Differential Privacy

Basic properties.

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(ε,δ)-Differential Privacy

Definition

Given $\varepsilon, \delta \geq 0$, a probabilistic query $Q: X^n \rightarrow R$ is (ε,δ)-differentially private iff for all adjacent database b_1, b_2 and for every $S \subseteq R$:

$$\Pr[Q(b_1) \in S] \leq \exp(\varepsilon)\Pr[Q(b_2) \in S] + \delta$$

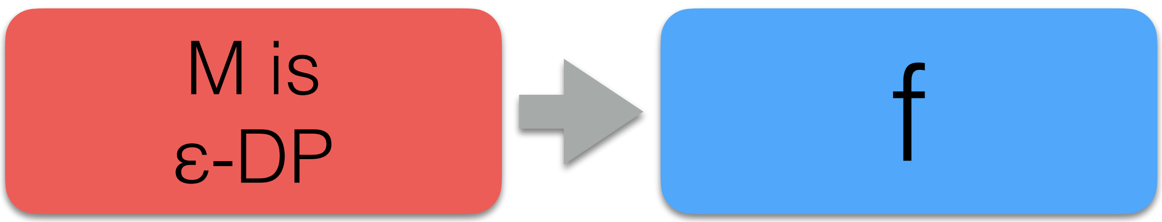
Some important properties

- Resilience to post-processing
- Group privacy
- Composition

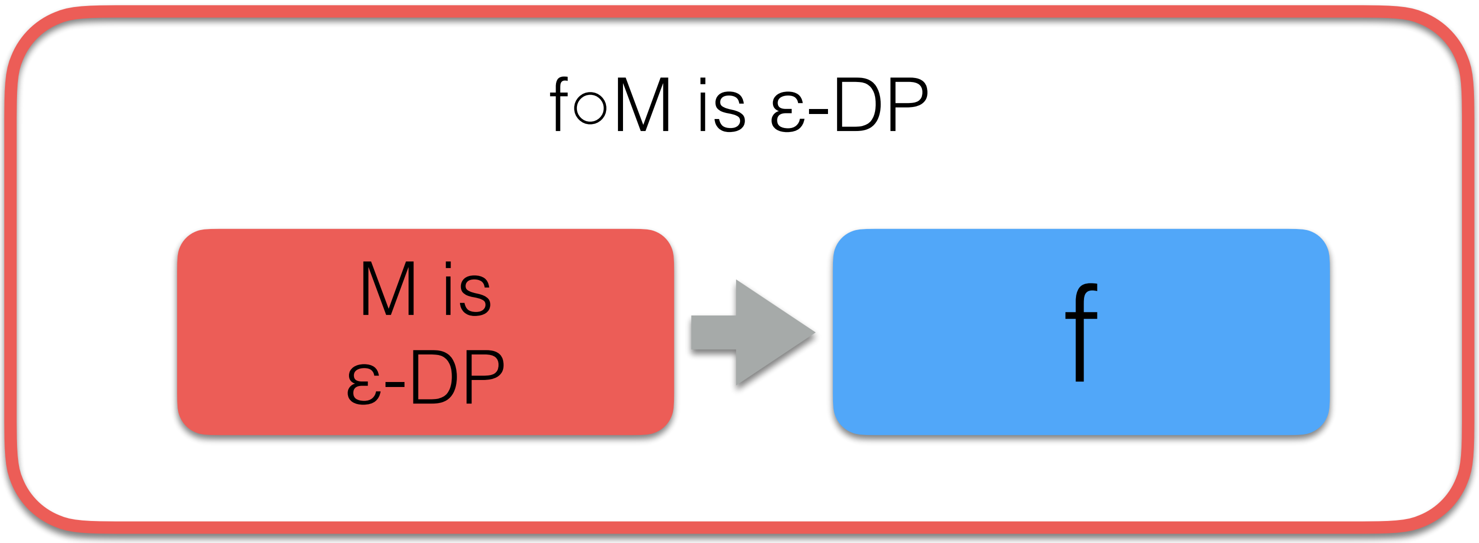
Resilience to Post-processing

M is
ε-DP

Resilience to Post-processing



Resilience to Post-processing



Resilience to Post-processing

Proposition 1.1 (Post-processing). Let $\mathcal{M} : \mathcal{X}^n \to R$ be a randomized algorithm that is ϵ-differentially private. Let $f : R \to R'$ be an arbitrary deterministic mapping. Then $f \circ \mathcal{M} : \mathcal{X}^n \to R'$ is also ϵ-differentially private.

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$$\Pr[f(\mathcal{M}(D)) \in S] \quad = \quad \Pr[\mathcal{M}(D) \in T]$$

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$$\begin{aligned} \Pr[f(\mathcal{M}(D)) \in S] &= \Pr[\mathcal{M}(D) \in T] \\ &\leq \exp(\epsilon) Pr[\mathcal{M}(D') \in T] \\ &= \exp(\epsilon) Pr[f(\mathcal{M}(D')) \in S] \end{aligned}$$

Resilience to Post-processing

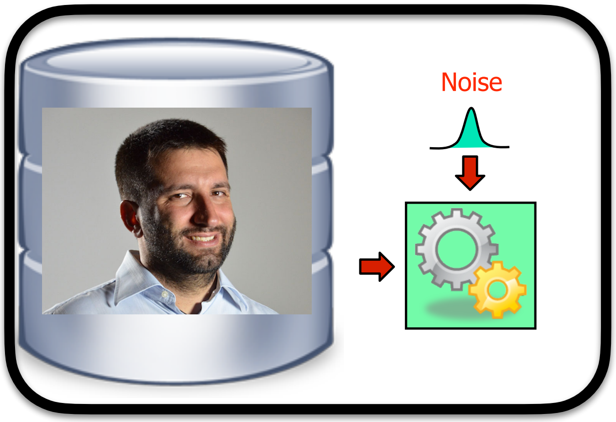
Question: Why is resilience to post-processing important?

Resilience to Post-processing

Question: Why is resilience to post-processing important?

Answer: Because it is what allows us to publicly release the result of a differentially private analysis!

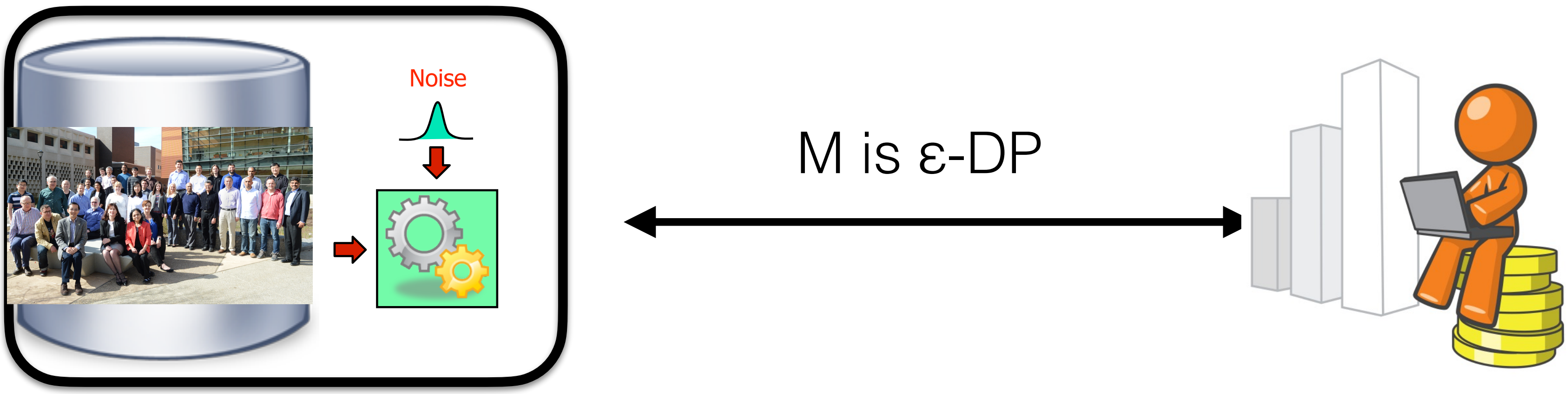
Group Privacy



M is ε-DP

$$\Pr[\mathcal{M}(D) = r] \leq e^{\epsilon} \Pr[\mathcal{M}(D') = r]$$

Group Privacy



$$\Pr[\mathcal{M}(D) \in S] \leq \exp(k\epsilon) \Pr[\mathcal{M}(D') \in S]$$

Group Privacy

Proposition 1.2 (Group Privacy). Let $\mathcal{M} : \mathcal{X}^n \to R$ be a randomized algorithm that is ϵ-differentially private. Then, $\mathcal{M}$ is $k\epsilon$-differentially private for groups of size k. That is, for datasets $D, D' \in \mathcal{X}^n$ such that $D \Delta D' \leq k$ and for all $S \subseteq R$ we have

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Proof. Fix any pair of databases D, D' with $D \Delta D' \leq k$. Then, we have databases $D_0, D_1, \ldots, D_k$ such that $D_0 = D$, $D_k = D'$ and $D_i \Delta D_{i+1} \leq 1$. Fix also any event $S \subseteq R'$. Then, we have have

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$$\begin{aligned} \Pr[\mathcal{M}(D) \in S] &= \Pr[\mathcal{M}(D_0) \in S] \\ &\leq \exp(\epsilon) \Pr[\mathcal{M}(D_1) \in S] \\ &\leq \exp(\epsilon)(\exp(\epsilon) \Pr[\mathcal{M}(D_2) \in S]) = \exp(2\epsilon) \Pr[\mathcal{M}(D_2) \in S] \end{aligned}$$

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$$\begin{aligned}
\Pr[\mathcal{M}(D) \in S] &= \Pr[\mathcal{M}(D_0) \in S] \\
&\leq \exp(\epsilon) \Pr[\mathcal{M}(D_1) \in S] \\
&\leq \exp(\epsilon)(\exp(\epsilon) \Pr[\mathcal{M}(D_2) \in S]) = \exp(2\epsilon) \Pr[\mathcal{M}(D_2) \in S] \\
&\leq \cdots \\
&\leq \exp(k\epsilon) \Pr[\mathcal{M}(D_k) \in S] = \exp(k\epsilon) \Pr[\mathcal{M}(D') \in S]
\end{aligned}$$

Group Privacy

Question: Why is group privacy important?

Group Privacy

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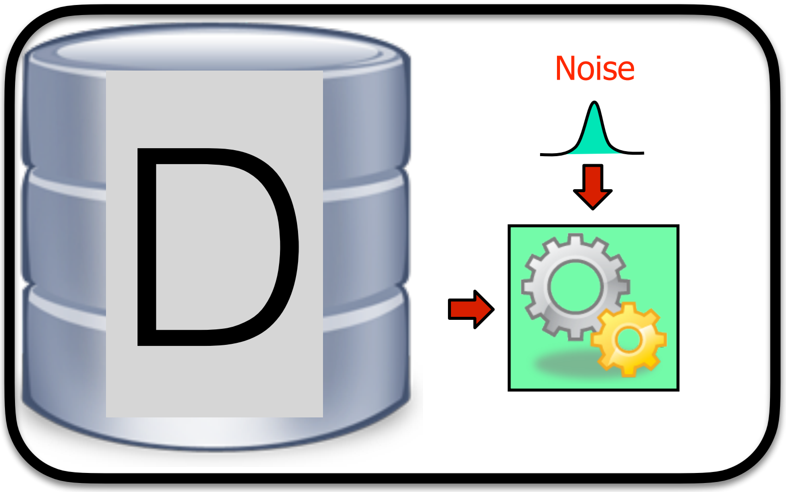
Answer: Because it allows to reason about privacy at different level of granularities!

Composition



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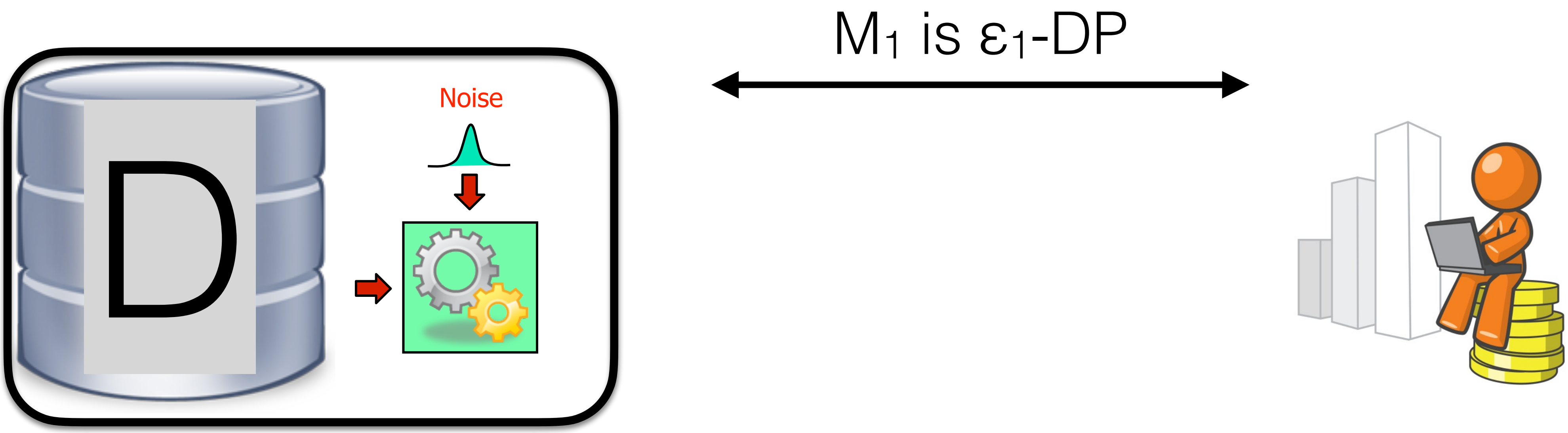

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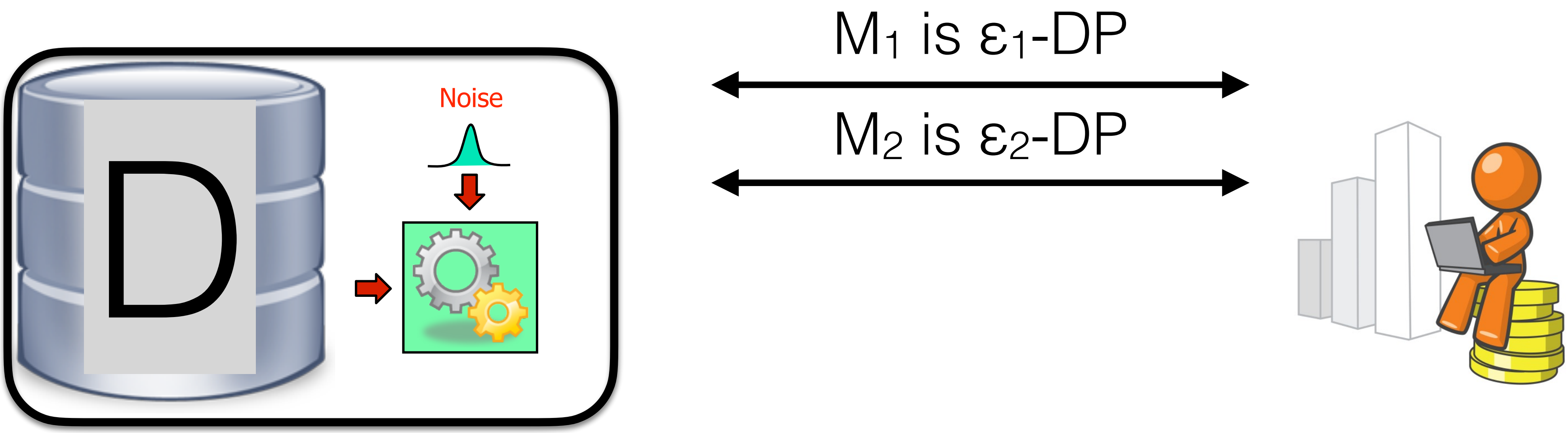

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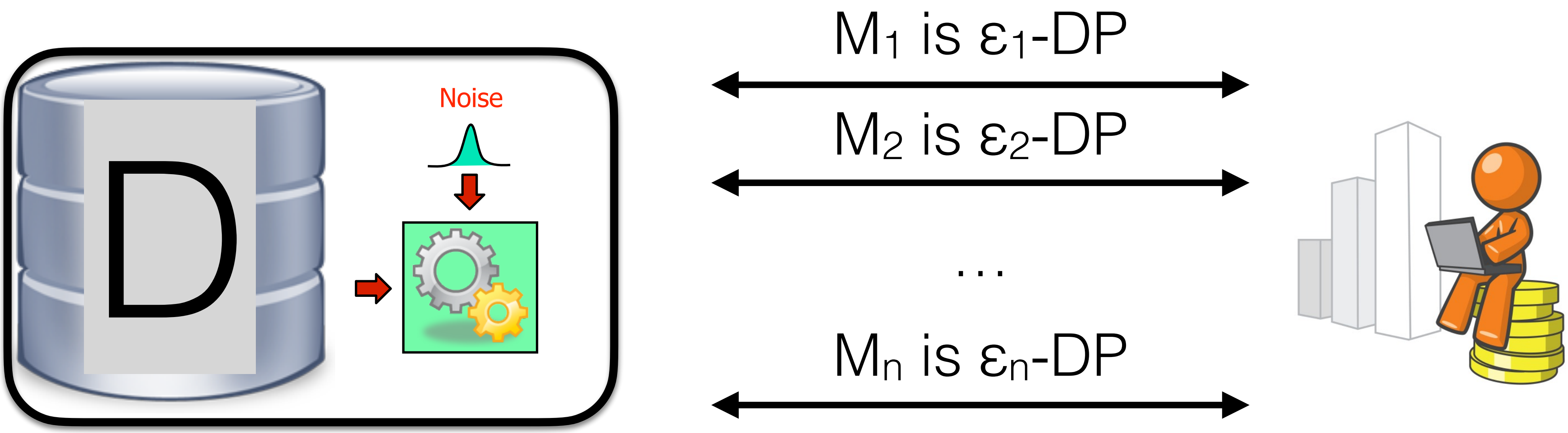

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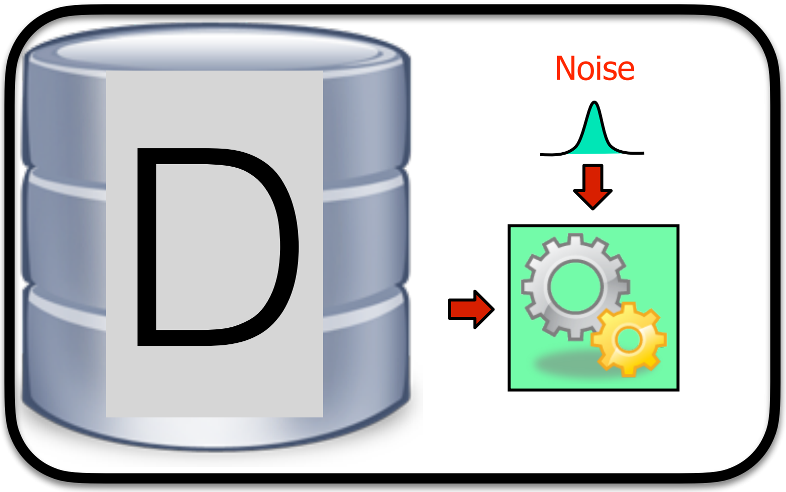

M_1 is ε_1-DP

M_2 is ε_2-DP

…

M_n is ε_n-DP

The overall process is $(\varepsilon_1+\varepsilon_2+\ldots+\varepsilon_n)$-DP

Composition

Theorem 1.7 (Standard composition for ϵ-differential privacy)**.** Let $\mathcal{M}_1 : \mathcal{X}^n \to R_1$ be an ϵ_1-differentially private algorithm and let $\mathcal{M}_2 : \mathcal{X}^n \to R_2$ be an ϵ_2-differentially private algorithm. Then their composition defined to be $\mathcal{M}_{1,2} : \mathcal{X}^n \to R_1 \times R_2$ by the mapping $\mathcal{M}_{1,2}(D) = (\mathcal{M}_1(D), \mathcal{M}_2(D))$ is $(\epsilon_1 + \epsilon_2)$-differentially private.

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Proof. Fix any pair of adjacent datasets $D \sim_1 D'$. Fix also a pair of output $(r_1, r_2) \in R_1 \times R_2$. We have:

$$\frac{\Pr[\mathcal{M}_{1,2}(D) = (r_1, r_2)]}{\Pr[\mathcal{M}_{1,2}(D') = (r_1, r_2)]} = \frac{(\Pr[\mathcal{M}_1(D), \mathcal{M}_2(D)) = (r_1, r_2)]}{(\Pr[\mathcal{M}_1(D'), \mathcal{M}_2(D')) = (r_1, r_2)]}$$

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$$\leq \exp(\epsilon_1) \exp(\epsilon_2) = \exp(\epsilon_1 + \epsilon_2).$$

Composition

Question: Why composition is important?

Composition

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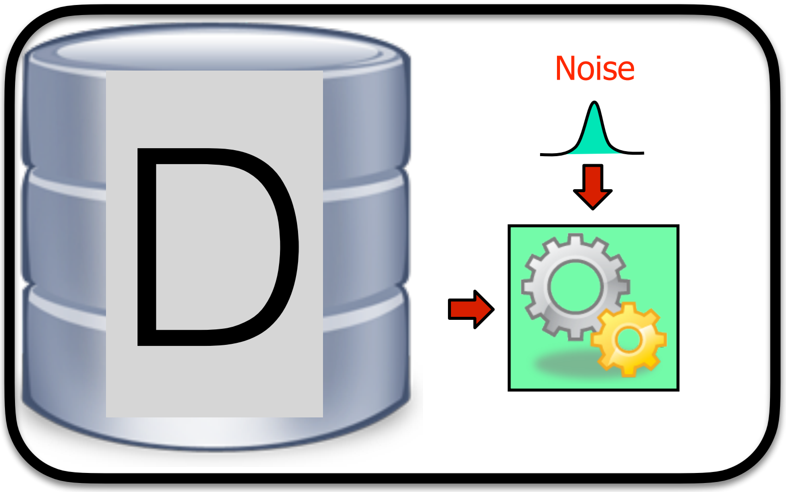
Answer: Because it allows to reason about privacy as a budget!

Composition

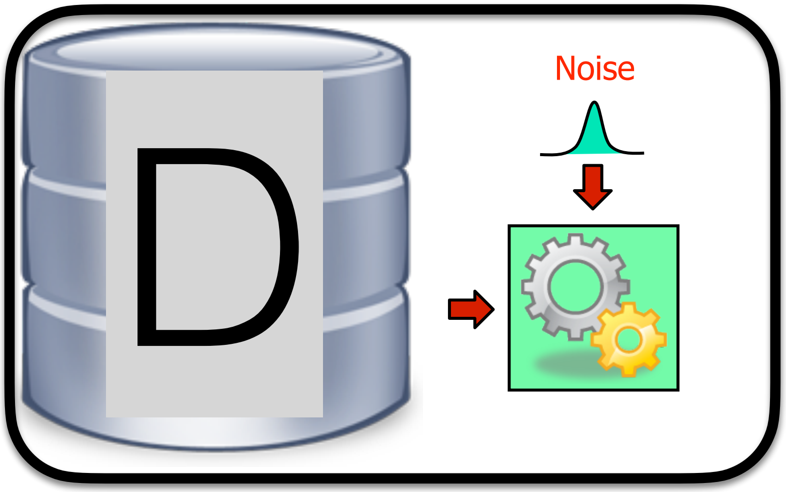
Budget=ε_{global}



Composition

Budget=ε_{global}

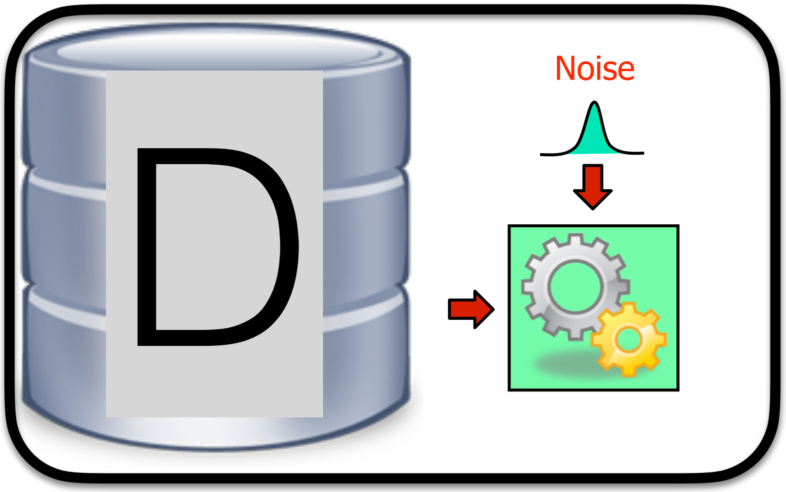
M_1 is ε_1-DP

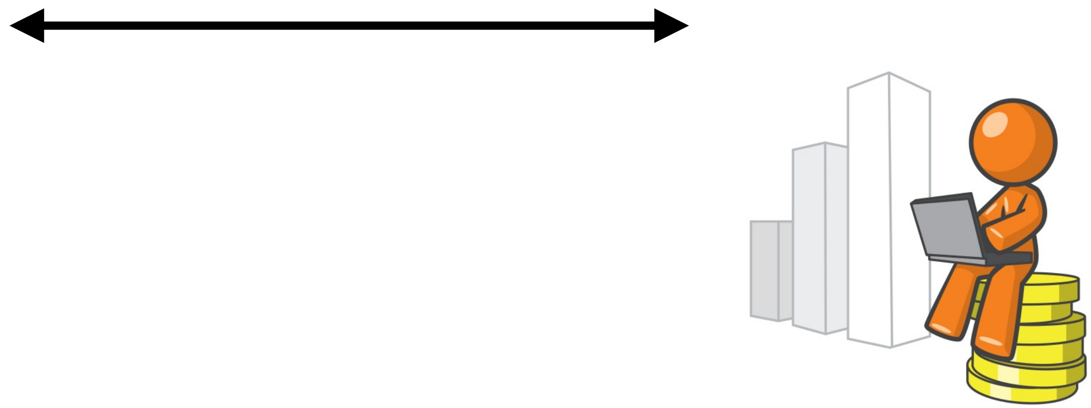


Composition

Budget=ε_{global} - ε_1

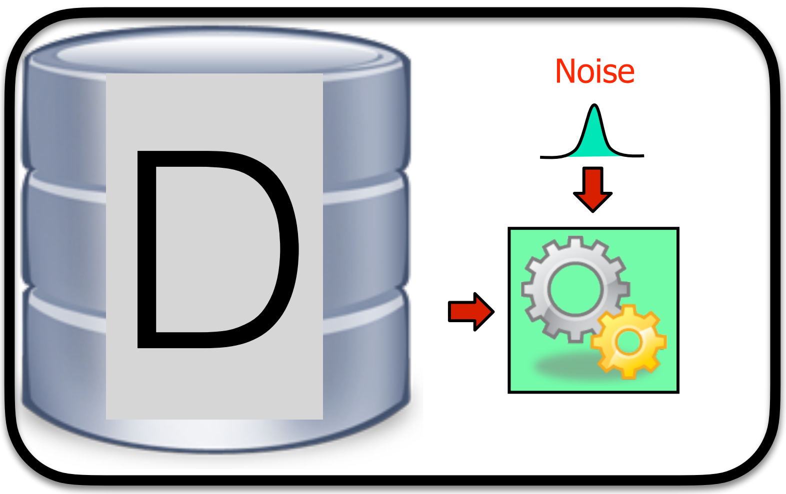
M_1 is ε_1-DP





Composition

Budget=ε_{global} - ε_1

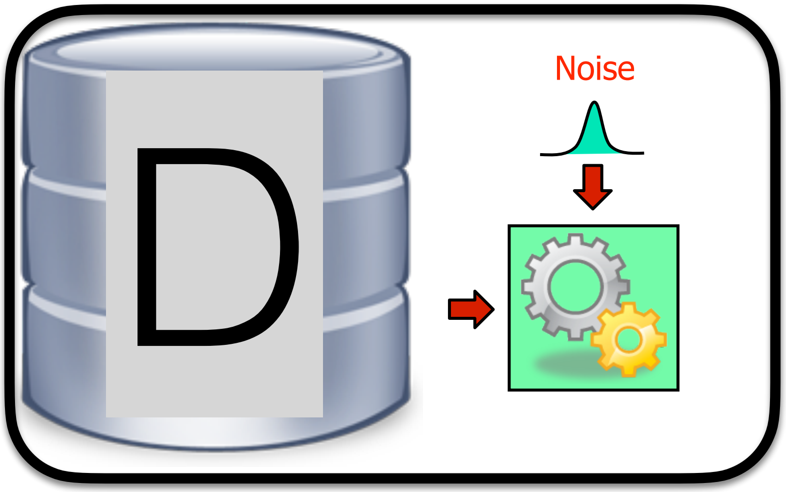


M_1 is ε_1-DP

M_2 is ε_2-DP

Composition

Budget= $\varepsilon_{global} - \varepsilon_1 - \varepsilon_2$

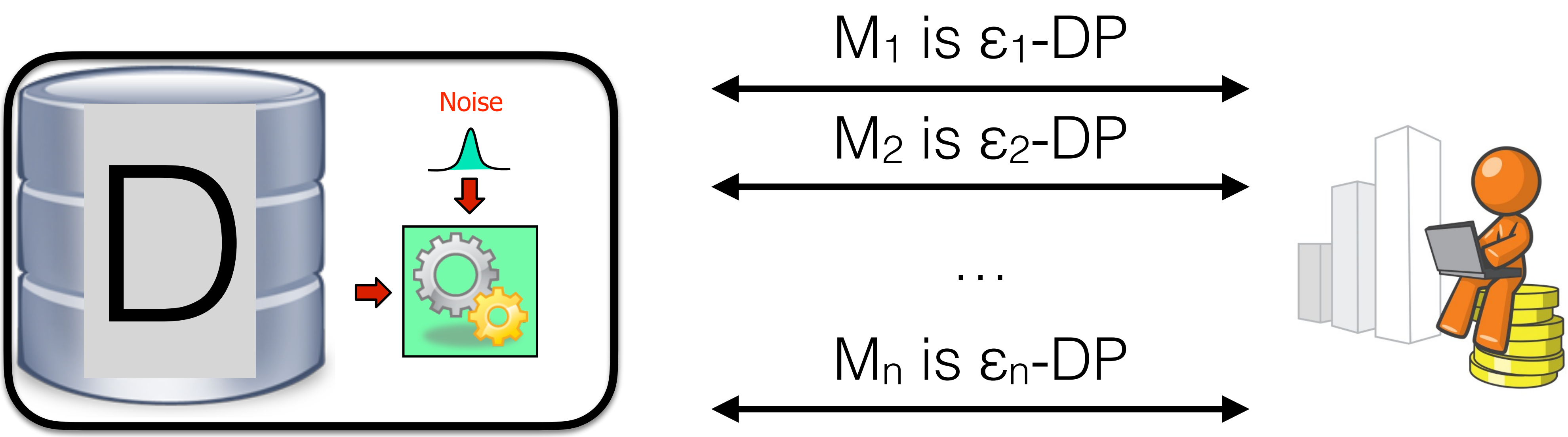


M_1 is ε_1-DP

M_2 is ε_2-DP

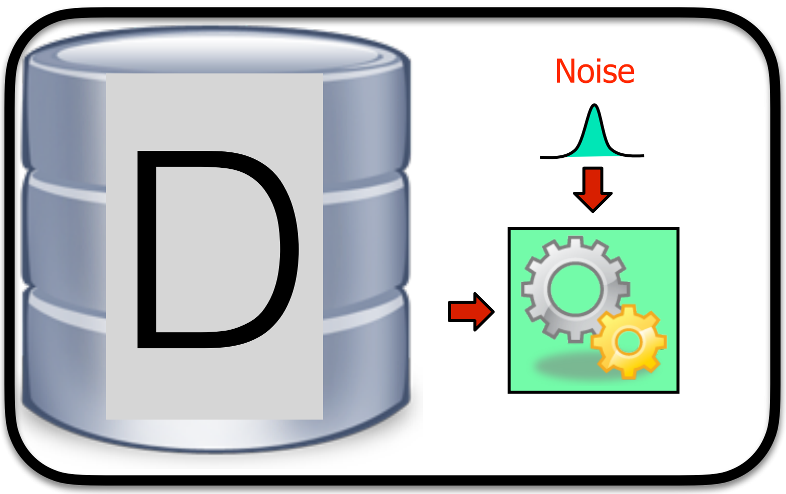
Composition

Budget=ε_{global} - ε_1 - ε_2 ...



Composition

Budget=ε_{global} - ε_1 - ε_2 … - ε_n



M_1 is ε_1-DP

M_2 is ε_2-DP

…

M_n is ε_n-DP

Example 1

Let's consider an arbitrary ordered universe domain $\mathcal{X}$ and let's consider the following predicate for $y \in \mathcal{X}$

$$q_y(x) = \begin{cases} 1 & \text{if } x \leq y \\ 0 & \text{otherwise} \end{cases}$$

we call a threshold function the associated counting query

$$q_y : \mathcal{X}^n \to [0, 1]$$

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Question: What is the sensitivity?

Example 1

Budget=ε_{global}

X={0,1}^{3} ordered wrt binary encoding.

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

Example I

$\text{Budget} = \varepsilon_{global} - \varepsilon_1$

$X = \{0,1\}^3$ ordered
wrt binary encoding.

$q^*_{000}(D) = .3 + L(1/n\varepsilon_1)$

$D \in X^{10} =$

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Example 1

Budget=ε_{global} - ε_1 - ε_2

X={0,1}^{3} ordered wrt binary encoding.

$q^*_{000}(D) = .3 + L(1/n\varepsilon_1)$

$q^*_{001}(D) = .4 + L(1/n\varepsilon_2)$

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

Example I

Budget=$\varepsilon_{global} - \varepsilon_1 - \varepsilon_2 - \varepsilon_3$

$X=\{0,1\}^3$ ordered wrt binary encoding.

$q^*_{000}(D) = .3+L(1/n\varepsilon_1)$

$q^*_{001}(D) = .4+L(1/n\varepsilon_2)$

$q^*_{010}(D) = .6+L(1/n\varepsilon_3)$

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Example I

Budget=$\varepsilon_{global} - \varepsilon_1 - \varepsilon_2 - \varepsilon_3 - \varepsilon_4$

X={0,1}^{3} ordered
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$q^*_{000}(D) = .3+L(1/n\varepsilon_1)$

$q^*_{001}(D) = .4+L(1/n\varepsilon_2)$

$q^*_{010}(D) = .6+L(1/n\varepsilon_3)$

$q^*_{011}(D) = .6+L(1/n\varepsilon_4)$

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I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

Example 1

Budget=$\varepsilon_{global} - \varepsilon_1 - \varepsilon_2 - \varepsilon_3 - \varepsilon_4 - \varepsilon_5$

$X=\{0,1\}^3$ ordered wrt binary encoding.

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

$q^*_{000}(D) = .3+L(1/n\varepsilon_1)$

$q^*_{001}(D) = .4+L(1/n\varepsilon_2)$

$q^*_{010}(D) = .6+L(1/n\varepsilon_3)$

$q^*_{011}(D) = .6+L(1/n\varepsilon_4)$

$q^*_{100}(D) = .6+L(1/n\varepsilon_5)$

Example I

Budget=$\varepsilon_{global} - \varepsilon_1 - \varepsilon_2 - \varepsilon_3 - \varepsilon_4 - \varepsilon_5 - \varepsilon_6$

$X=\{0,1\}^3$ ordered wrt binary encoding.

$q^*_{000}(D) = .3+L(1/n\varepsilon_1)$

$q^*_{001}(D) = .4+L(1/n\varepsilon_2)$

$q^*_{010}(D) = .6+L(1/n\varepsilon_3)$

$q^*_{011}(D) = .6+L(1/n\varepsilon_4)$

$q^*_{100}(D) = .6+L(1/n\varepsilon_5)$

$q^*_{101}(D) = .9+L(1/n\varepsilon_6)$

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

Example I

Budget=$\varepsilon_{global} - \varepsilon_1 - \varepsilon_2 - \varepsilon_3 - \varepsilon_4 - \varepsilon_5 - \varepsilon_6 - \varepsilon_7$

$X=\{0,1\}^3$ ordered
wrt binary encoding.

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

$q^*_{000}(D) = .3+L(1/n\varepsilon_1)$

$q^*_{001}(D) = .4+L(1/n\varepsilon_2)$

$q^*_{010}(D) = .6+L(1/n\varepsilon_3)$

$q^*_{011}(D) = .6+L(1/n\varepsilon_4)$

$q^*_{100}(D) = .6+L(1/n\varepsilon_5)$

$q^*_{101}(D) = .9+L(1/n\varepsilon_6)$

$q^*_{110}(D) = 1+L(1/n\varepsilon_7)$

Example I

Budget=ε_{global} - ε_1 - ε_2 - ε_3 - ε_4 - ε_5 - ε_6 - ε_7 - ε_8

X={0,1}^{3} ordered wrt binary encoding.

$D \in X^{10}$ =

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

$q^*_{000}(D) = .3+L(1/n\varepsilon_1)$

$q^*_{001}(D) = .4+L(1/n\varepsilon_2)$

$q^*_{010}(D) = .6+L(1/n\varepsilon_3)$

$q^*_{011}(D) = .6+L(1/n\varepsilon_4)$

$q^*_{100}(D) = .6+L(1/n\varepsilon_5)$

$q^*_{101}(D) = .9+L(1/n\varepsilon_6)$

$q^*_{110}(D) = 1+L(1/n\varepsilon_7)$

$q^*_{111}(D) = 1+L(1/n\varepsilon_8)$

Example I

Budget=ε_{global} - ε_1 - ε_2 - ε_3 - ε_4 - ε_5 - ε_6 - ε_7 - ε_8

$X=\{0,1\}^3$ ordered wrt binary encoding.

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

$q^*_{000}(D) = .3+L(1/n\varepsilon_1)$

$q^*_{001}(D) = .4+L(1/n\varepsilon_2)$

$q^*_{010}(D) = .6+L(1/n\varepsilon_3)$

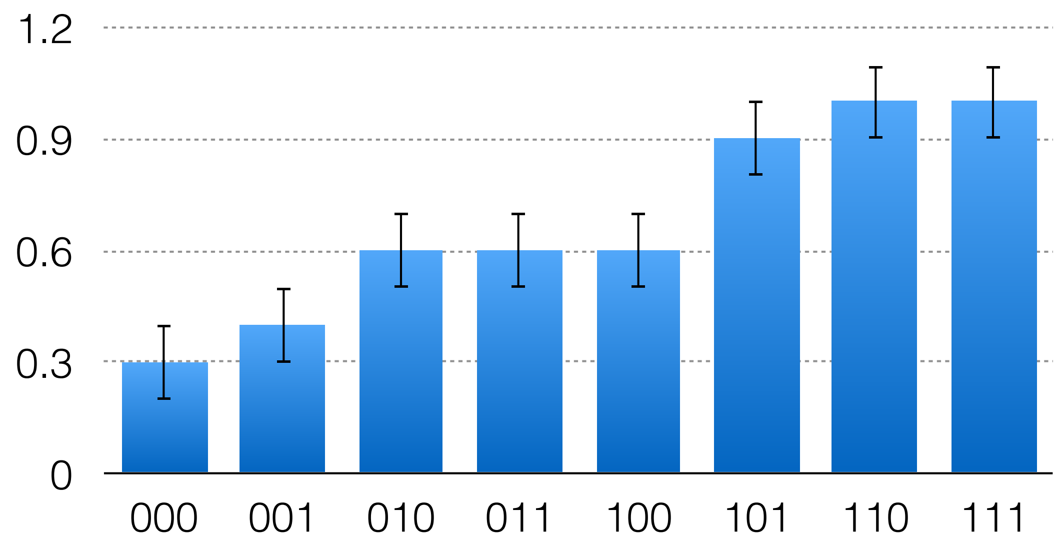
$q^*_{011}(D) = .6+L(1/n\varepsilon_4)$

$q^*_{100}(D) = .6+L(1/n\varepsilon_5)$

$q^*_{101}(D) = .9+L(1/n\varepsilon_6)$

$q^*_{110}(D) = 1+L(1/n\varepsilon_7)$

$q^*_{111}(D) = 1+L(1/n\varepsilon_8)$



Example II

Let's consider the universe domain $\mathcal{X} = \{0,1\}^d$ and let's consider the following predicate for an index $1 \leq j \leq d$

$$q_j(x) = x_j$$

we call an attribute mean function the associated counting query

$$q_j : \mathcal{X}^n \rightarrow [0,1]$$

Example II

Let’s consider the universe domain $\mathcal{X} = \{0,1\}^d$ and let’s consider the following predicate for an index $1 \leq j \leq d$

$$q_j(x) = x_j$$

we call an attribute mean function the associated counting query

$$q_j : \mathcal{X}^n \rightarrow [0,1]$$

Question: What is the sensitivity?

Example II

Budget=ε_{global}

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

Example II

Budget=ε_{global} - ε_1

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

$q^*_1(D) = .4 + L(1/n\varepsilon_1)$

Example II

Budget=ε_{global} - ε_1 - ε_2

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

$q^*_1(D) = .4+L(1/n\varepsilon_1)$

$q^*_2(D) = .3+L(1/n\varepsilon_2)$

Example II

Budget=ε_{global} - ε_1 - ε_2 - ε_3

$D \in X^{10}$ =

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

$q^*_1(D) = .4+L(1/n\varepsilon_1)$

$q^*_2(D) = .3+L(1/n\varepsilon_2)$

$q^*_3(D) = .4+L(1/n\varepsilon_3)$

Example II

Budget=$\varepsilon_{global} - \varepsilon_1 - \varepsilon_2 - \varepsilon_3$

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1
margin	$4+Y_1$	$3+Y_2$	$4+Y_3$

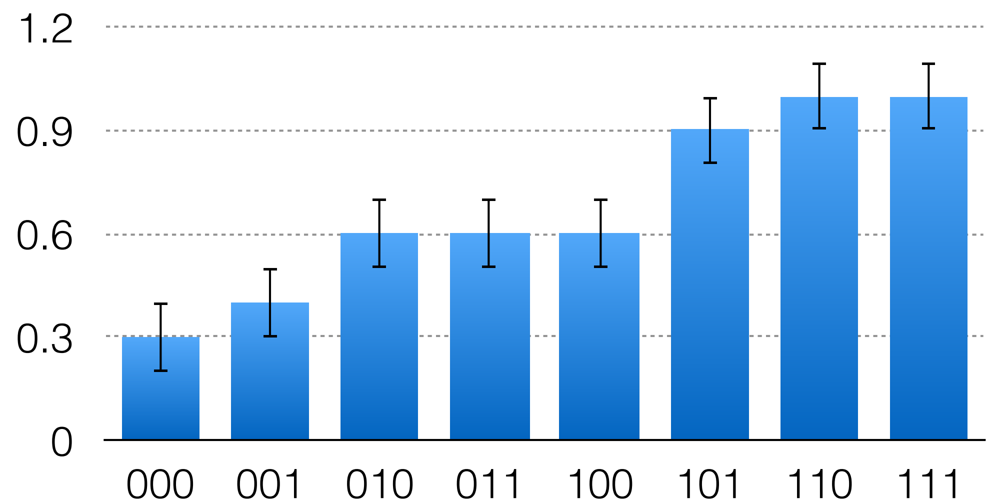
$q^*_1(D) = .4+L(1/n\varepsilon_1)$

$q^*_2(D) = .3+L(1/n\varepsilon_2)$

$q^*_3(D) = .4+L(1/n\varepsilon_3)$

Example II

Budget= $\varepsilon_{global} - \varepsilon_1 - \varepsilon_2 - \varepsilon_3 - \varepsilon_4 - \varepsilon_5 - \varepsilon_6 - \varepsilon_7 - \varepsilon_8$



Budget= $\varepsilon_{global} - \varepsilon_1 - \varepsilon_2 - \varepsilon_3$

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1
margin	$4+Y_1$	$3+Y_2$	$4+Y_3$

Privacy Budget vs Epsilon

Sometimes is more convenient to think in terms of Privacy Budget: Budget=$\varepsilon_{global} - \sum \varepsilon_{local}$

Sometimes is more convenient to think in terms of epsilon: $\varepsilon_{global} = \sum \varepsilon_{local}$

Privacy Budget vs Epsilon

Sometimes is more convenient to think in terms of Privacy Budget: Budget=$\varepsilon_{global} - \sum \varepsilon_{local}$

Sometimes is more convenient to think in terms of epsilon: $\varepsilon_{global} = \sum \varepsilon_{local}$

Making them uniforms is sometimes more informative.

Privacy Budget vs Epsilon

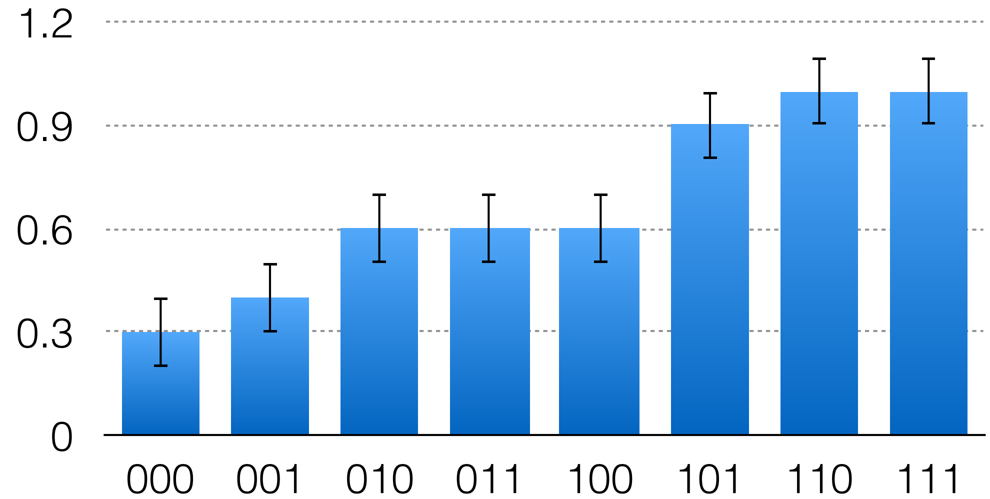
Sometimes is more convenient to think in terms of Privacy Budget: Budget=$\varepsilon_{global} - \sum \varepsilon_{local}$

Sometimes is more convenient to think in terms of epsilon: $\varepsilon_{global} = \sum \varepsilon_{local}$

Making them uniforms is sometimes more informative.

Note: There are situations where the two are not equivalent.

Example II



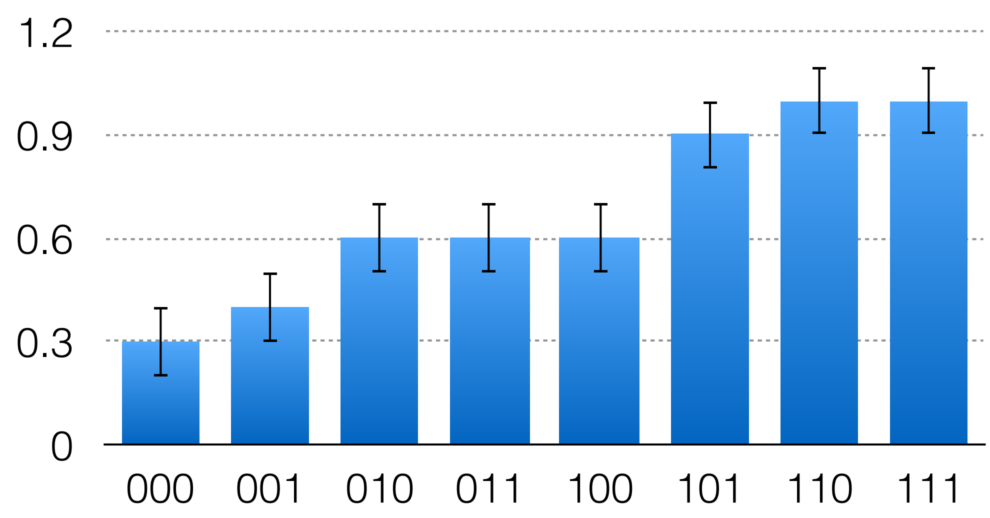
$\text{Budget} = \varepsilon_{global} - \varepsilon_1 - \varepsilon_2 - \varepsilon_3 - \varepsilon_4 - \varepsilon_5 - \varepsilon_6 - \varepsilon_7 - \varepsilon_8$

$\text{Budget} = \varepsilon_{global} - \varepsilon_1 - \varepsilon_2 - \varepsilon_3$

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1
margin	$4+Y_1$	$3+Y_2$	$4+Y_3$

Example II



Budget= ε_{global} - ε_1 - ε_2 - ε_3 - ε_4 - ε_5 - ε_6 - ε_7 - ε_8

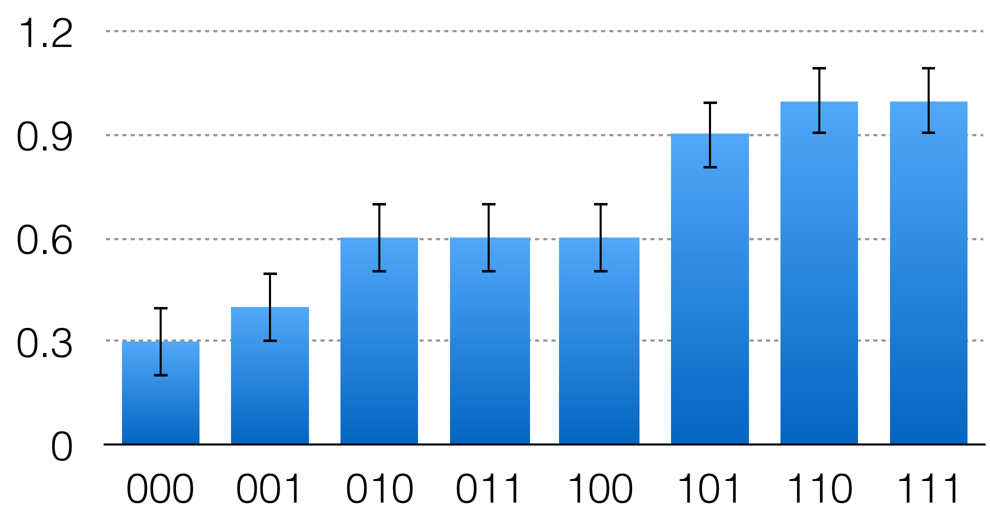
$\varepsilon_{global} = \varepsilon+\varepsilon+\varepsilon+\varepsilon+\varepsilon+\varepsilon+\varepsilon+\varepsilon = 8\varepsilon$

Budget= ε_{global} - ε_1 - ε_2 - ε_3

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1
margin	$4+Y_1$	$3+Y_2$	$4+Y_3$

Example II



Budget= $\varepsilon_{global} - \varepsilon_1 - \varepsilon_2 - \varepsilon_3 - \varepsilon_4 - \varepsilon_5 - \varepsilon_6 - \varepsilon_7 - \varepsilon_8$

$\varepsilon_{global} = \varepsilon+\varepsilon+\varepsilon+\varepsilon+\varepsilon+\varepsilon+\varepsilon+\varepsilon = 8\varepsilon$

Budget= $\varepsilon_{global} - \varepsilon_1 - \varepsilon_2 - \varepsilon_3$

$\varepsilon_{global} = \varepsilon+\varepsilon+\varepsilon = 3\varepsilon$

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1
margin	$4+Y_1$	$3+Y_2$	$4+Y_3$

Composition

Question: How about histograms?

Example III

Let's consider an arbitrary universe domain $\mathcal{X}$ and let's consider the following predicate for $y \in \mathcal{X}$

$$q_y(x) = \begin{cases} 1 & \text{if } y = x \\ 0 & \text{otherwise} \end{cases}$$

we call a point function the associated counting query

$$q_y : \mathcal{X}^n \to [0, 1]$$

Example III

Let's consider an arbitrary universe domain $\mathcal{X}$ and let's consider the following predicate for $y \in \mathcal{X}$

$$q_y(x) = \begin{cases} 1 & \text{if } y = x \\ 0 & \text{otherwise} \end{cases}$$

we call a point function the associated counting query

$$q_y : \mathcal{X}^n \to [0, 1]$$

Question: What is the sensitivity?

Example III

Budget=ε_{global}

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

Example III

Budget=ε_{global} - ε

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

$q^*_{000}(D) = .3+L(1/n\varepsilon)$

Example III

Budget=ε_{global} - ε - ε

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

$q^*_{000}(D) = .3+L(1/n\varepsilon)$

$q^*_{001}(D) = .1+L(1/n\varepsilon)$

Example III

$\text{Budget} = \varepsilon_{global} - \varepsilon \quad - \varepsilon \quad - \varepsilon$

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

$q^*_{000}(D) = .3 + L(1/n\varepsilon)$

$q^*_{001}(D) = .1 + L(1/n\varepsilon)$

$q^*_{010}(D) = .2 + L(1/n\varepsilon)$

Example III

Budget=ε_{global} - ε - ε - ε - ε

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

$q^*_{000}(D) = .3+L(1/n\varepsilon)$

$q^*_{001}(D) = .1+L(1/n\varepsilon)$

$q^*_{010}(D) = .2+L(1/n\varepsilon)$

$q^*_{011}(D) = 0+L(1/n\varepsilon)$

Example III

Budget=ε_{global} - ε - ε - ε - ε - ε

$D \in X^{10}$ =

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

$q^*_{000}(D) = .3+L(1/n\varepsilon)$

$q^*_{001}(D) = .1+L(1/n\varepsilon)$

$q^*_{010}(D) = .2+L(1/n\varepsilon)$

$q^*_{011}(D) = 0+L(1/n\varepsilon)$

$q^*_{100}(D) = 0+L(1/n\varepsilon)$

Example III

Budget=ε_{global} - ε - ε - ε - ε
- ε - ε

$D \in X^{10}$ =

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

$q^*_{000}(D) = .3+L(1/n\varepsilon)$

$q^*_{001}(D) = .1+L(1/n\varepsilon)$

$q^*_{010}(D) = .2+L(1/n\varepsilon)$

$q^*_{011}(D) = 0+L(1/n\varepsilon)$

$q^*_{100}(D) = 0+L(1/n\varepsilon)$

$q^*_{101}(D) = .3+L(1/n\varepsilon)$

Example III

Budget=ε_{global} - ε - ε - ε - ε
- ε - ε - ε

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

$q^*_{000}(D) = .3+L(1/n\varepsilon)$

$q^*_{001}(D) = .1+L(1/n\varepsilon)$

$q^*_{010}(D) = .2+L(1/n\varepsilon)$

$q^*_{011}(D) = 0+L(1/n\varepsilon)$

$q^*_{100}(D) = 0+L(1/n\varepsilon)$

$q^*_{101}(D) = .3+L(1/n\varepsilon)$

$q^*_{110}(D) = .1+L(1/n\varepsilon)$

Example III

Budget=ε_{global} - ε - ε - ε - ε
- ε - ε - ε - ε

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

$q^*_{000}(D) = .3+L(1/n\varepsilon)$

$q^*_{001}(D) = .1+L(1/n\varepsilon)$

$q^*_{010}(D) = .2+L(1/n\varepsilon)$

$q^*_{011}(D) = 0+L(1/n\varepsilon)$

$q^*_{100}(D) = 0+L(1/n\varepsilon)$

$q^*_{101}(D) = .3+L(1/n\varepsilon)$

$q^*_{110}(D) = .1+L(1/n\varepsilon)$

$q^*_{111}(D) = 0+L(1/n\varepsilon)$

Example III

Budget= ε_{global} - ε - ε - ε - ε - ε - ε - ε - ε

$D \in X^{10}$ =

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

$q^*_{000}(D) = .3+L(1/n\varepsilon)$

$q^*_{001}(D) = .1+L(1/n\varepsilon)$

$q^*_{010}(D) = .2+L(1/n\varepsilon)$

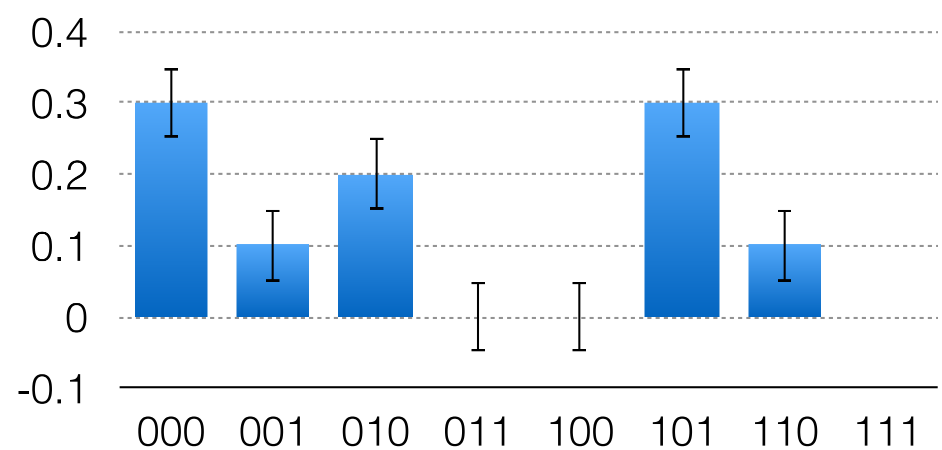
$q^*_{011}(D) = 0+L(1/n\varepsilon)$

$q^*_{100}(D) = 0+L(1/n\varepsilon)$

$q^*_{101}(D) = .3+L(1/n\varepsilon)$

$q^*_{110}(D) = .1+L(1/n\varepsilon)$

$q^*_{111}(D) = 0+L(1/n\varepsilon)$



Example III

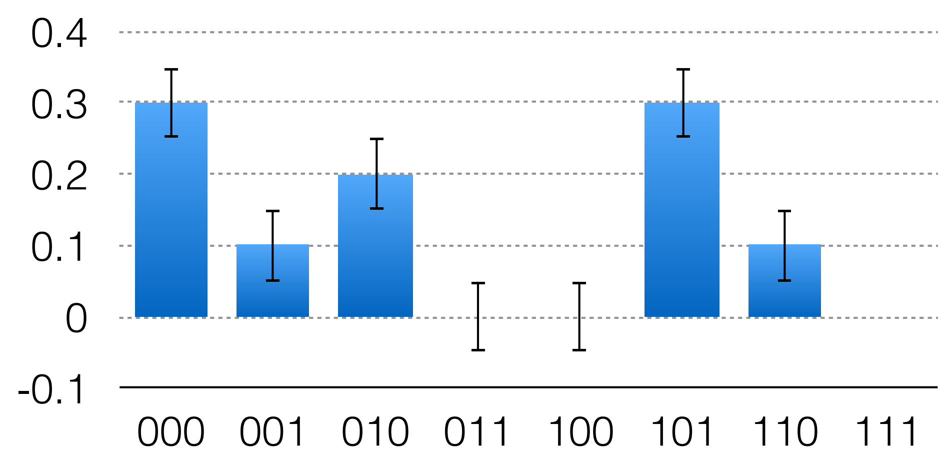
Budget=ε_{global} - ε - ε - ε - ε
- ε - ε - ε - ε

Can we do better?

$$q^*_{000}(D) = .3+L(1/n\varepsilon)$$
$$q^*_{001}(D) = .1+L(1/n\varepsilon)$$
$$q^*_{010}(D) = .2+L(1/n\varepsilon)$$
$$q^*_{011}(D) = 0+L(1/n\varepsilon)$$
$$q^*_{100}(D) = 0+L(1/n\varepsilon)$$
$$q^*_{101}(D) = .3+L(1/n\varepsilon)$$
$$q^*_{110}(D) = .1+L(1/n\varepsilon)$$
$$q^*_{111}(D) = 0+L(1/n\varepsilon)$$

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1



Example III

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

$D' \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	1	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

Example III

$D \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	0	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

$D' \in X^{10} =$

	D1	D2	D3
I1	0	0	0
I2	1	0	1
I3	0	1	0
I4	1	0	1
I5	0	1	0
I6	0	0	1
I7	1	1	0
I8	0	0	0
I9	0	1	0
I10	1	0	1

$q_{000}(D) = .3$

Example III

$D \in X^{10} =$

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I1	0	0	0
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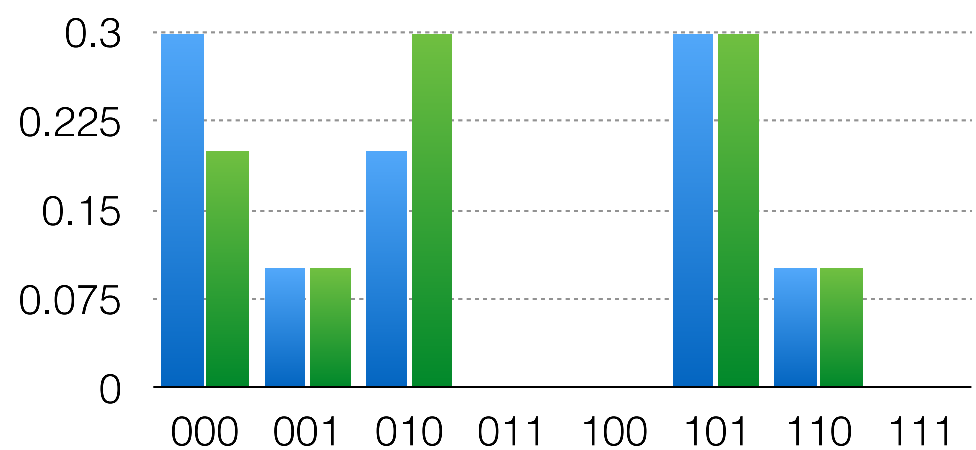

Example III

Budget=ε_{global} - 2ε

Can we do better?

$q^*_{000}(D) = .3+L(1/n\varepsilon)$

$q^*_{001}(D) = .1+L(1/n\varepsilon)$

$q^*_{010}(D) = .2+L(1/n\varepsilon)$

$q^*_{011}(D) = 0+L(1/n\varepsilon)$

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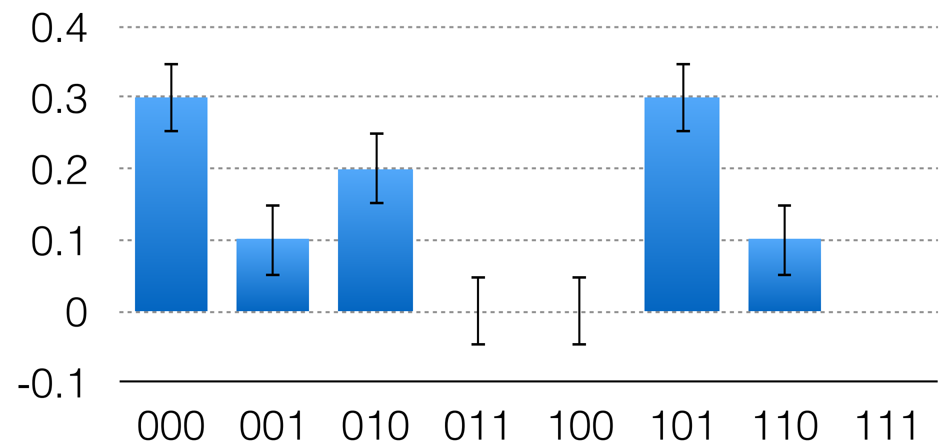

Releasing partial sums

```
DummySum(d : {0,1} list) : real list
  i:= 0;
  s:= 0;
  r:= [];
  while (i<size d)
     s:= s + d[i]
     z:=$ s + Lap(1/eps)
     r:= r ++ [z];
     i:= i+1;
  return r
```

What is the global epsilon here?

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     s:= s + z
     r:= r ++ [s];
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```

What is the global epsilon here?

Parallel Composition

Let M_1:DB →R be a (ε_1,δ_1)-differentially private program and M_2:DB →R be a (ε_2,δ_2)-differentially private program. Suppose that we partition D in a data-independent way into two datasets D_1 and D_2. Then, the composition $M_{1,2}$:DB→R defined as

$$MP_{1,2}(D)=(M_1(D_1),M_2(D_2))$$

is $(\max(\varepsilon_1,\varepsilon_2),\max(\delta_1,\delta_2))$-differentially private.

Composition

Question: how much perturbation do we have if we want to answer n queries under ε-DP?

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Question: how much perturbation do we have if we want to answer n counting queries under ε_{global}-DP?

We can split the privacy budget uniformly:

$$\epsilon = \frac{\epsilon_{\text{global}}}{n}$$

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Laplace accuracy: with high probability we have:

$$\left| q(D) - r \right| \leq O\left(\frac{1}{\epsilon n}\right)$$

Composition

Question: how much perturbation do we have if we want to answer n counting queries under $\varepsilon_{\text{global}}$-DP?

By putting them together (hiding some details) we have as a max error

$$O\Big(\frac{n}{\epsilon_{\text{global}} n}\Big) = O\Big(\frac{1}{\epsilon_{\text{global}}}\Big)$$

Composition

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By putting them together (hiding some details) we have as a max error

$$O\Big(\frac{n}{\epsilon_{\text{global}} n}\Big) = O\Big(\frac{1}{\epsilon_{\text{global}}}\Big)$$

Notice that if we don't renormalize this is of the order of

$$O\Big(\frac{n}{\epsilon_{\text{global}}}\Big)$$

bigger than the sample error.

Composition

Question: how many counting queries can we answer with small error under ε_{global}-DP?

Let's now target an error similar to sample error. How many queries we can answer?

If we want a non-normalized error of:

$$O\left(\frac{\sqrt{n}}{\epsilon_{\text{global}}}\right)$$

we can answer at most $\sqrt{n}$ queries.

Composition

Question: Can we do better?

Composition

Question: how much perturbation do we have if we want to answer n queries under (ε,δ)-DP?

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[DworkRothblumVadhan10, SteinkeUllman16]

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If we don't renormalize this is of the order of

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comparable to the sample error.

[DworkRothblumVadhan10, SteinkeUllman16]

Summary

- Resilience to post-processing
- Group privacy
- Composition