Lecture outline

- Decision-tree classification

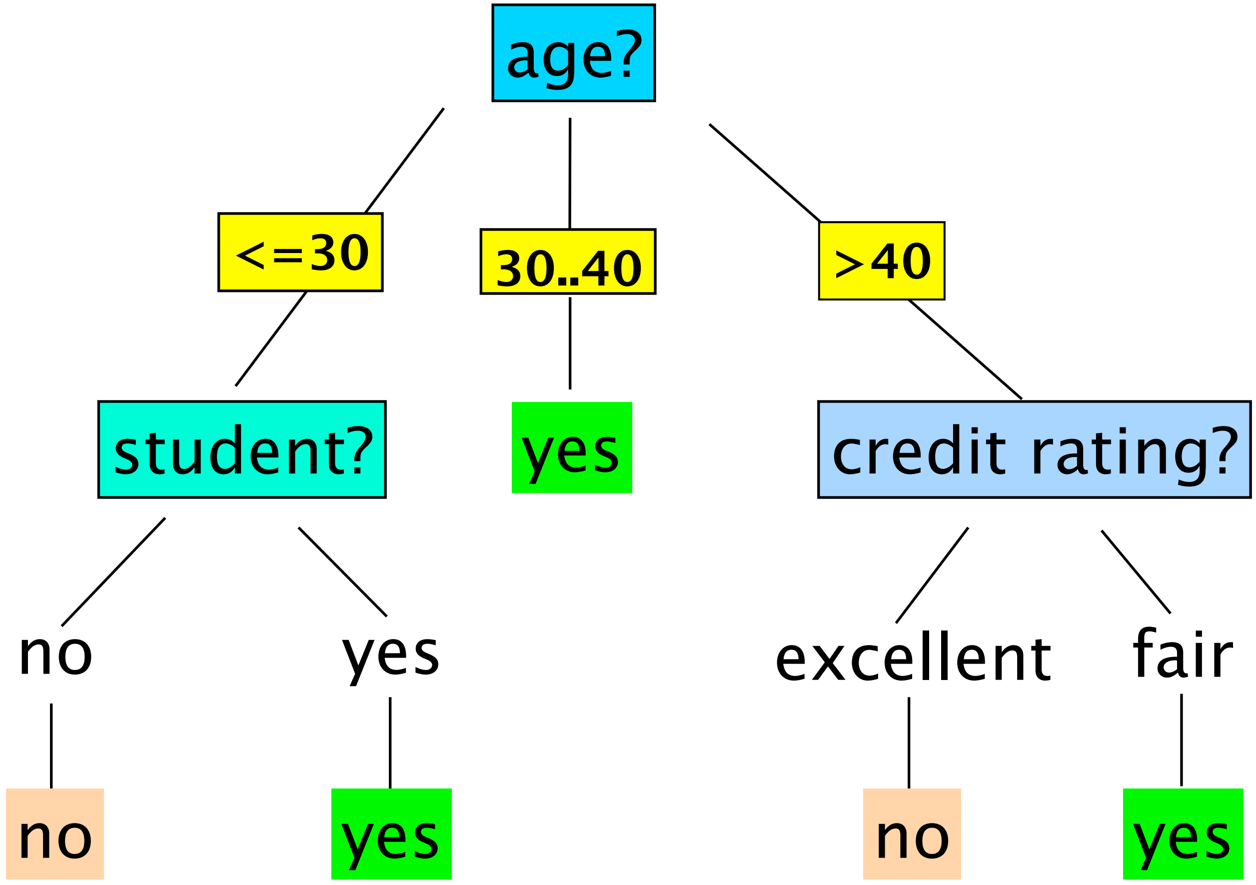
Decision Trees

- Decision tree
  - A flow-chart-like tree structure
  - Internal node denotes a test on an attribute
  - Branch represents an outcome of the test
  - Leaf nodes represent class labels or class distribution
- Decision tree generation consists of two phases
  - **Tree construction**
    - At start, all the training examples are at the root
    - Partition examples recursively based on selected attributes
  - **Tree pruning**
    - Identify and remove branches that reflect noise or outliers
- Use of decision tree: Classifying an unknown sample
  - Test the attribute values of the sample against the decision tree

Training

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

Output: A Decision Tree for



Constructing decision trees

- Exponentially many decision trees can be constructed from a given set of attributes

- Finding the most accurate tree is NP-hard

- **In practice: greedy algorithms**
  - Grow a decision tree by making a series of **locally optimum decisions on which attributes to use** for partitioning the data

Constructing decision trees: the Hunt's algorithm

- X_t: the set of training records for node **t**
- $y=\{y_1,\ldots,y_c\}$: class labels
- **Step 1:** If all records in X_t belong to the same class y_t, then t is a leaf node labeled as y_t
- **Step 2:** If X_t contains records that belong to more than one class,
  - select **attribute test condition** to partition the records into smaller subsets
  - Create a **child node** for each outcome of test condition
  - Apply algorithm **recursively** for each **child**

Decision-tree construction (Example)

Tid	Home Owner (binary)	Marital Status (categorical)	Annual Income (continuous)	Defaulted Borrower (class)
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Figure 4.6. Training set for predicting borrowers who will default on loan payments.

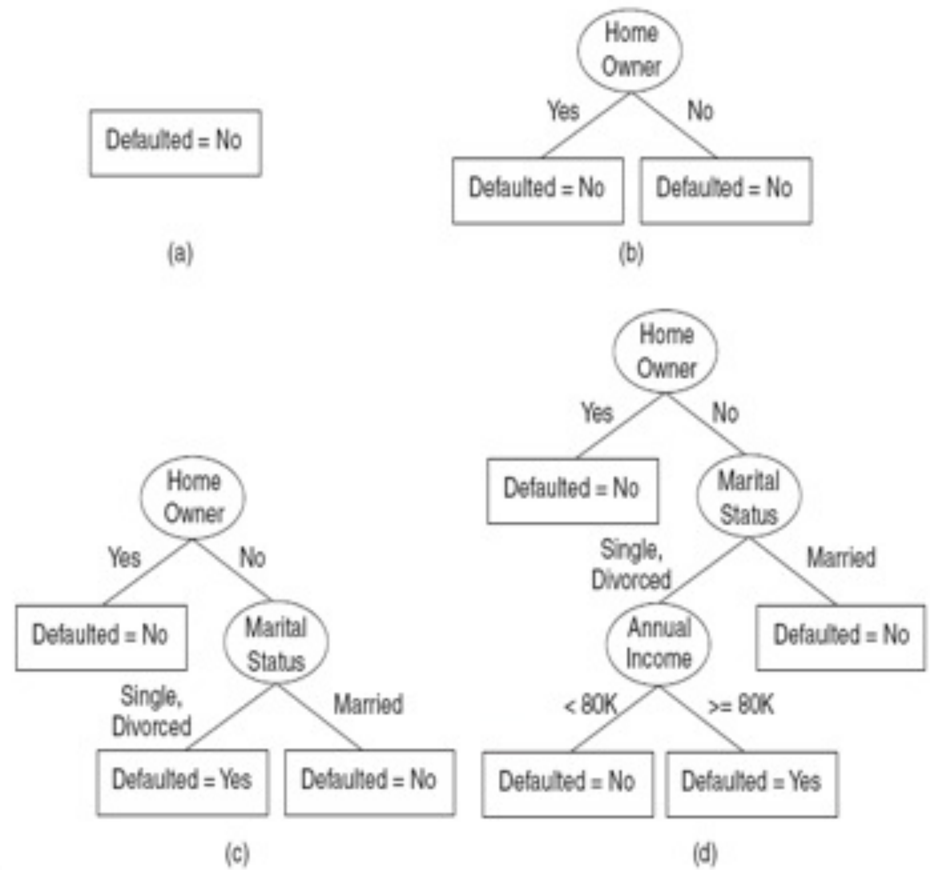


Figure 4.7. Hunt's algorithm for inducing decision trees.

Design issues

- How should the training records be split?

- How should the splitting procedure stop?

Splitting methods

- Binary attributes

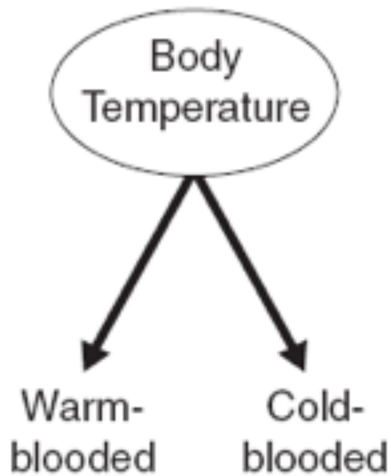


Figure 4.8. Test condition for binary attributes.

Splitting methods

- Nominal attributes

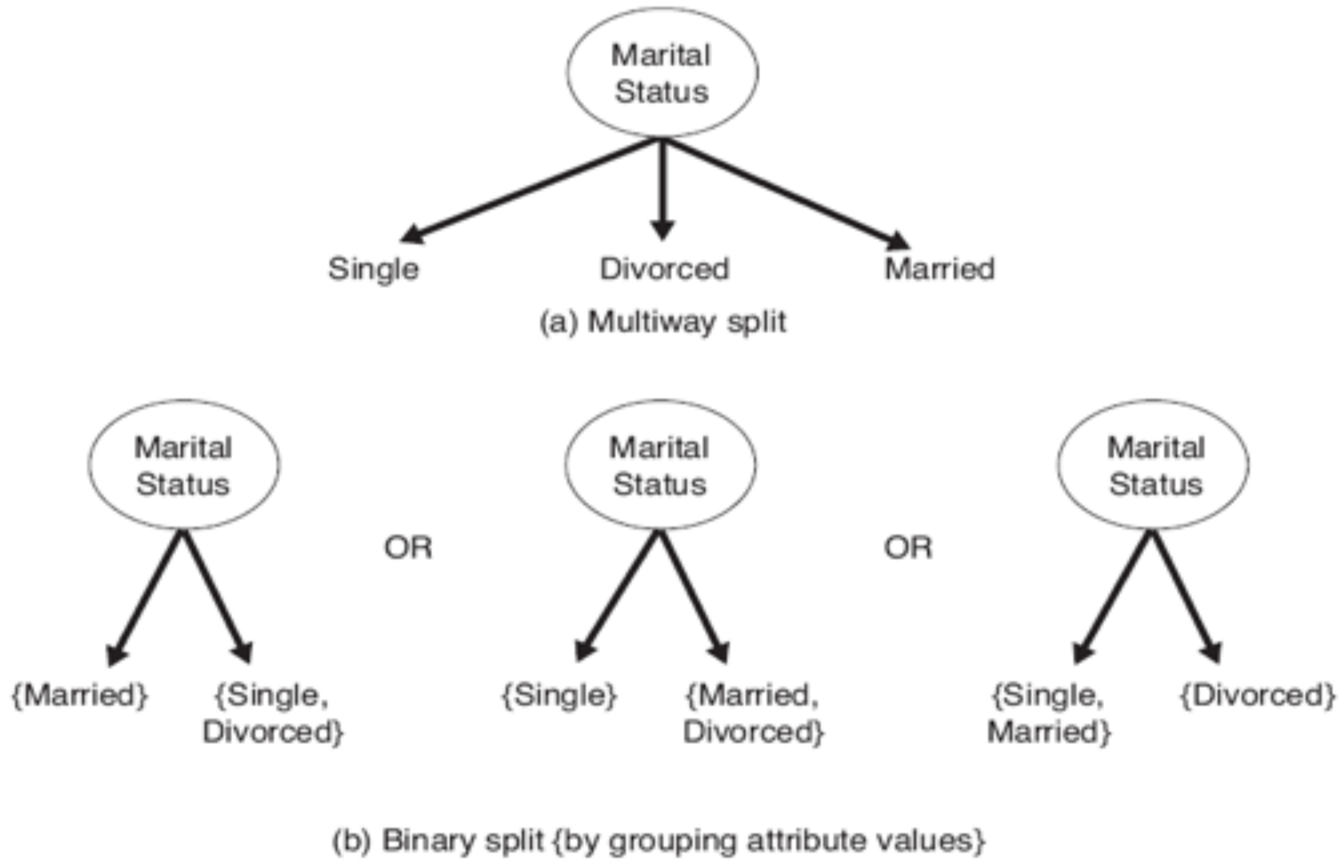


Figure 4.9. Test conditions for nominal attributes.

Splitting methods

- Ordinal attributes

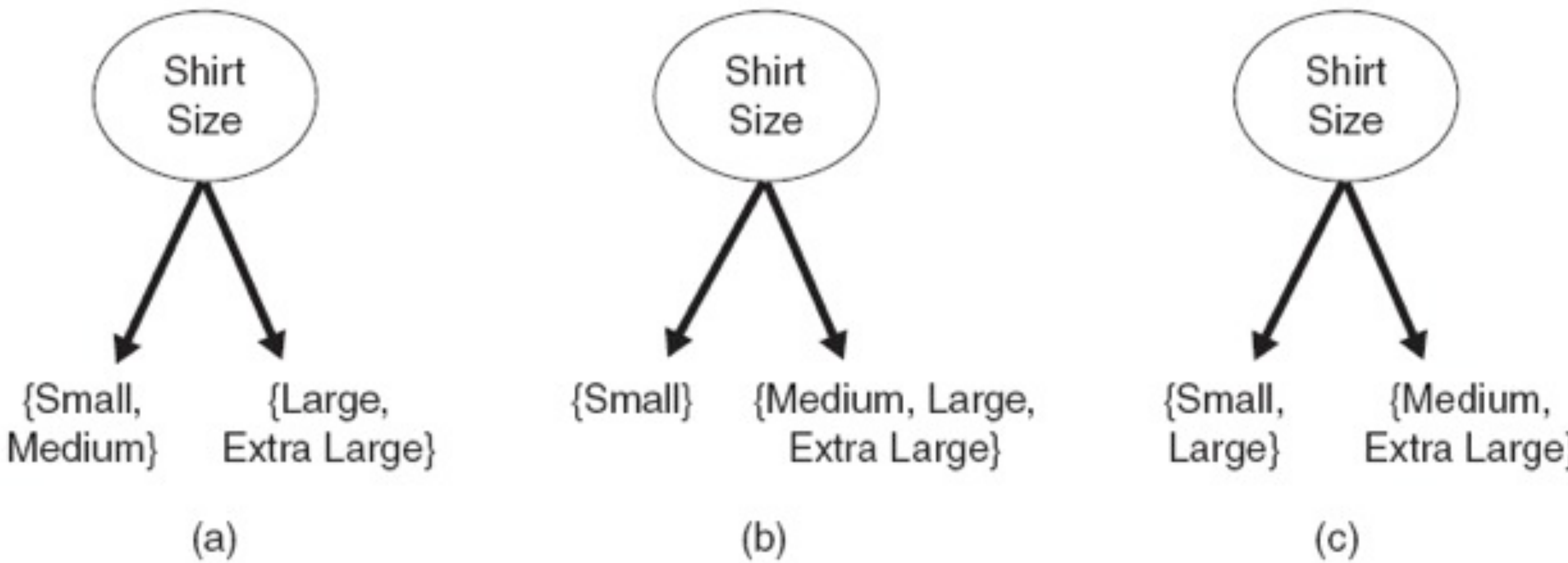


Figure 4.10. Different ways of grouping ordinal attribute values.

Splitting methods

- Continuous attributes

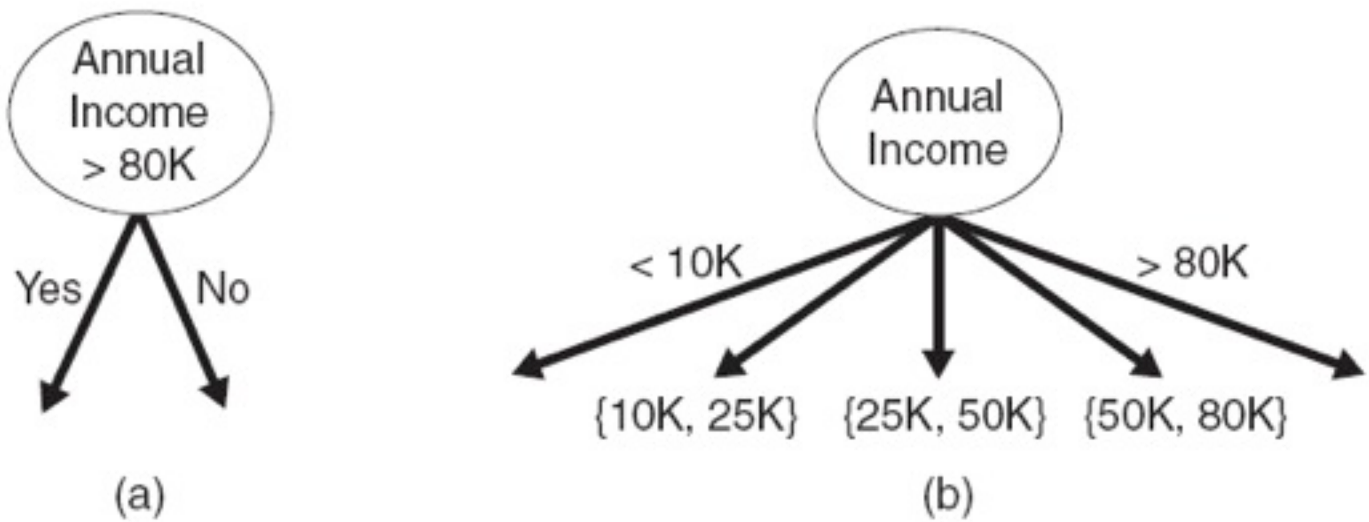


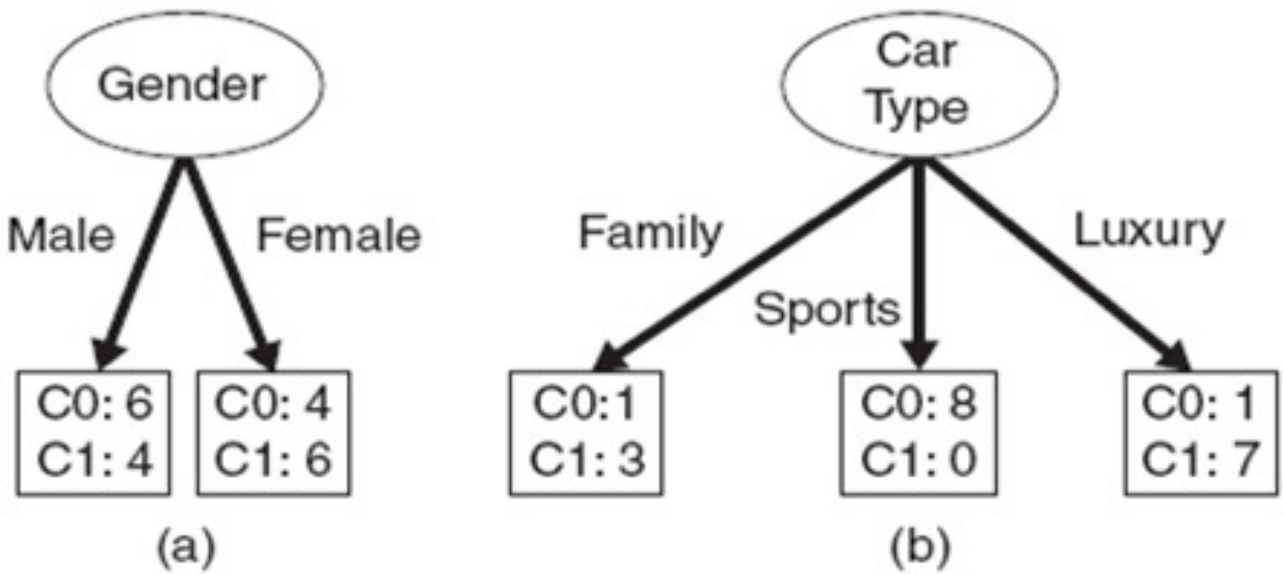
Figure 4.11. Test condition for continuous attributes.

Selecting the best split

- **p(i|t)**: fraction of records belonging to class **i**
- **Best split** is selected based on the degree of **impurity** of the child nodes
  - Class distribution **(0,1)** has **high purity**
  - Class distribution **(0.5,0.5)** has the **smallest purity (highest impurity)**

- **Intuition:** high purity → small value of impurity measures → better split

Selecting the best split



Selecting the best split: Impurity measures

- **p(i|t)**: fraction of records associated with node **t** belonging to class **i**

$$\text{Entropy}(t) = -\sum_{i=1}^{c} p(i|t) \log p(i|t)$$

$$\text{Gini}(t) = 1 - \sum_{i=1}^{c} [p(i|t)]^2$$

$$\text{Classification-Error}(t) = 1 - \max_i [p(i|t)]$$

Range of impurity measures

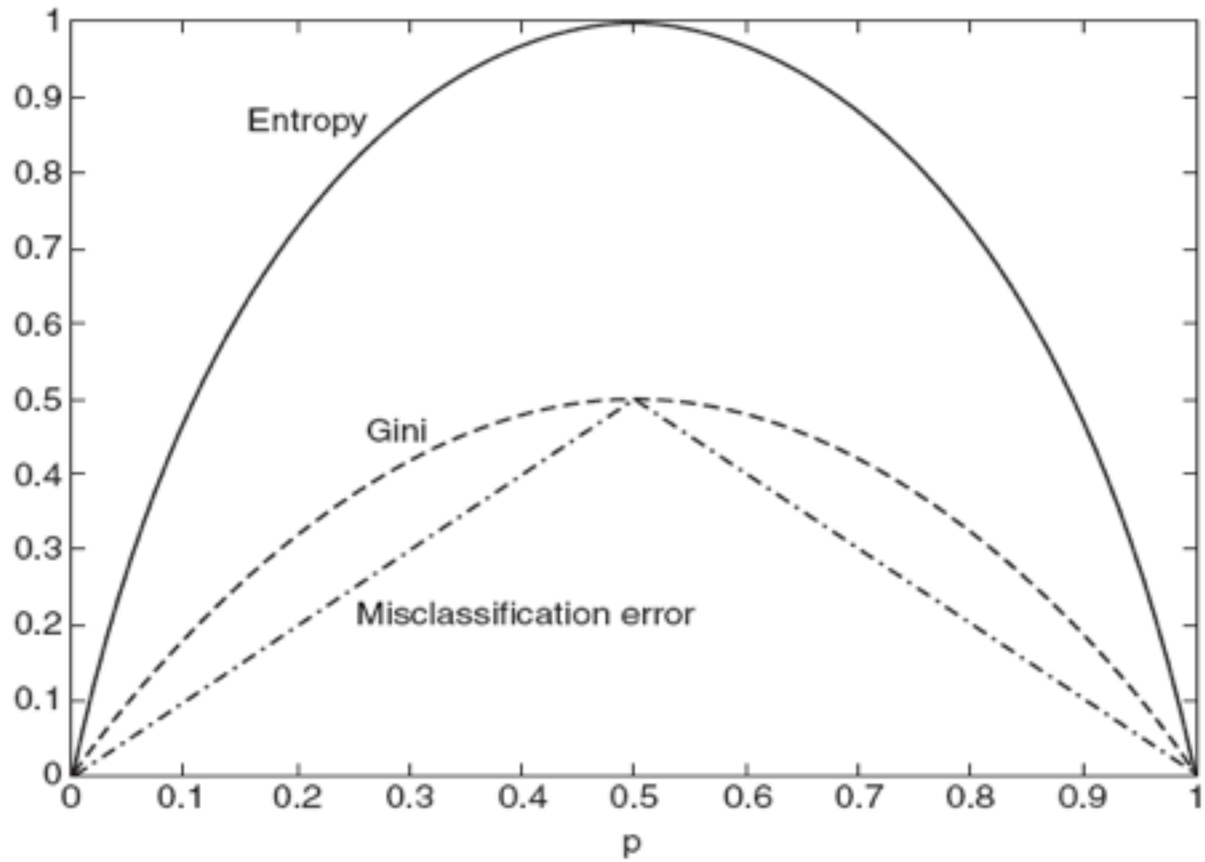


Figure 4.13. Comparison among the impurity measures for binary classification problems.

Impurity measures

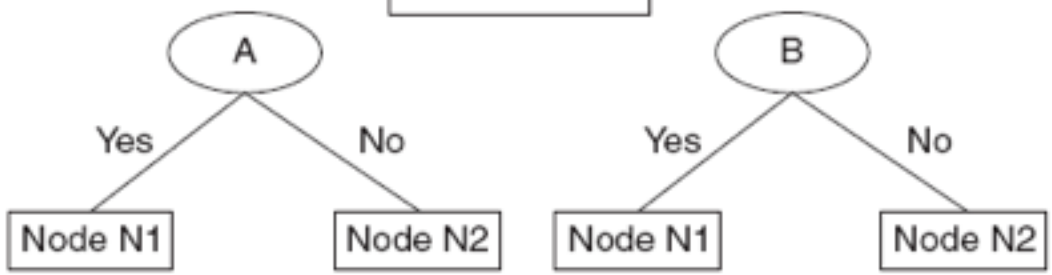
- In general the different impurity measures are **consistent**
- **Gain of a test condition:** compare the impurity of the parent node with the impurity of the child nodes

$$\Delta = I(\text{parent}) - \sum_{j=1}^{k} \frac{N(v_j)}{N} I(v_j)$$

- Maximizing the gain == minimizing the weighted average impurity measure of children nodes
- If **I() = Entropy(),** then Δ_{info} is called **information gain**

Computing gain: example

	Parent
C0	6
C1	6
Gini = 0.500	



	N1	N2
C0	4	2
C1	3	3
Gini = 0.486		

	N1	N2
C0	1	5
C1	4	2
Gini = 0.375		

Figure 4.14. Splitting binary attributes.

Is minimizing impurity/ maximizing Δ enough?

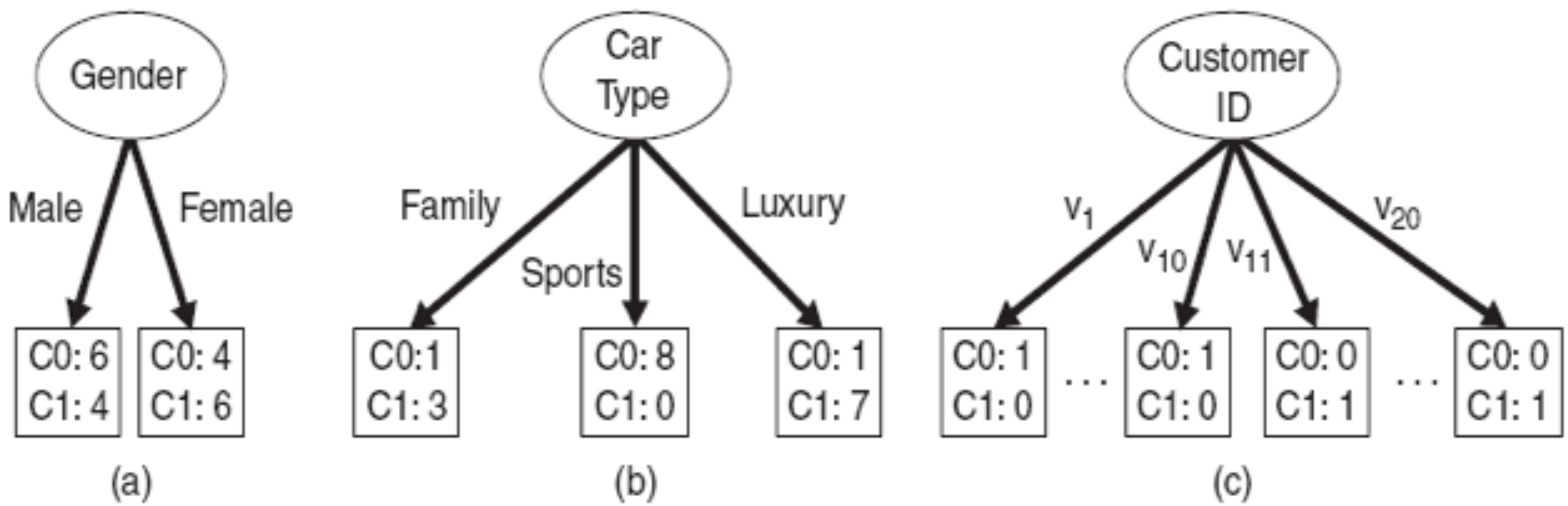


Figure 4.12. Multiway versus binary splits.

Is minimizing impurity/ maximizing Δ enough?

- Impurity measures favor attributes with large number of values

- A test condition with large number of outcomes may not be desirable
  - # of records in each partition is too small to make predictions

Gain ratio

- **Gain ratio = Δ_{info}/Splitinfo**

- **SplitInfo = $-\Sigma_{i=1\ldots k} p(v_i)\log(p(v_i))$**
- **k:** total number of splits
- If each attribute has the same number of records, **SplitInfo = logk**
- Large number of splits → large **SplitInfo** → small gain ratio

Constructing decision-trees (pseudocode)

GenDecTree(Sample **S**, Features **F**)

1. **If stopping_condition(S,F)** = true **then**
   a. **leaf = createNode()**
   b. **leaf.label= Classify(S)**
   c. **return leaf**
2. **root = createNode()**
3. **root.test_condition = findBestSplit(S,F)**
4. **V = {v| v a** possible outcome of **root.test_condition}**
5. **for each** value **v∈V**:
   a. S_v: = {s | **root.test_condition**(s) = v and s ∈ S};
   b. **child = TreeGrowth**(S_v ,F) ;
   c. Add **child** as a descent of **root** and label the edge **(root→child) as** v

Stopping criteria for tree induction

- Stop expanding a node when all the records belong to the same class
- Stop expanding a node when all the records have similar attribute values
- Early termination

Advantages of decision trees

- Inexpensive to construct
- Extremely fast at classifying unknown records
- Easy to interpret for small-sized trees
- Accuracy is comparable to other classification techniques for many simple data sets

Example: C4.5 algorithm

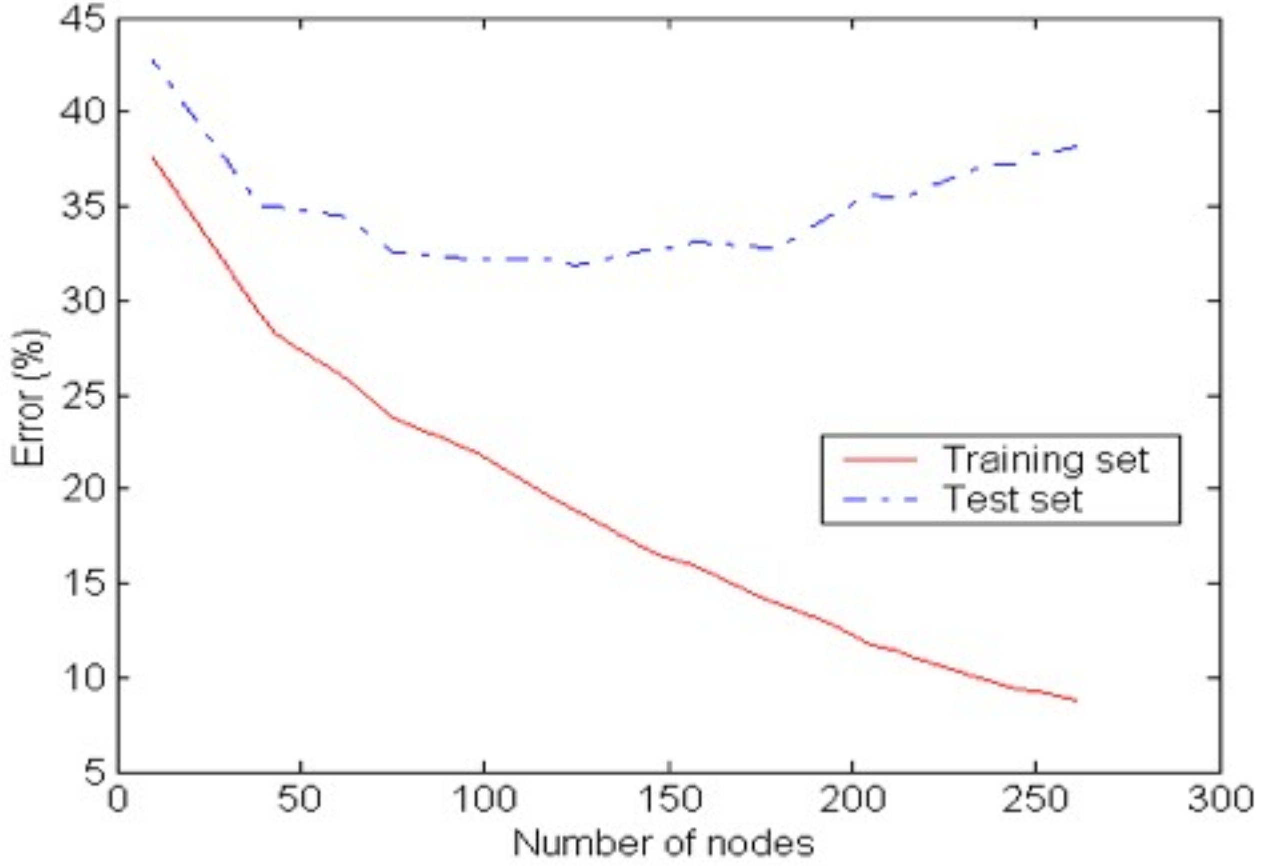
- Simple depth-first construction.
- Uses Information Gain
- Sorts Continuous Attributes at each node.
- Needs entire data to fit in memory.
- Unsuitable for Large Datasets.

- You can download the software from: http://www.cse.unsw.edu.au/~quinlan/c4.5r8.tar.gz

Practical problems with classification

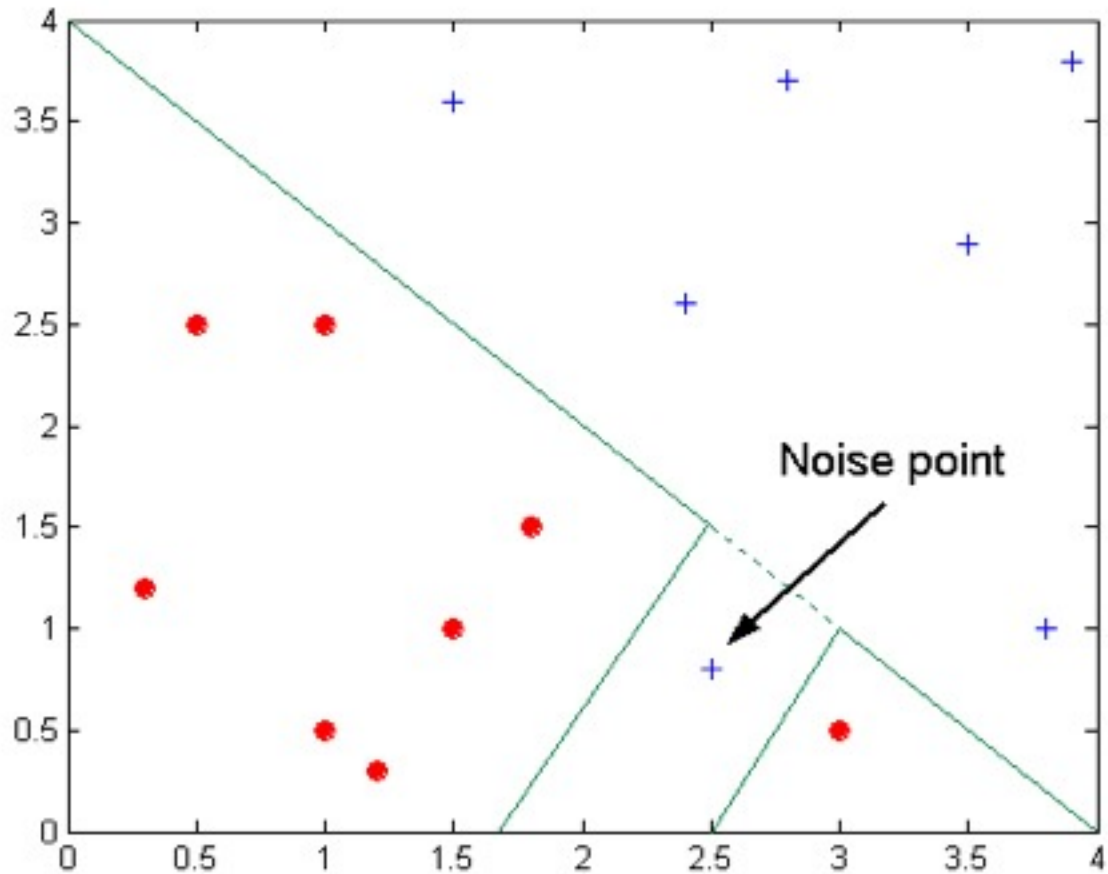
- Underfitting and overfitting
- Missing values
- Cost of classification

Overfitting and underfitting



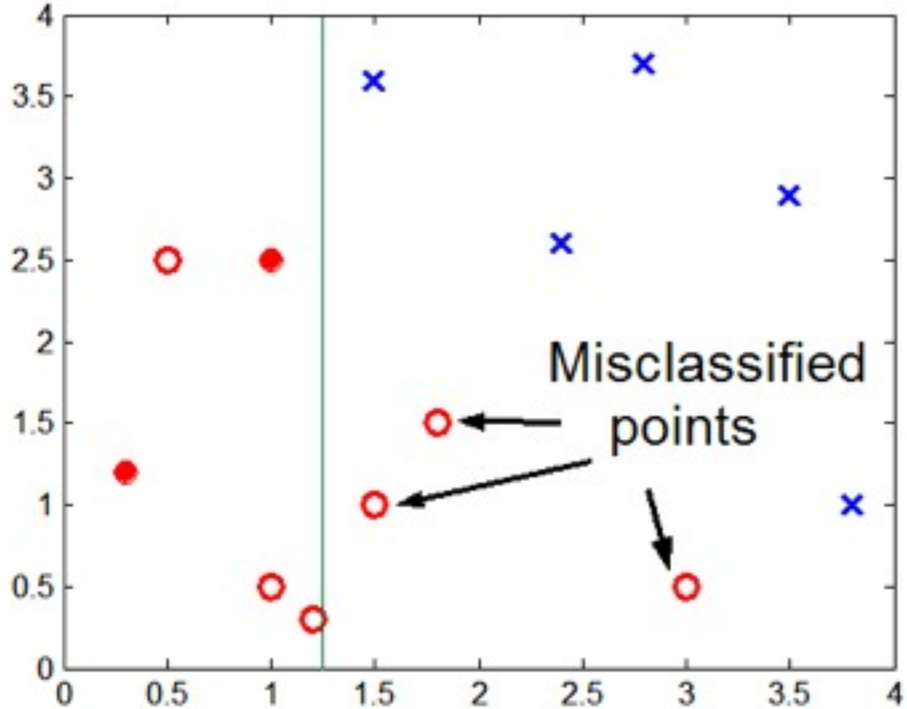
Underfitting: when model is too simple, both training and test errors are large

Overfitting due to noise



Decision boundary is distorted by noise point

Underfitting due to insufficient samples



Lack of data points in the lower half of the diagram makes it difficult to predict correctly the class labels of that region

– Insufficient number of training records in the region causes the decision tree to predict the test examples using other training records that are irrelevant to the classification task

Overfitting: course of action

- Overfitting results lead to decision trees that are more complex than necessary

- Training error no longer provides a good estimate of how well the tree will perform on previously unseen records

- Need new ways for estimating errors

Methods for estimating the error

- Re-substitution errors: error on training (Σ **e(t)**)
- Generalization errors: error on testing (Σ **e'(t)**)

- Methods for estimating generalization errors:
  - Optimistic approach: **e'(t) = e(t)**
  - Pessimistic approach:
    - For each leaf node: **e'(t) = (e(t)+0.5)**
    - Total errors: **e'(T) = e(T) + N × 0.5** (N: number of leaf nodes)
    - For a tree with 30 leaf nodes and 10 errors on training (out of 1000 instances):

      Training error = 10/1000 = 1%

      Generalization error = (10 + 30×0.5)/1000 = 2.5%
  - Reduced error pruning (REP):
    - uses **validation data set** to estimate generalization error

Addressing overfitting: Occam's razor

- Given two models of similar generalization errors, one should prefer the simpler model over the more complex model
- For complex models, there is a greater chance that it was fitted accidentally by errors in data
- Therefore, one should include model complexity when evaluating a model

Addressing overfitting: postprunning

- Grow decision tree to its entirety
- Trim the nodes of the decision tree in a bottom-up fashion
- If generalization error improves after trimming, replace sub-tree by a leaf node.
- Class label of leaf node is determined from majority class of instances in the sub-tree
- Can use MDL for post-pruning

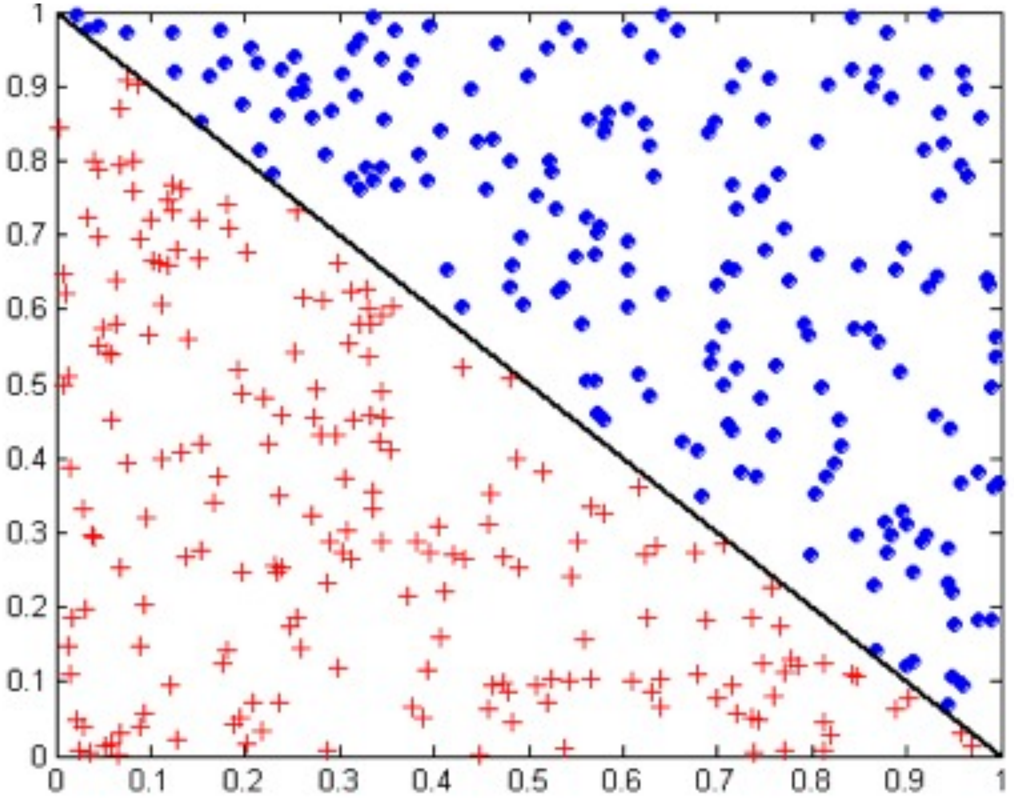
Addressing overfitting: preprunning

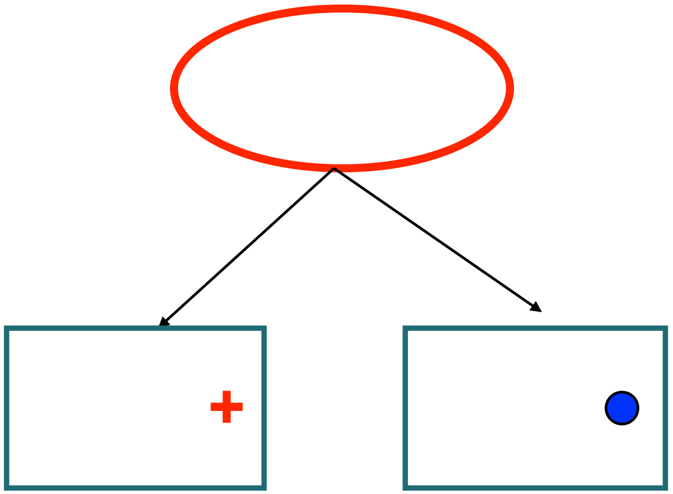
- Stop the algorithm before it becomes a fully-grown tree
- Typical stopping conditions for a node:
    - Stop if all instances belong to the same class
    - Stop if all the attribute values are the same
- More restrictive conditions:
    - Stop if number of instances is less than some user-specified threshold
    - Stop if expanding the current node does not improve impurity
      measures (e.g., Gini or information gain).

Decision boundary for decision trees

- Border line between two neighboring regions of different classes is known as **decision boundary**
- **Decision boundary in decision trees** is parallel to axes because test condition involves a single attribute at-a-time

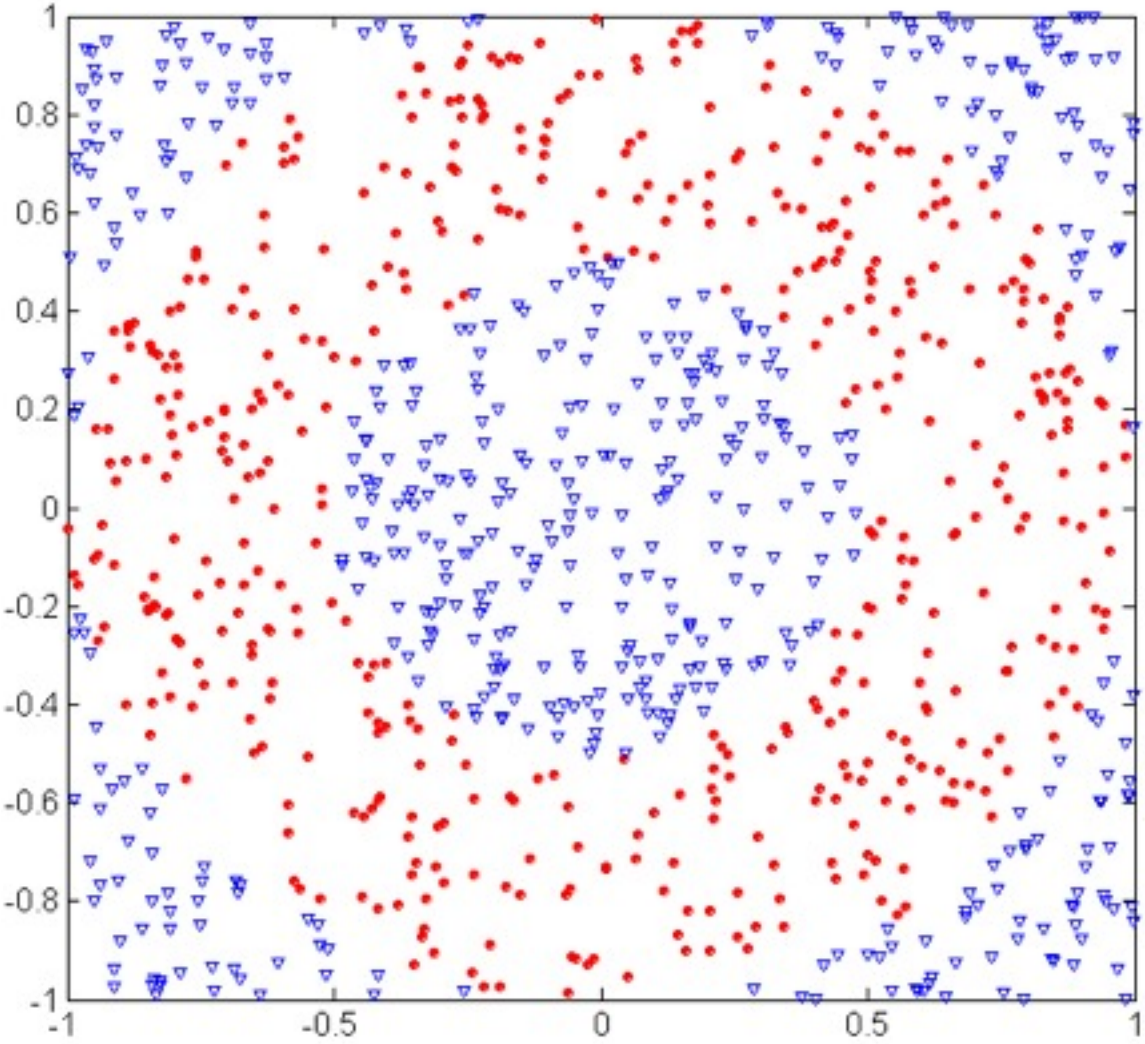
Oblique Decision Trees





Not all datasets can be partitioned optimally using test conditions involving single attributes!

Oblique Decision Trees



Circular points:

$0.5 \leq \text{sqrt}(x_1^2+x_2^2) \leq 1$

Triangular points:

$\text{sqrt}(x_1^2+x_2^2) > 1$ or

$\text{sqrt}(x_1^2+x_2^2) < 0.5$