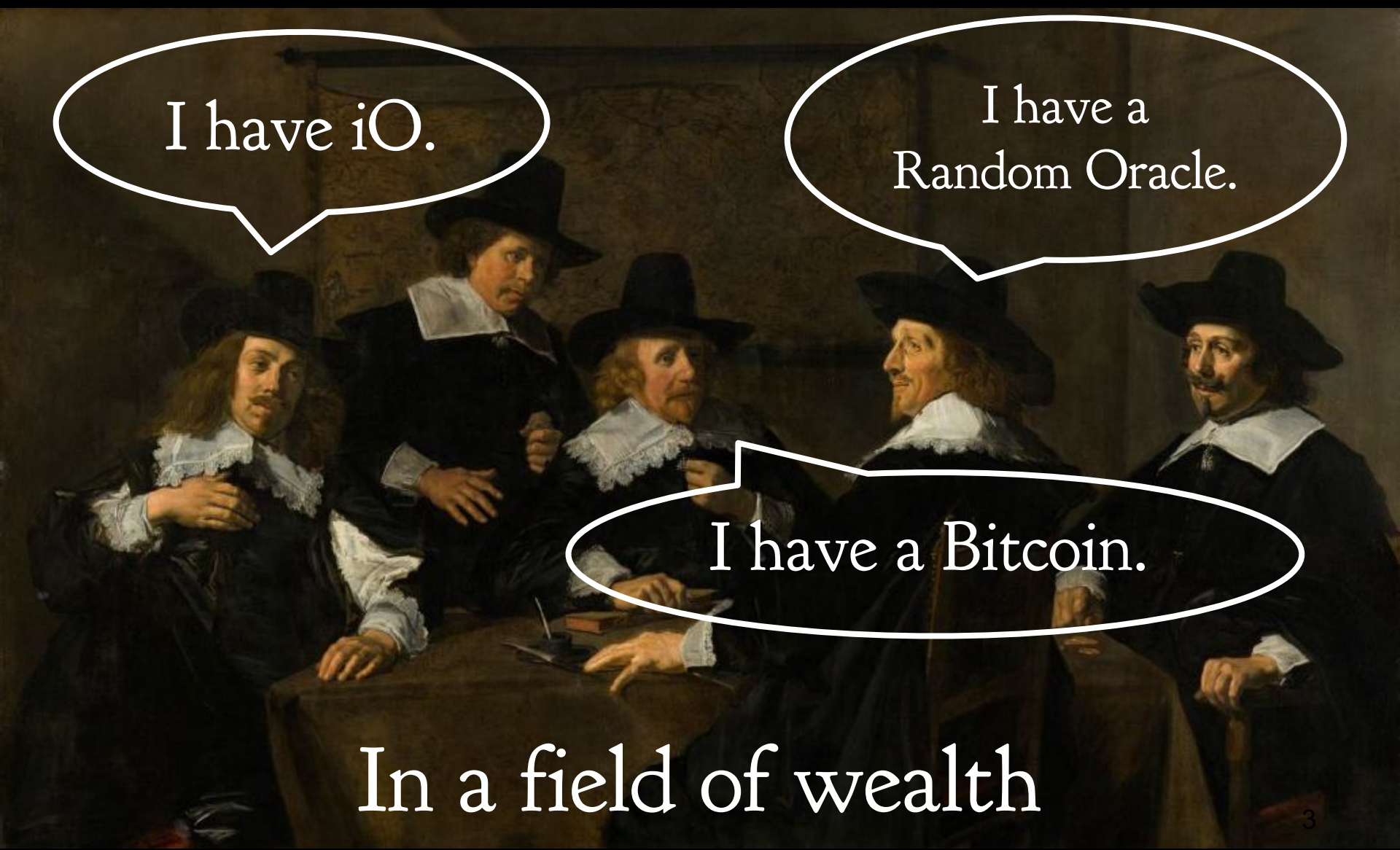


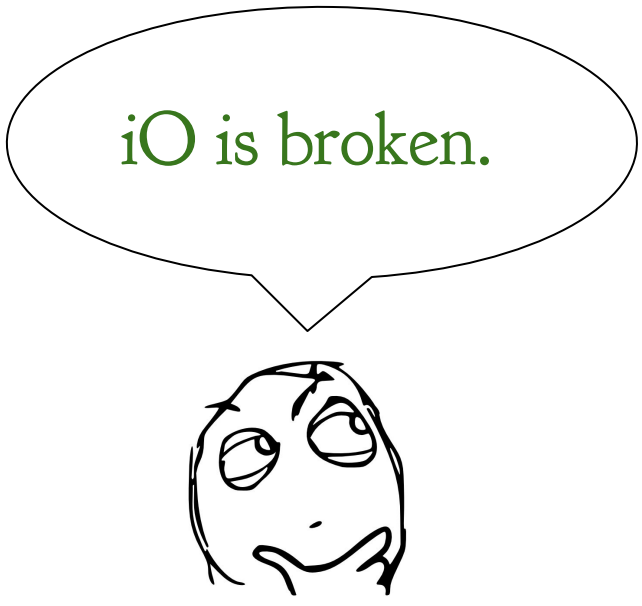
Fiat-Shamir and correlation intractability from strong KDM secure encryptions

Ran Canetti
Yilei Chen
Leonid Reyzin
Ron Rothblum

Trailer



In a field of wealth



They had nothing

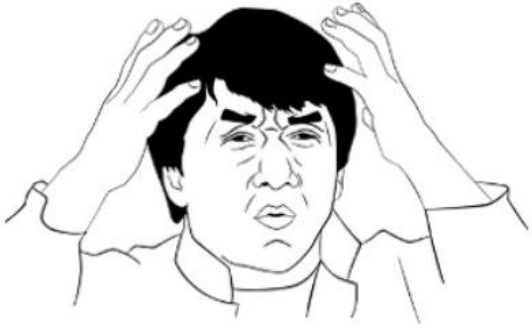
Equal number of Bitcoin for every member!

Feel free to assume iO.

Free access to Random oracle!

On the land of liberty

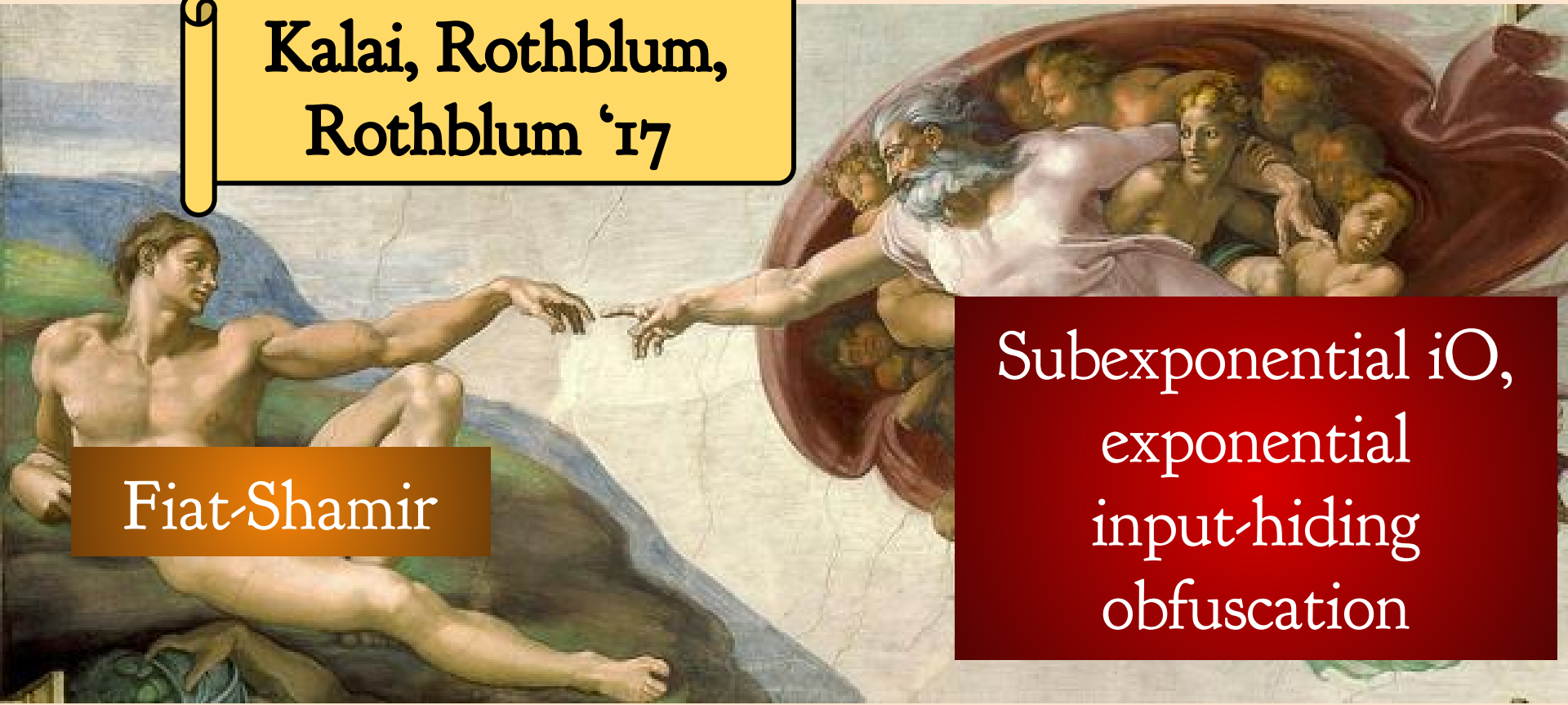
Practical Identity-Based Encryption without Random Oracles



Functional Encryption without Obfuscation

They are enslaved

At the moment of glory



At the moment of glory

Fiat-Shamir and correlation intractability

from

~~subexponentially secure iO and~~

~~exponentially secure input-hiding obfuscation~~

They raise,

Fiat-Shamir and correlation intractability

from

~~subexponentially secure iO and~~

~~exponentially secure input-hiding obfuscation~~

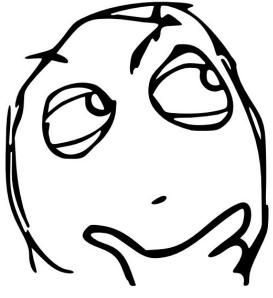
exponentially KDM secure encryption schemes

They raise, united

Directors

Ran Canetti
Yilei Chen
Leonid Reyzin
Ron Rothblum

How to capture the properties of a “good enough” cryptographic hash function?



Fiat-Shamir ‘86, Bellare-Rogaway ‘93:

Can model cryptographic hash functions as “Random Oracles”

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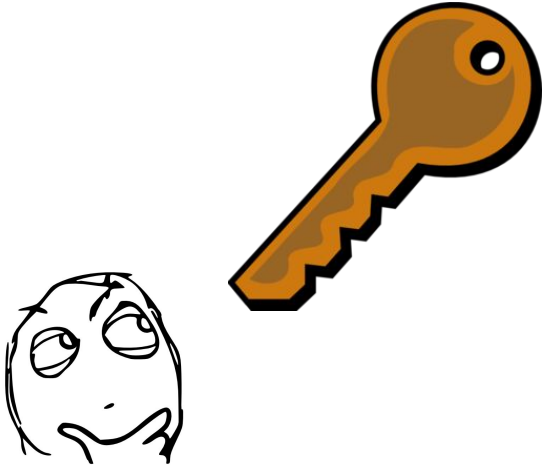

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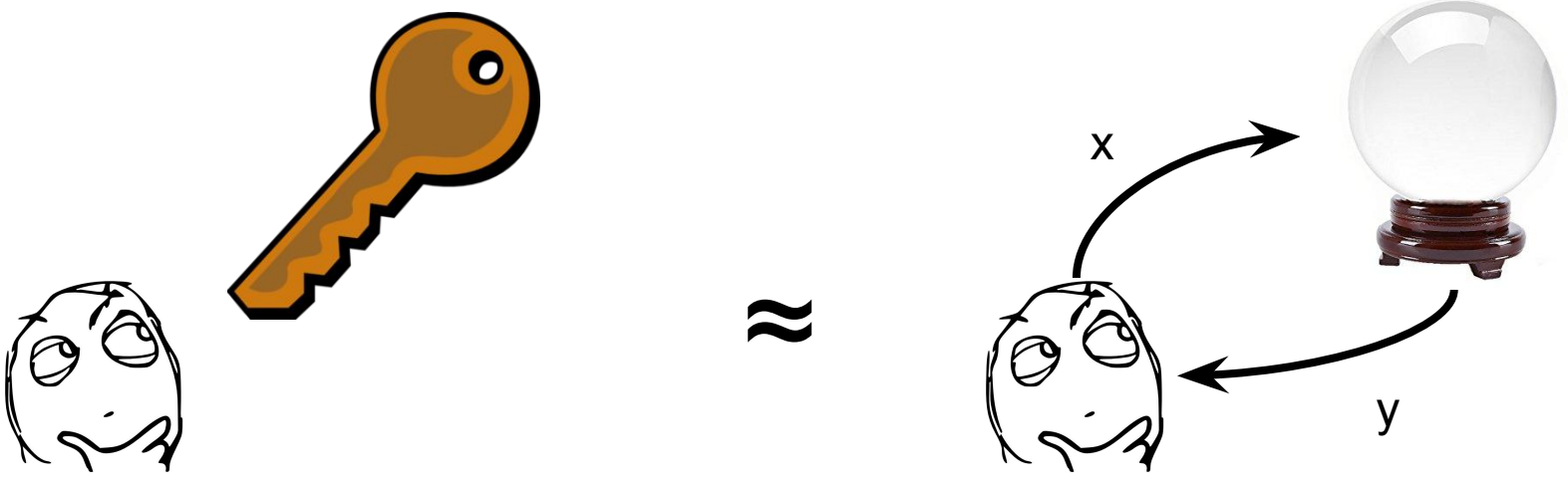

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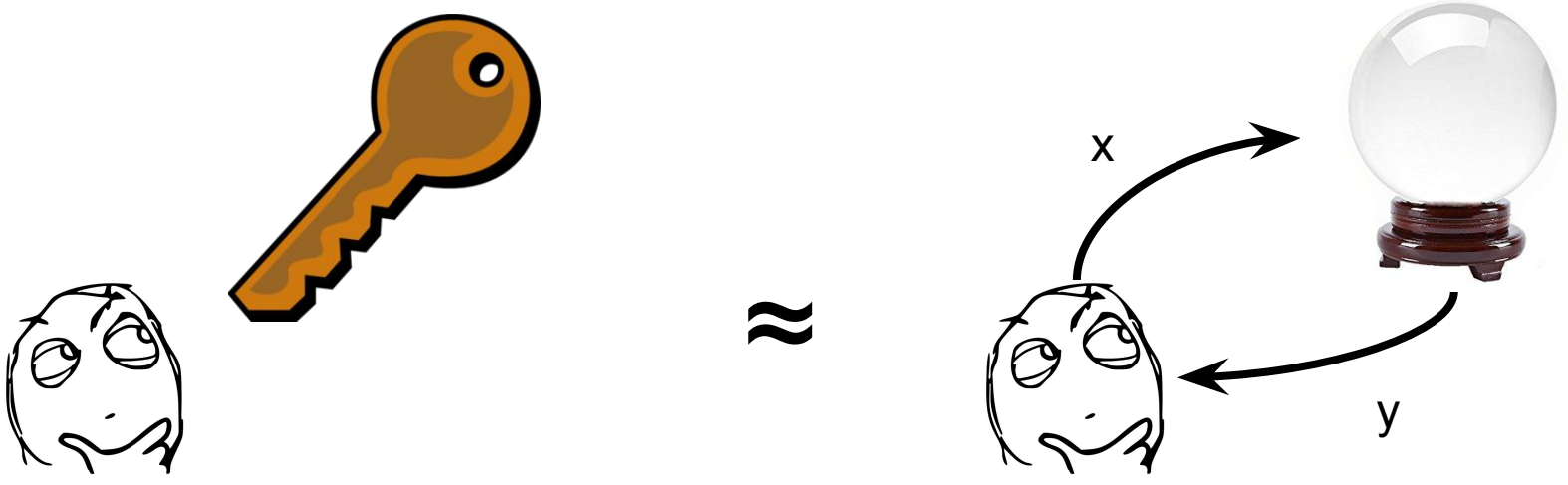
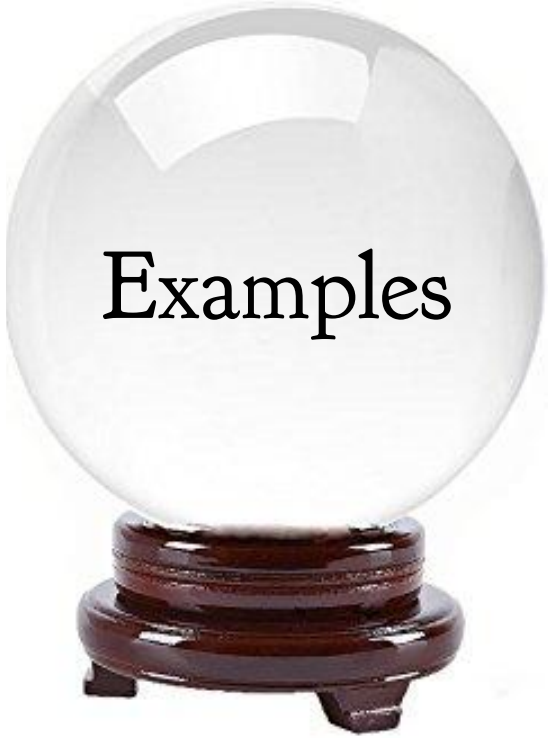


Build efficient crypto schemes (secure under heuristics):

- Efficient CCA secure encryptions
- Hash-and-sign paradigm
- Many applications


Examples

d

H(X)=000000000110….110010111101

Desired property:
Find an input X such that H(X) has a prefix of d 0s takes roughly 2^d steps.

Fiat, Shamir 86: Turning an interactive protocol => non-interactive

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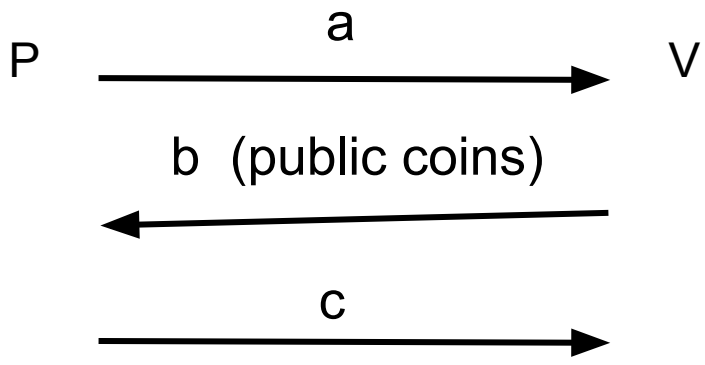
An interactive protocol for a language L and an instance x:



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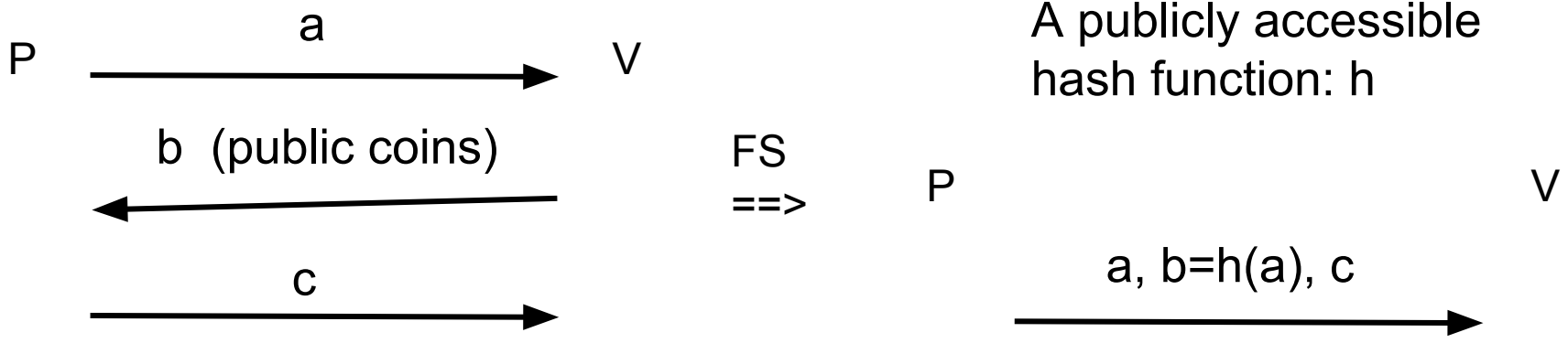

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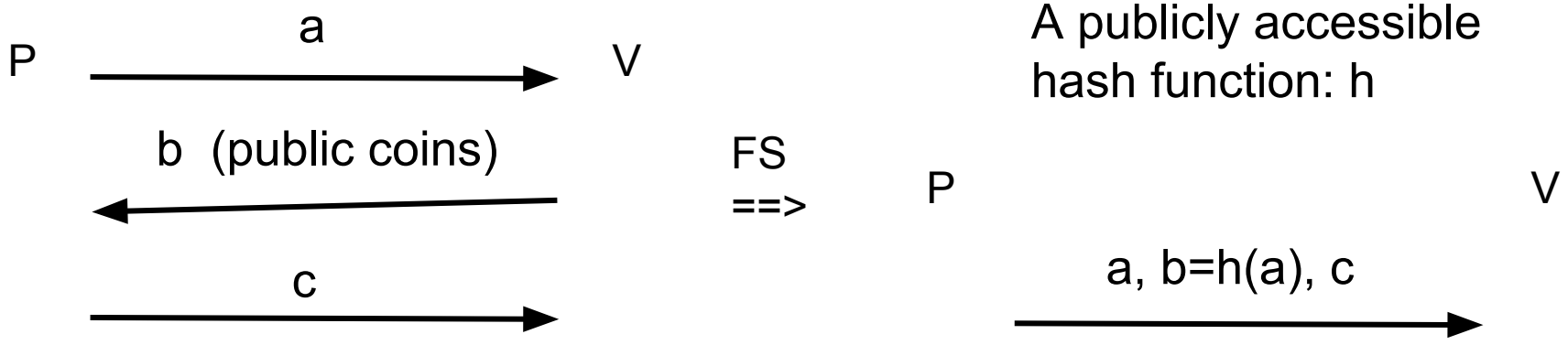
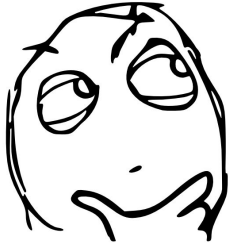


Goal: preserve the original properties of the protocol, e.g. completeness and soundness.

Fiat, Shamir

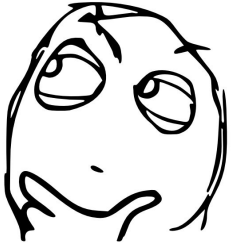
Does the Random oracle model oversimplifies the problem in the reality?



Fiat, Shamir

Does the Random oracle model oversimplifies the problem in the reality?

Can we define a more concrete property that captures what we want?



Today:

Correlation Intractability

"infeasibility of finding 'sparse' input-output relations"

--- Canetti, Goldreich, Halevi 1998

Sparse Relations

“For each input (x),
the fraction of outputs (y) in the relation is negligible”

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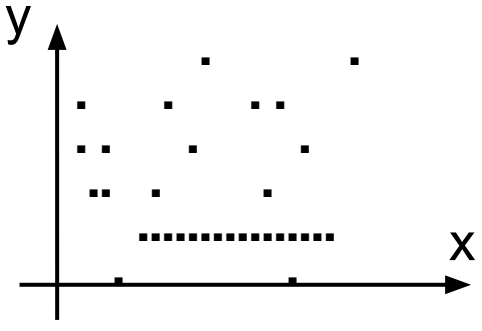

Sparse Relations

"For each input (x),
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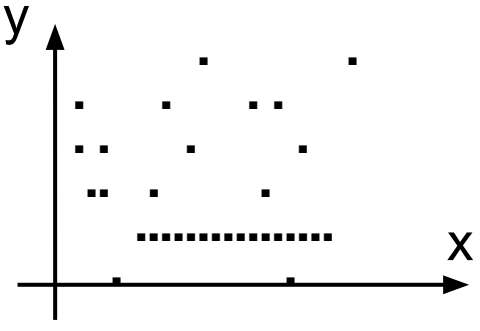
Implicitly: relations that are intractable on truly random functions

For all (non-uniform) p.p.t. Adversary:

$$Pr_{Adv,\ O}[\ Adv^{O} \rightarrow x: R(x, O(x))=1\ ] < negl.$$



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***Can naturally generalize to multi-input-output relations**

For all (non-uniform) p.p.t. Adversary:

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Sparse Relations

"For each input (x),
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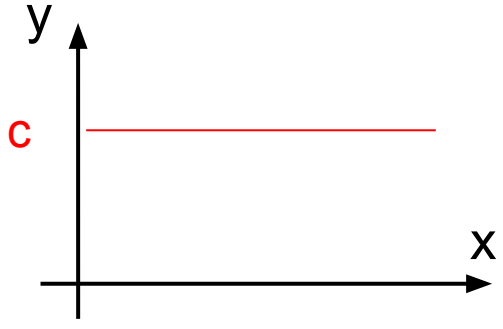
Examples: Interesting sparse relations

Constant relation: R(x, y) = 1, if y=c



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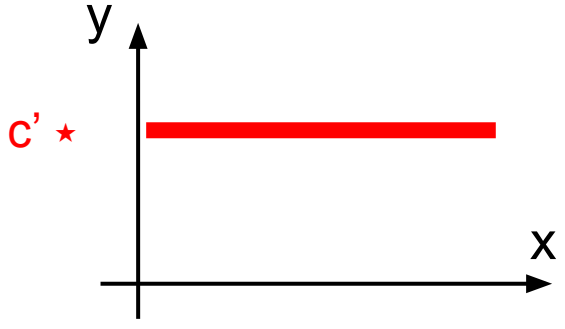
Constant relation: R(x, y) = 1, if y=c
Partial constant relation: R(x, y) = 1, if the first half of y=c'



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*Examples for interesting multiple-input-output relations

Collision relation: R(x1, y1, x2, y2) = 1, if y1=y2 and (not x1=x2)

Sparse Relations

"For each input (x),
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Correlation intractability [Canetti, Goldreich, Halevi '98]

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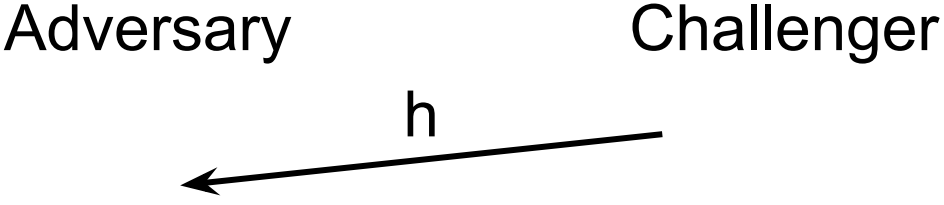

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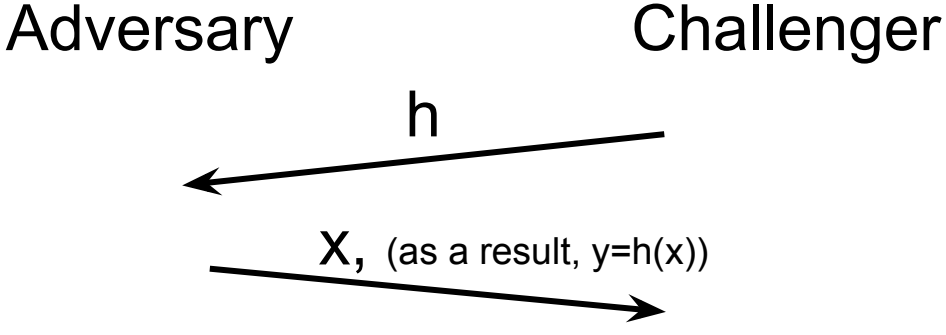

Adversary wins if R(x, h(x))=1

Examples: Interesting sparse relations

Constant relation: $R(x, y) = 1$, if $y=c$

Partial constant relation: $R(x, y) = 1$, if the first half of $y=c'$

Exercise: Prove CI w.r.t. simple relations from your favorite hash function.

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Bitcoin

H(???...?)=000000….XYZ3d83h

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For all relations of negligible density, all polynomial adversaries succeed with negligible probability.

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Correlation intractability [Canetti, Goldreich, Halevi 98]
For all relations of negligible density, all polynomial adversaries succeed with negligible probability.

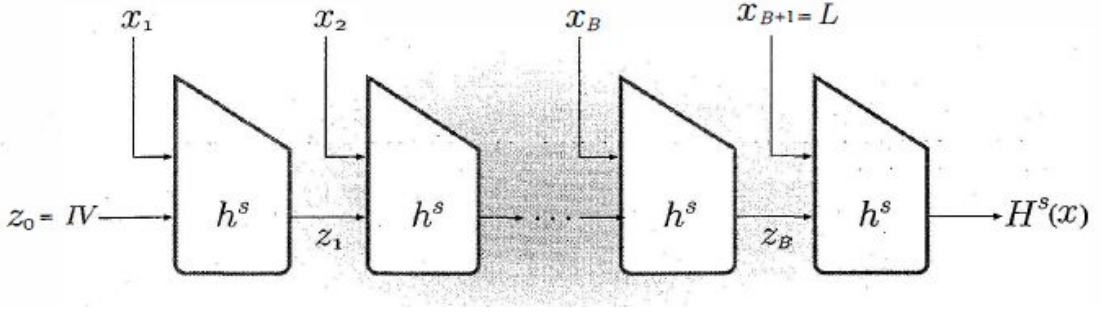
Quantitative correlation intractability [This work]
For all relations of density D, all adversaries running in time T succeed with probability f(D, T).

The smallest possible f to hope for: f(D,T) = DT

H(???...?)=000000….XYZ3d83h

In fact, SHA256 doesn’t satisfy the best-possible quantitative CI.

AsicBoost takes advantage of Merkle-Damgard to speed up bitcoin mining.



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Bitcoin **need quantitative hardness**

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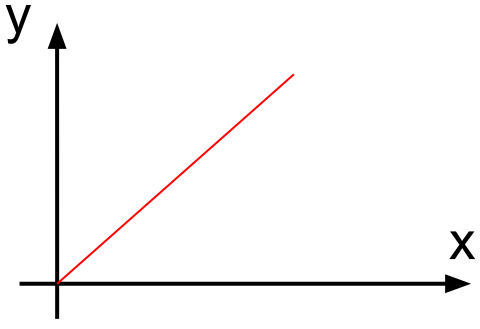

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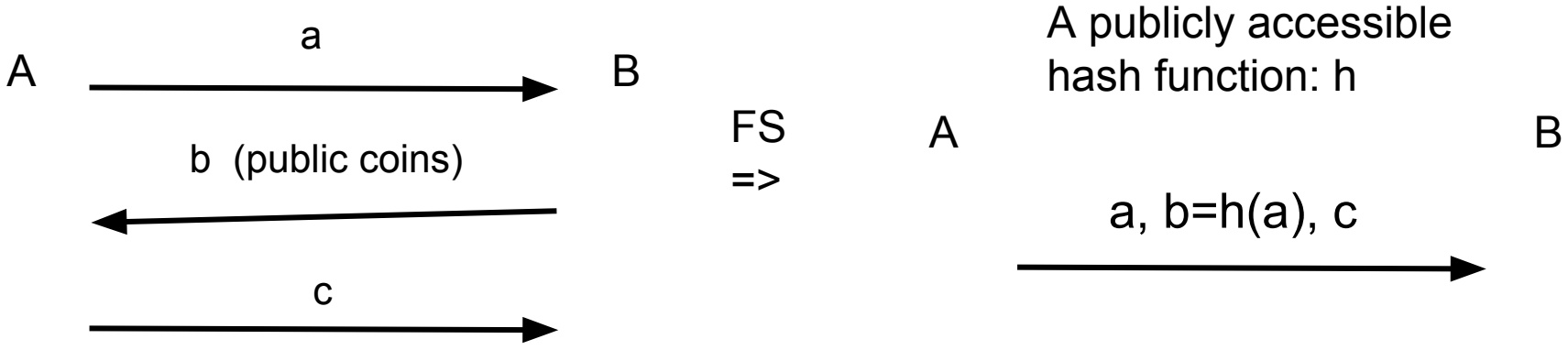
Fiat-Shamir: Long story ...

An interactive protocol for a language L and an instance x:



Fiat, Shamir 86: 3 round proof system => 1 round argument

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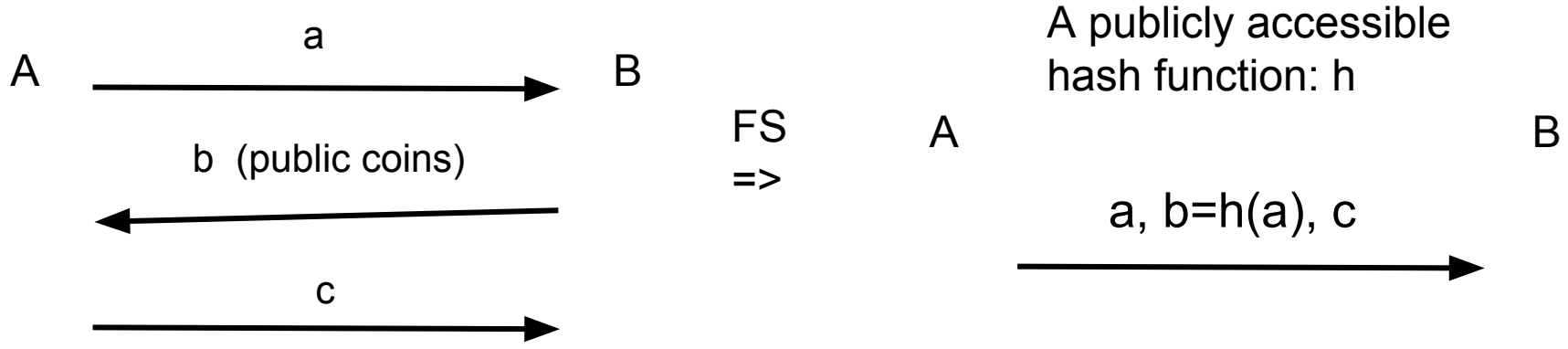


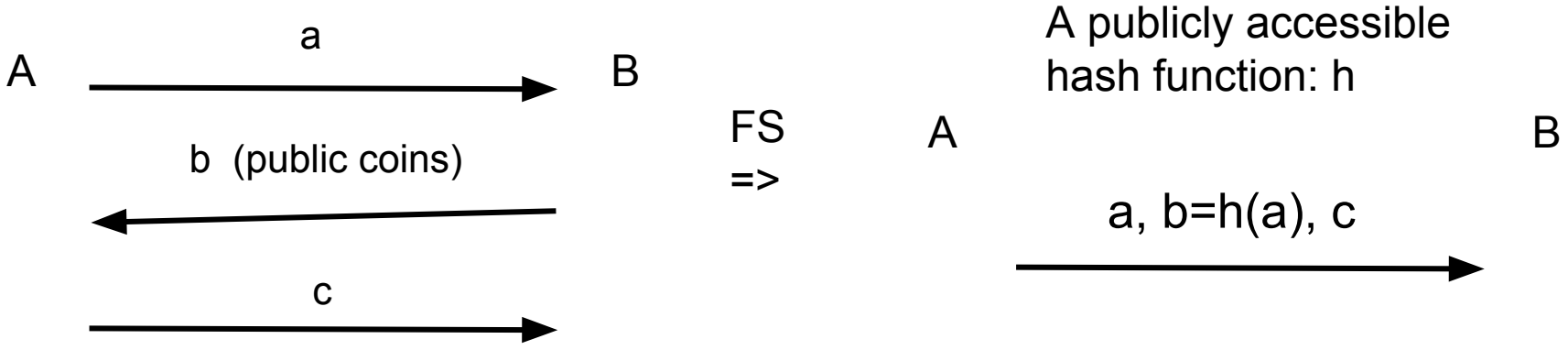
Fiat, Shamir 86: 3 round proof system => 1 round argument

Fiat, Shamir relation: [Hada, Tanaka 99, Dwork et al 03]

R(a, b)=1 if $x \notin L$ and $\exists$ c s.t. Verifier(x, a, b, c) accepts

R is a sparse relation following the soundness of the 3 round proof system.
The membership of R is typically not polynomial-time decidable due to "$\exists$ c".

An interactive protocol for a language L and an instance x:



Fiat, Shamir 86: 3 round proof system => 1 round argument

Difficulty to prove the Fiat-Shamir property:

[Goldwasser, Kalai ‘03] impossibility for arguments.
[Barak, Lindell, Vadhan ‘06] Define “Entropy preserving”, it implies FS for proofs.
[Dodis, Ristenpart, Vadhan ‘12] “Entropy preserving” is necessary for FS for proofs.
[Bitansky et al. ‘13] for proof systems, impossible from black-box reductions to falsifiable assumptions.

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Bitcoin **need quantitative hardness**

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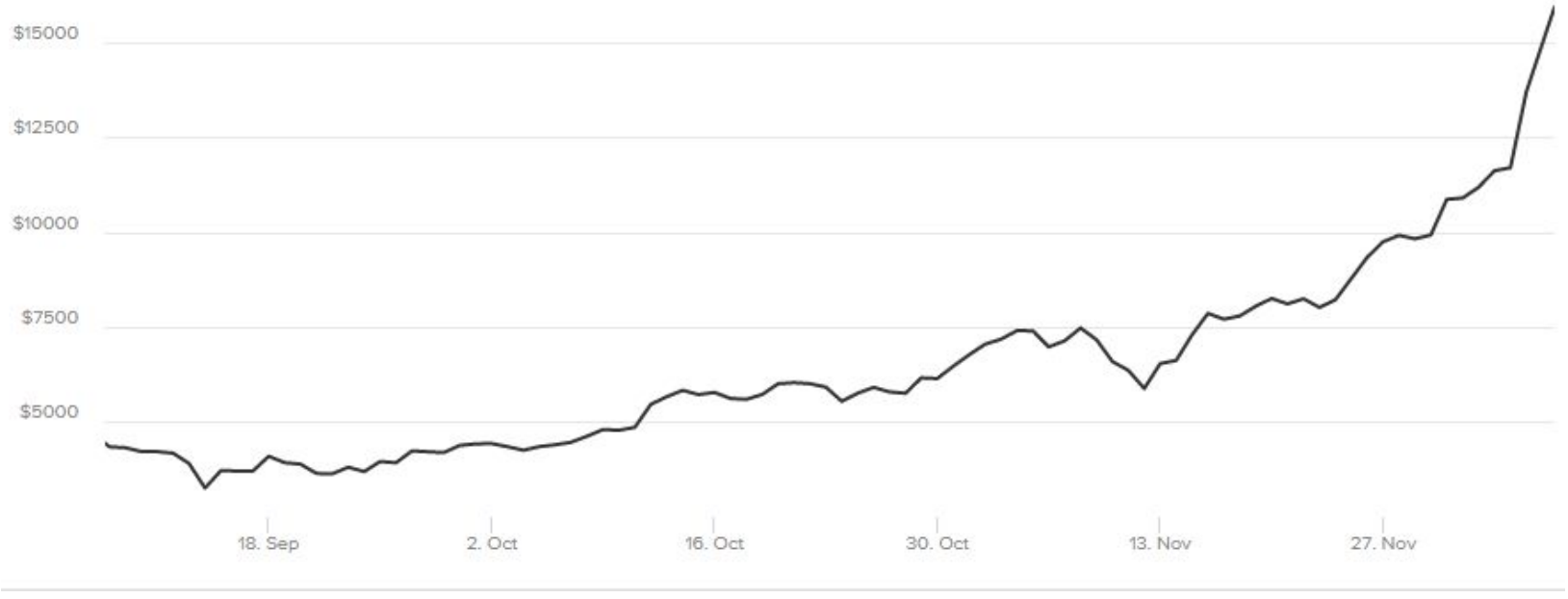
[CGH 98]

Correlation intractability is impossible to achieve

Bitcoin price
live



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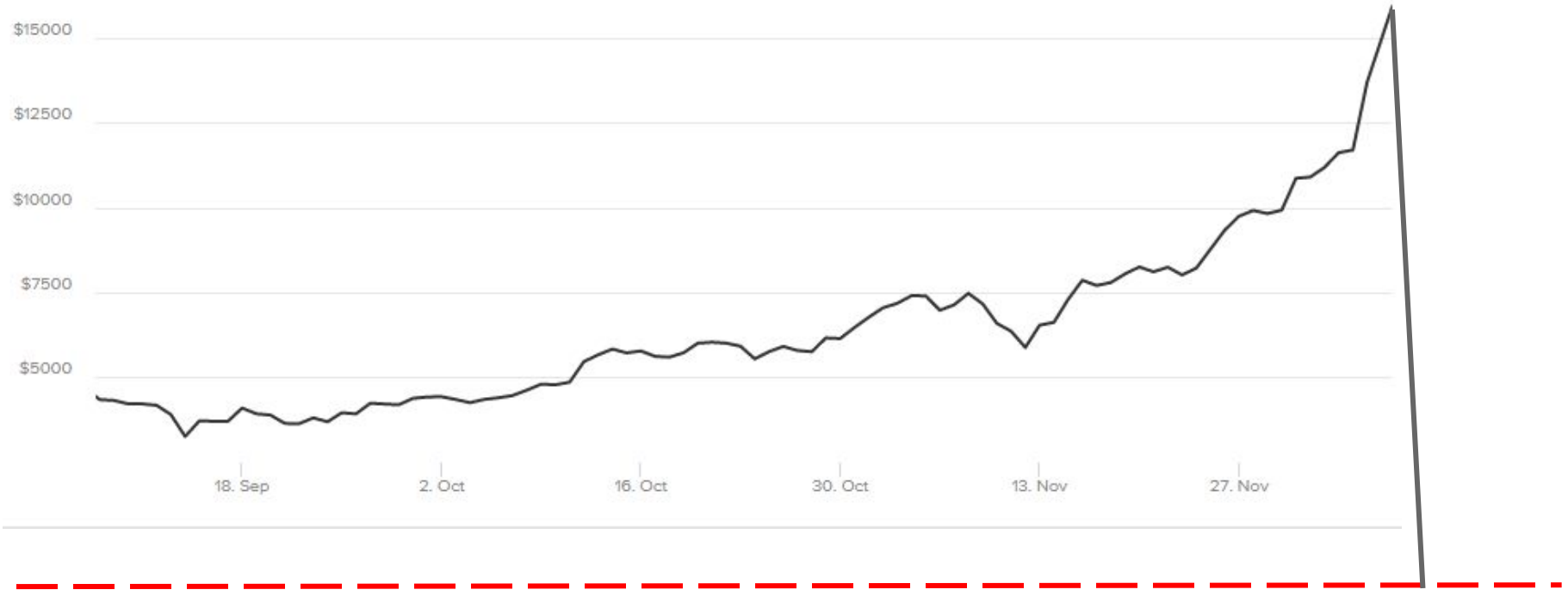

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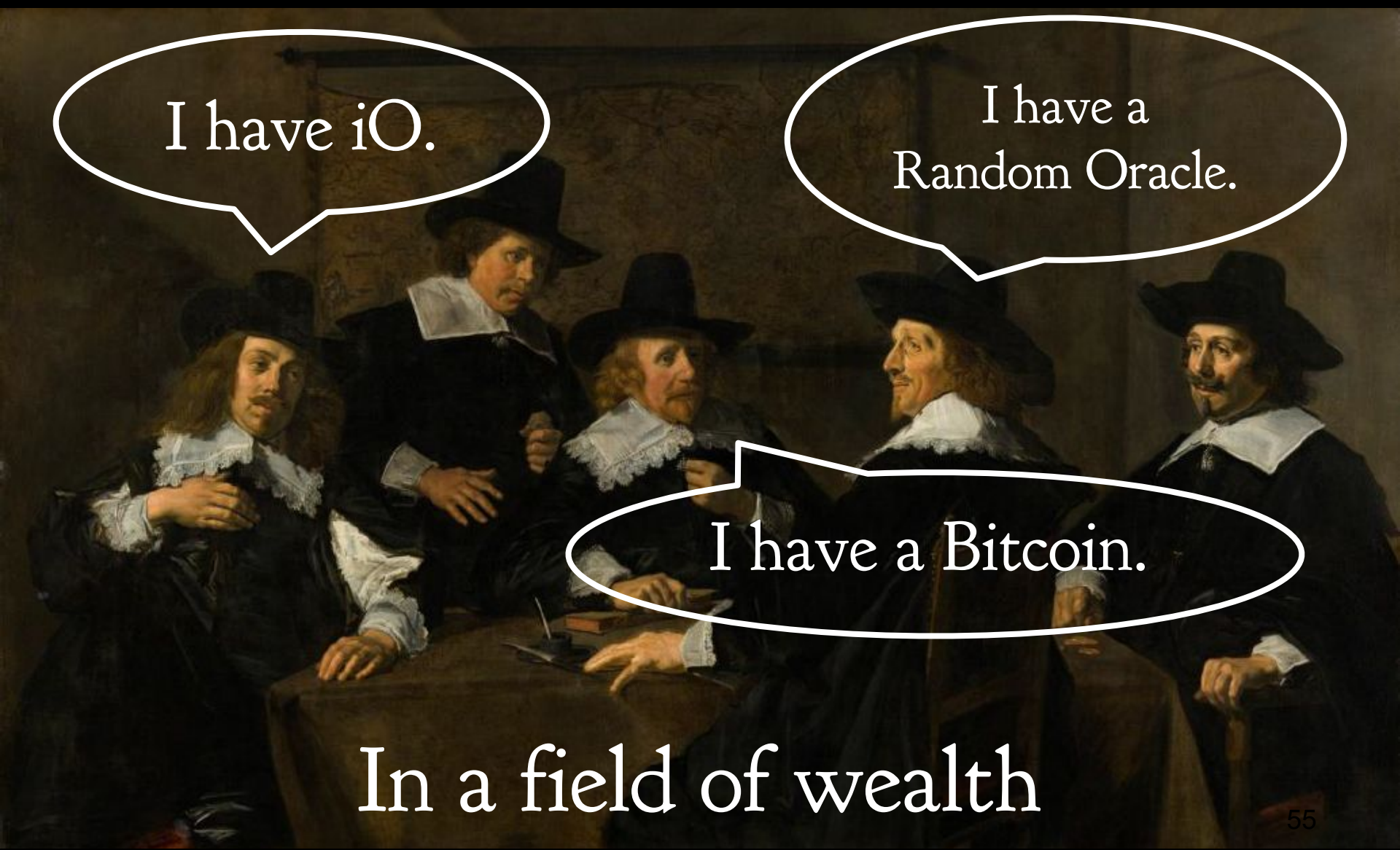
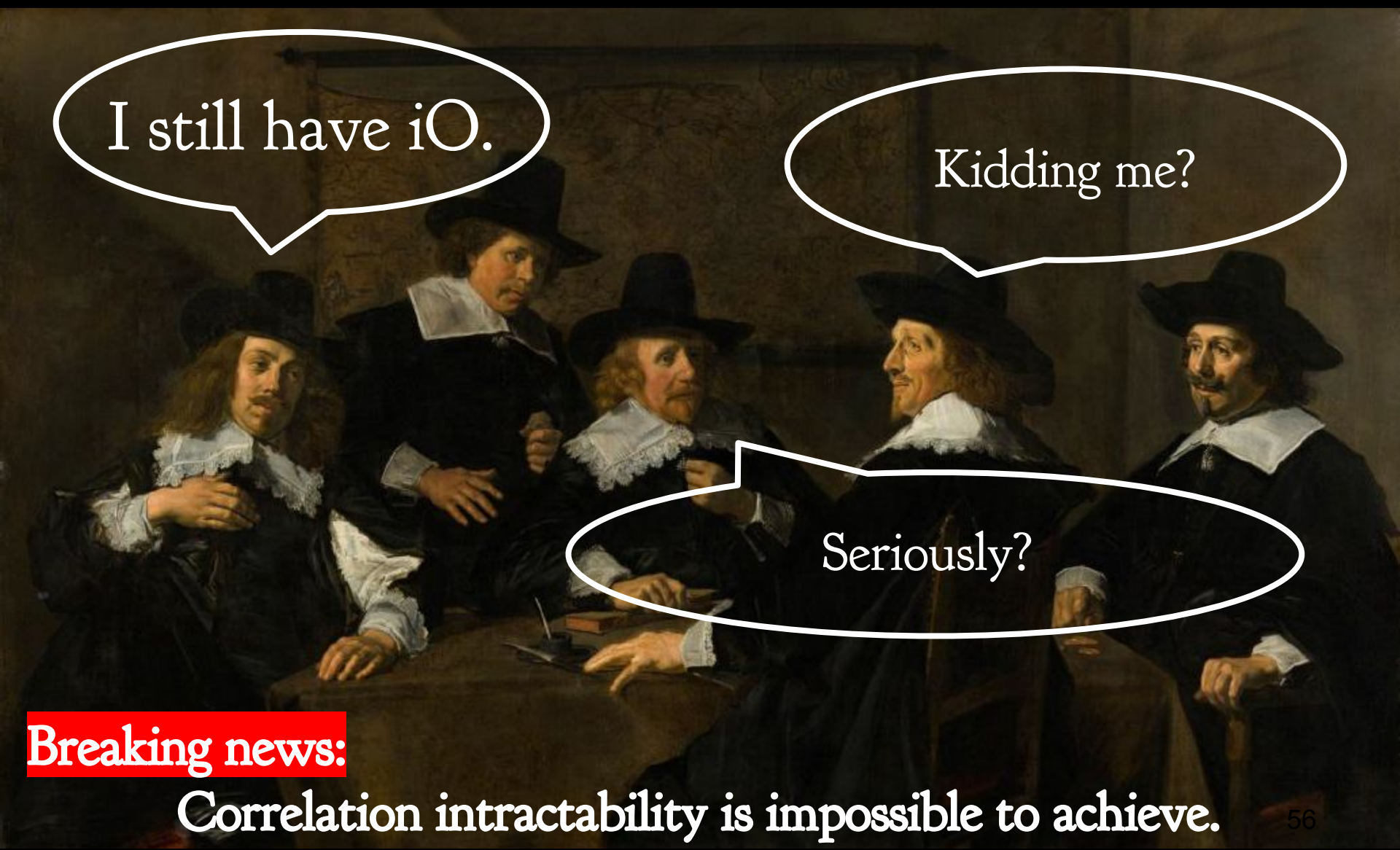


In a field of wealth



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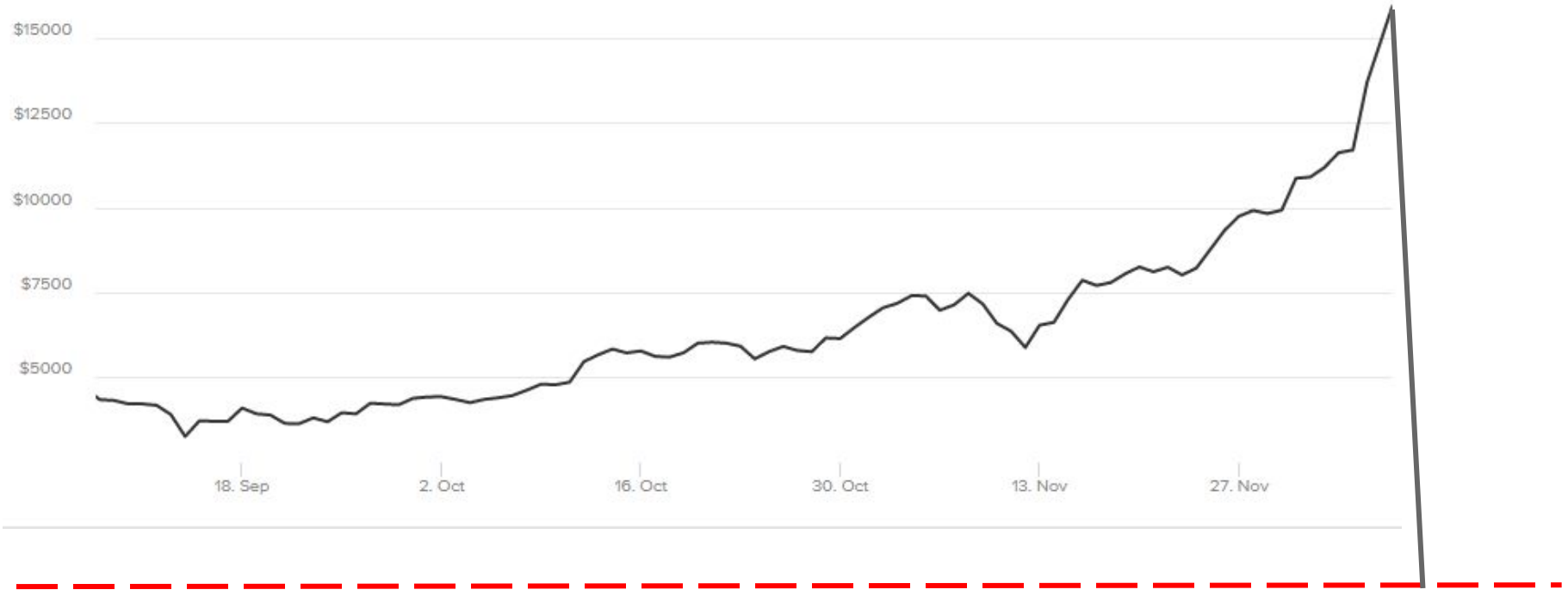
[CGH 98]

Correlation intractability is impossible to achieve … in some cases.

Bitcoin price
live



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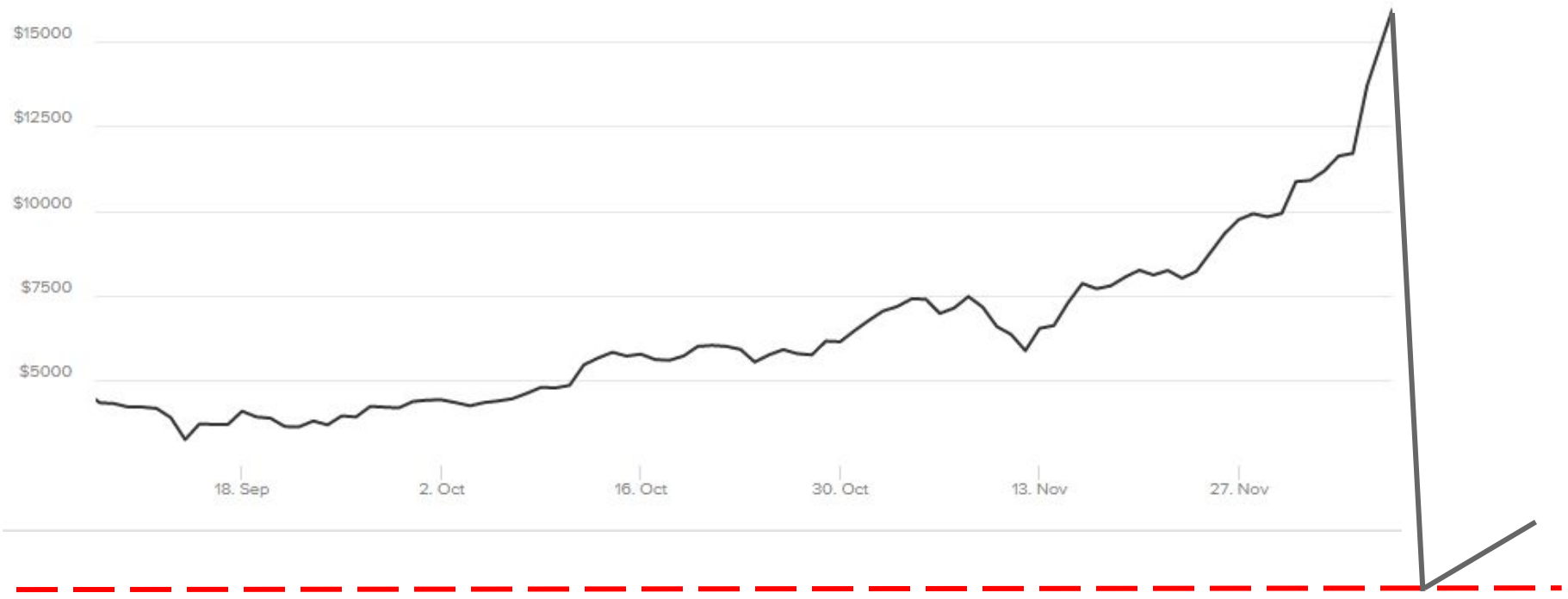

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Canetti, Goldreich, Halevi:

H cannot be correlation intractable if **the key is short** !!!

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Consider the “Diagonal” relation:

$$R^H(x, y)=1 \text{ iff } y=x(x)$$

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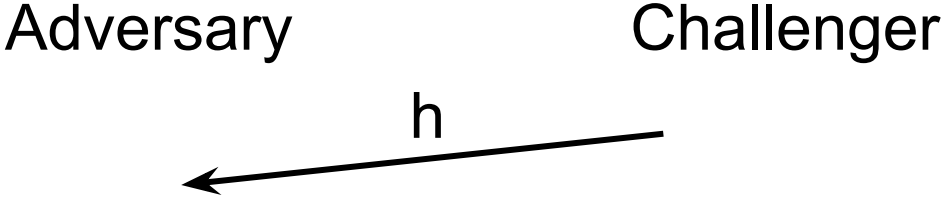

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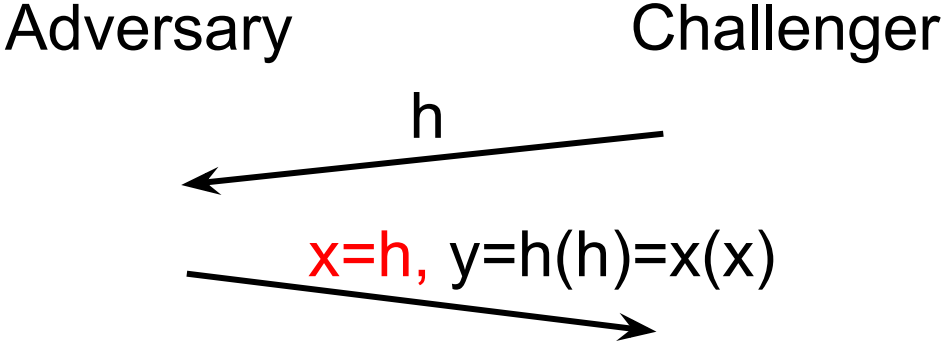
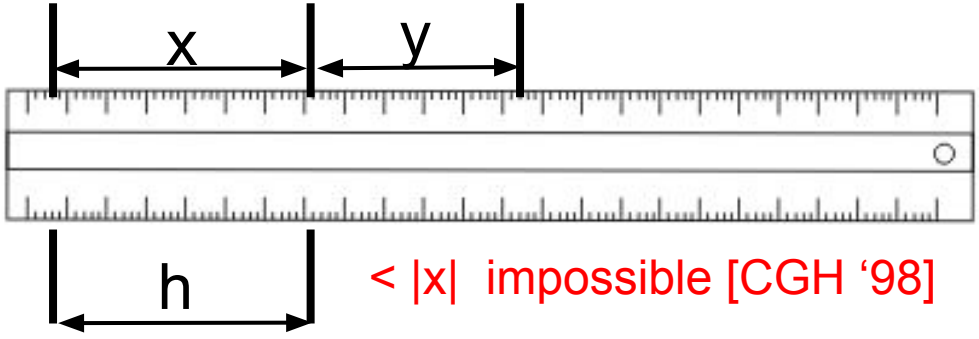
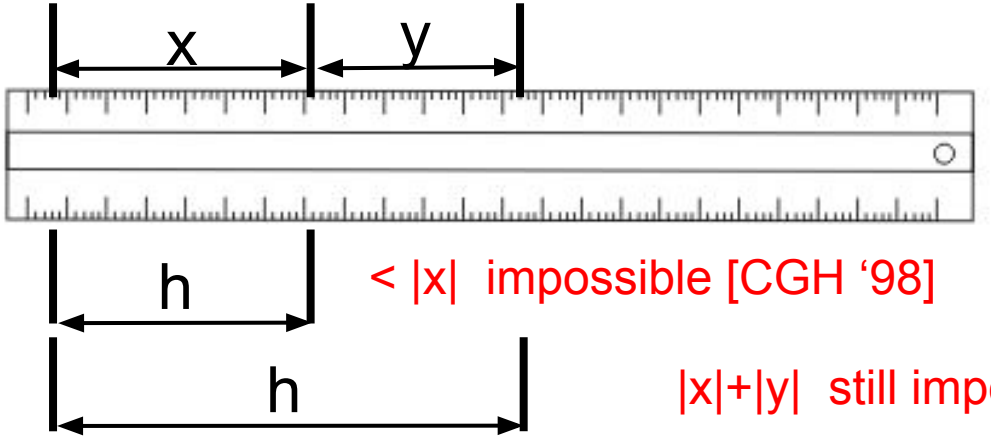


Adversary wins if $R^H(x, x(x))=1$



< |x| impossible [CGH ‘98]



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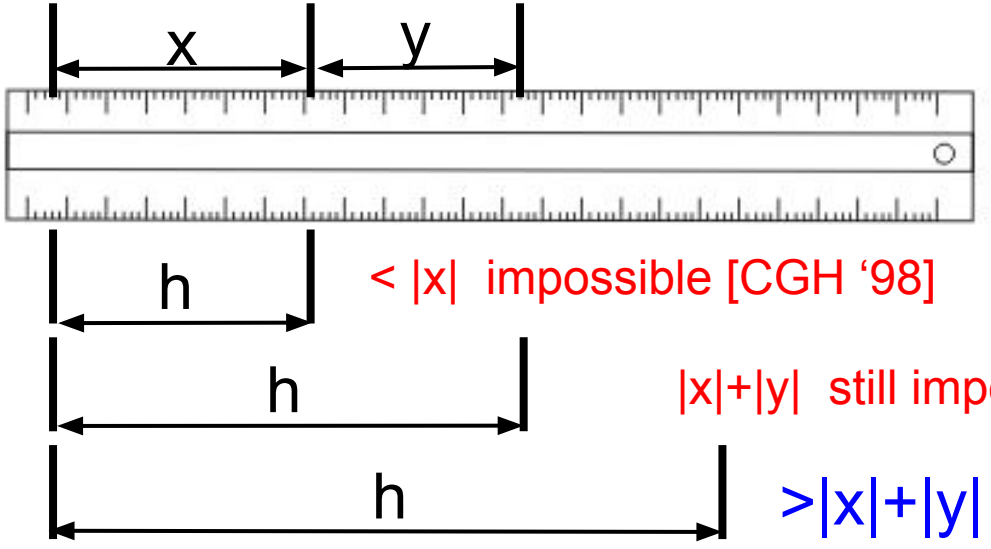

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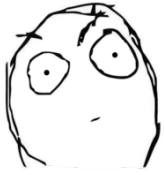

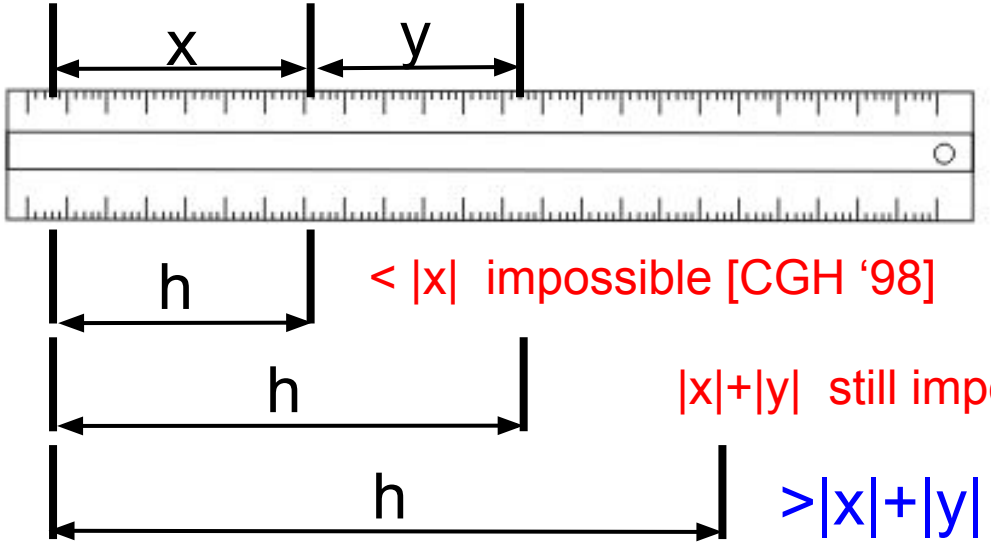

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Possible for hash functions with even just 'slightly' longer keys… not too bad.

Functions from $\{0,1\}^* \rightarrow \{0,1\}^m$ can never be correlation intractable.

Examples: Interesting sparse relations

Constant relation: R(x, y) = 1, if y=c

Partial constant relation: R(x, y) = 1, if the first half of y=c'

Bitcoin **need quantitative hardness**

Fixed point relation: R(x, y) = 1 if y=x (or prefix of x if |y|<|x|)

Fiat-Shamir: Long story ...

"Diagonal relation": $R^H(x, y) = 1$ if $y=x(x)$ [CGH 98]

"Small family relation": $R^H(x, y) = 1$ if $\exists\ h \in H$ s.t. $y=h(x)$ [this work]

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Fiat-Shamir for proofs: $R(a, b) = 1$ if $x \notin L$ and $\exists\ c$ s.t. $Verifier(x, a, b, c)=1$

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Constraints: the key must be longer than $|x|+|y|$, the seed must be longer than $|x|$.
Obvious difficulty: prove "weird" relations.

Other concrete properties for RO-like hash functions:

Perfect one-wayness [Canetti 97, Canetti, Micciancio, Reingold 98]

Non-malleability [Boldyreva, Cash, Fischlin, Warinschi 09]

Magic Functions [Dwork, Naor, Reingold, Stockmeyer 03]

Entropy preservation [Barak, Lindell, Vadhan 04]

Seed-incompressible CI [Halevi, Myers, Rackoff 08]

Correlated-Input security [Goyal, O'Neill, Rao 11]

Universal Computational Extractor [Bellare, Hoang, Keelveedhi 13]

ELF [Zhandry 16]

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Assuming subexponential iO, subexponential owf, input-hiding obfuscation for evasive functions.

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the existence of a sub-exponentially secure one-way function.

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- For all key k*, a random ciphertext CT decrypts to a random message m.
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For all f (possibly inefficient), any polytime Adv, for all superpoly function s

$$\Pr_k[\, \text{Adv}(\, \text{Enc}(k, f(k))\,) \rightarrow k\,] < \frac{s(n)}{2^n} \text{ , where } n = \text{secp} = |k|$$

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We provide parameters where ElGamal and Regev encryptions plausibly satisfy that level of security.

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For the instantiations from ElGamal or Regev, the assumptions are morally:
Discrete-log where polynomial time adv succeeds with probability
no-better-than-guessing + KDM for any f.
LWE where polynomial time adv succeeds in key-recovery with probability
no-better-than-guessing + KDM for any f.

H(???...?)=000000….XYZ3d83h

The instantiations from ElGamal or Regev do not suffice for the quantitative correlation intractability required for Bitcoin (will explain later).

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How do we get around the black-box lower bound of [Bitansky et al. 13]?

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NIZK for NP. (soundness is from FS; zero-knowledge need to prove)

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[Reingold, Rothblum, Rothblum ‘16]: Constant round doubly efficient IP for any language computable in polytime and fixed polynomial space.
=> a non-interactive one
(for non-interactive delegation with public verifiability, previous results assume RO, or iO, or “mmaps-looking FHE” by [Paneth-Rothblum ‘17])

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Construction of the hash function.

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[Fiat-Shamir, '86]

3.2 Security

The formal proof of security in this extended abstract assumes that n is sufficiently large and that f is a truly random function. Consequently, there can be no generic attack which breaks the scheme for any n and f unless factoring is easy. Practical implementations which use particular moduli n_0 and psuedo-random functions f_0 may still be vulnerable to specialized attacks, but they mearly show that n_0 is too small or that f_0 is demonstrably non-random. When n_0 is at least 512 bits long and f_0 is sufficiently strong (*e.g.*, multiple DES with a fixed cleartext and variable key), such attacks are quite unlikely.

Explain the assumption.
Explain the proof idea.
Possible relaxations & generalizations.

Instantiation via Regev's scheme.

Parameters: n = security parameter, B = O(n), $q=O(n^3)$, n' s.t. $(2B+1)^{n'} = 2^n$

sk: $s \in [-B, B]^{n'}$;

Enc(sk, b): sample $a \in Z_q^n$; sample $e \in [0, q/2)$. CT: a, $z = \langle a,s \rangle + b \cdot q/2 + e$

Dec(sk, CT): $z - \langle a,s \rangle$, then round by 2.

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(it is not a RO-like function due to approx. linearity.)

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Lattice open problem: find a polynomial time algorithm for LWE with polynomial modulus that achieves better-than-guessing success probability.

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Is there a group where this assumption is plausible?

Discrete log over finite field (of size roughly 2^n)	T(n) time, ~ 1 success probability	Polytime, P success probability
Pollard's rho algorithm	$T(n) = \exp(n/2)$	$P = \text{poly}(n) \cdot 2^{-n}$
Index calculus algorithm	$T(n) = \exp(n^{1/3} (\log n)^{2/3})$	???

The success prob. of index calculus in polynomial time:

We don't know how to achieve
better-than-guessing success probability.

Exercise: find a polytime algorithm for discrete-log over finite field that achieves better-than-guessing success probability.

In the online-offline model:

The offline phase gets g, F_q, runs infinite time, keep polysize advice.

The online phase gets $h = g^x$, and the advice, runs in polytime.

Claim: index-calculus achieves $2^{-n/C}$ success probability for any constant C.

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Claim: index-calculus achieves $2^{-n/C}$ success probability for any constant C.

How:
The offline phase gets g, F_q, picks a polynomial bound $B=n^C$, computes all the
$\log_g(p)$ for p in {2, 3, 5, 7, …, B}

The online phase gets $h = g^x$, picks a random r, compute $w = h \cdot g^r = g^{x+r} \bmod q$.
Then see if all the factors of w are below B. [Rankin 1938: $2^{-n/C}$]
If so, $w = 2^{x2} \cdot 3^{x3} \cdot 5^{x5} \cdot \ldots \cdot B^{xB}$
Then $x = \log_g(2) \cdot x2 + \log_g(3) \cdot x3 + \ldots + \log_g(B) \cdot xB - r \bmod \varphi(q)$.

Used also in the Logjam attack [Adrian et al., CCS15]

Discrete log over elliptic curve groups:

("bad" means subexponential time algorithms are known)
MOV 93: supersingular is bad.
ADH 94: hyperelliptic is bad.
GHS 02: Composite order extension over F_2 is bad.
Semeav 04: Summation polynomial is useful but I don't know how.
Gaudry 09 and Diem 11: Yes it is useful to attack some fields.
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Still no algorithm achieves non-trivial running time or success probability for $E(F_q)$ where q is a prime, $\#E(F_q)$ has a large prime factor.

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For Bitcoin, T time ~ T^2 success probability doesn't suffice for "decentralize"

Explain the assumption. ✓

Explain the proof idea.

Possible relaxations & generalizations.

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Again, [Kalai, Rothblum, Rothblum ‘17] is right!

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KRR17: some property = finding the input in the Input-hiding obfuscation. Transition is done by iO + puncturing (computational).

This work: some property = key recovery in a KDM ciphertext. Transition is done via statistical properties.

Keygen: $CT \leftarrow_r CTspace$. $H: \{0,1\}^n \rightarrow \{0,1\}^l$, $H_{CT}(k) = Dec(k, CT)$.

From any R, pad it to R' so that
(1) R' is still sparse,
(2) every x has almost equal number of y.

Keygen: $CT \leftarrow_r CTspace$. $H: \{0,1\}^n \to \{0,1\}^l$, $H_{CT}(k) = Dec(k, CT)$.

Fix a sparse relation R with density d

0. Suppose by contradiction:
$Pr_{CT}[\ Adv(CT) \to k \text{ and } (k, Dec(k, CT)) \in R\ ] > v.$

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1. Averaging over one of the 2^n possible inputs k*:
$Pr_{CT, k^*}[\ Adv(CT) \to k \text{ and } k=k^* \text{ and } (k^*, Dec(k^*, CT)) \in R\ ] > v \cdot 2^{-n}.$

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2. k*, m* randomly, CT' s.t. Dec(k*, CT')=m*

-- For all k*, a random CT decrypts to a random msg.

$Pr_{k^*, m^*, CT'}[\ Adv(CT') \to k \text{ and } k=k^* \text{ and } (k^*, m^*) \in R\ ] > v \cdot 2^{-n}.$

Keygen: $CT <-_r$ CTspace. $H: \{0,1\}^n \to \{0,1\}^l$, $H_{CT}(k) = Dec(k, CT)$.

Fix a sparse relation R with density d

0. Suppose by contradiction:
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-- Exponential KDM

-- For all k*,m*, CT'=Enc(k*,m*) is stat. close to CT' s.t. Dec(k*, CT')=m*

Possible relaxations & generalizations.

Bigger success probability?
Weaker KDM assumptions?

What if someone achieves success prob. $2^{-n/10}$ for DLOG over $E(F_q)$?

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Impact on DLOG over elliptic curve groups:

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Impact on our result.

Fun fact: $2^{-n/10} >> \text{poly}(n) \cdot 2^{-n}$, ☹

Condition to form a contradiction:

$2^{-n/10} < \text{non-negl}/(d \cdot 2^n) \Rightarrow d > 2^{-9n/10}$

=> the density of a non-trivial relation has to be larger than $2^{-9n/10}$

=> |output| > 9n/10, where n = |input|

=> Implies e.g. Fiat-Shamir for protocols where

|first msg| = n, |second msg|=9n/10, soundness = $2^{-9n/10}$

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Assuming KDM security for
Enc(k, m) = Ext(owf(k)) + m?

Summary:

Correlation intractability for all sparse relations (implies Fiat-Shamir) from symmetric encryption schemes with
(1) Natural statistical properties;
(2) Exponentially hard KDM security for all key-dependency function f.

Multiple-input-output relations

Quantitatively correlation intractable.

Future directions

Correlation intractability for multiple-input-output relations.

It will be more useful.

For example, lots of Fiat-Shamir application in practices starts from an argument.

As another example, Gennaro-Halevi-Rabin signature.

Of course, need to bypass the impossibility results by CGH and Goldwasser-Kalai.

Maybe start from “simple” relations.

Quantitative correlation intractability.

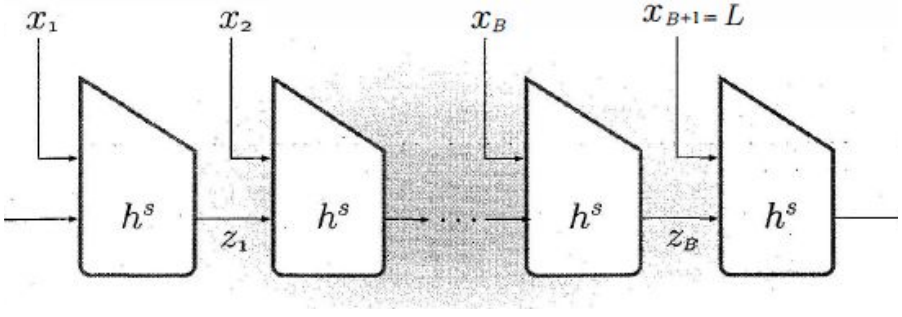
[Ball, Rosen, Sabin, Vasudevan 17] “proof of useful work”, can we do it for hash functions?

The CGH impossibility results for f: $\{0,1\}^* \rightarrow \{0,1\}^m$ also holds for relatively sparse CI.

Corollary: Domain extension techniques (like Merkle-Damgard) might preserve collision resistance, but do not preserve correlation intractability.

In fact, AsicBoost takes advantage of Merkle-Damgard to speed up bitcoin mining.
*(but that’s not because of CGH impossibility)

Achieve f: $\{0,1\}^* \rightarrow \{0,1\}^m$ that is quantitatively CI for “bitcoin relation”?




Thanks for
your time!