CS 599: Formal Methods in Security and Privacy

Differential Privacy

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Data

Releasing the mean of Some Data

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Mean(d : private data) : public real
   i:=0;
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   while (i<size(d))
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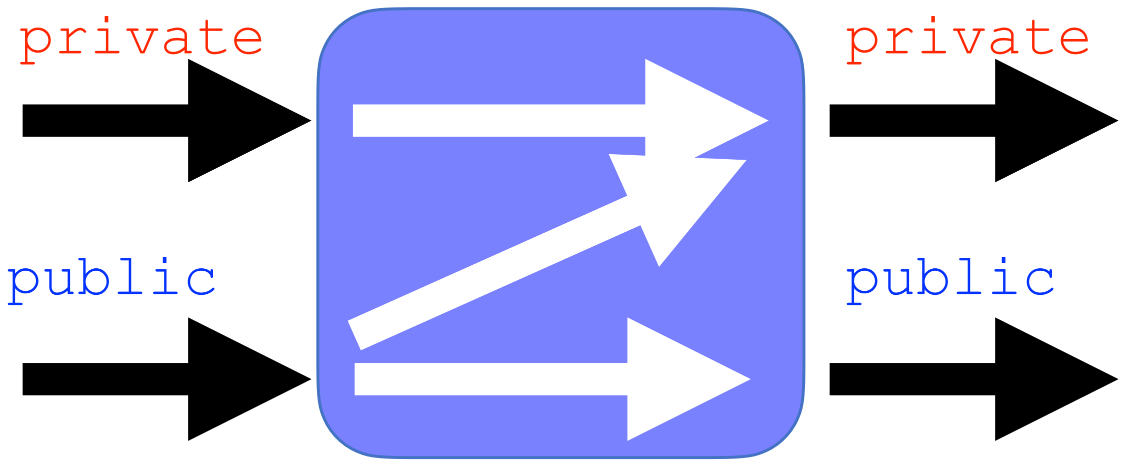

Privacy-preserving data analysis?

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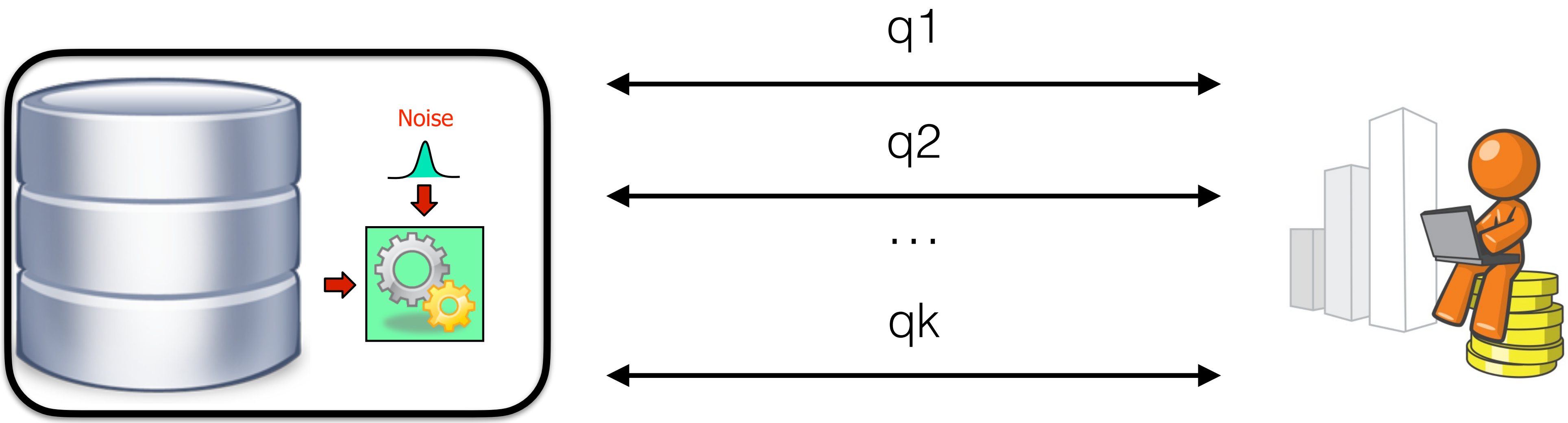
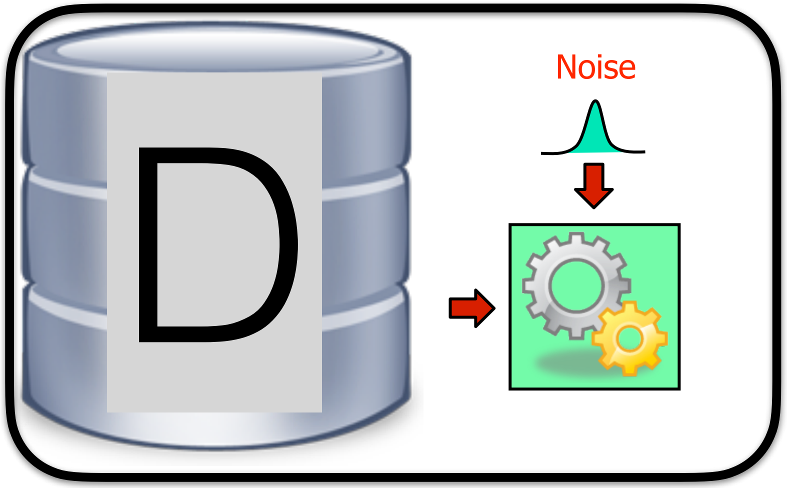


Data analyst

Fundamental Law of Information Reconstruction

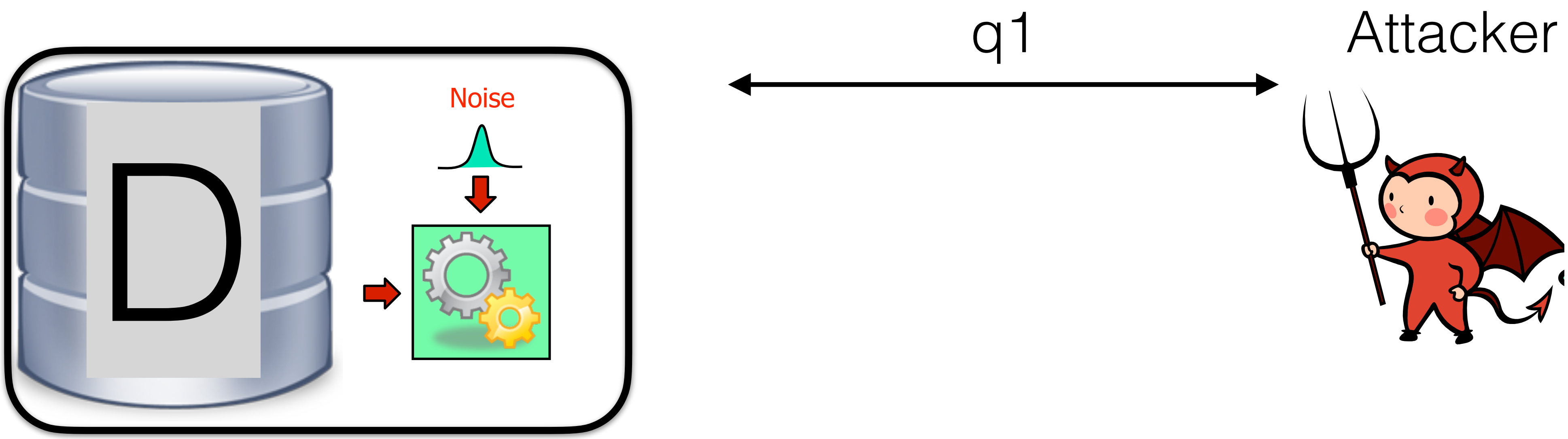
The release of too many overly accurate statistics permits reconstruction attacks.

Reconstruction attack

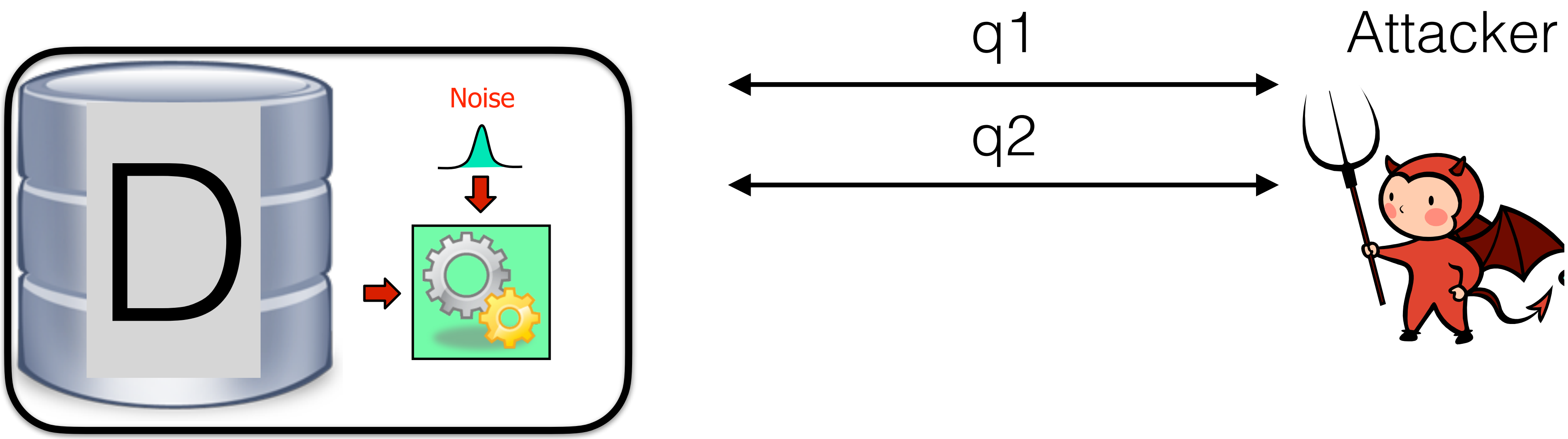


Attacker

Reconstruction attack



Reconstruction attack



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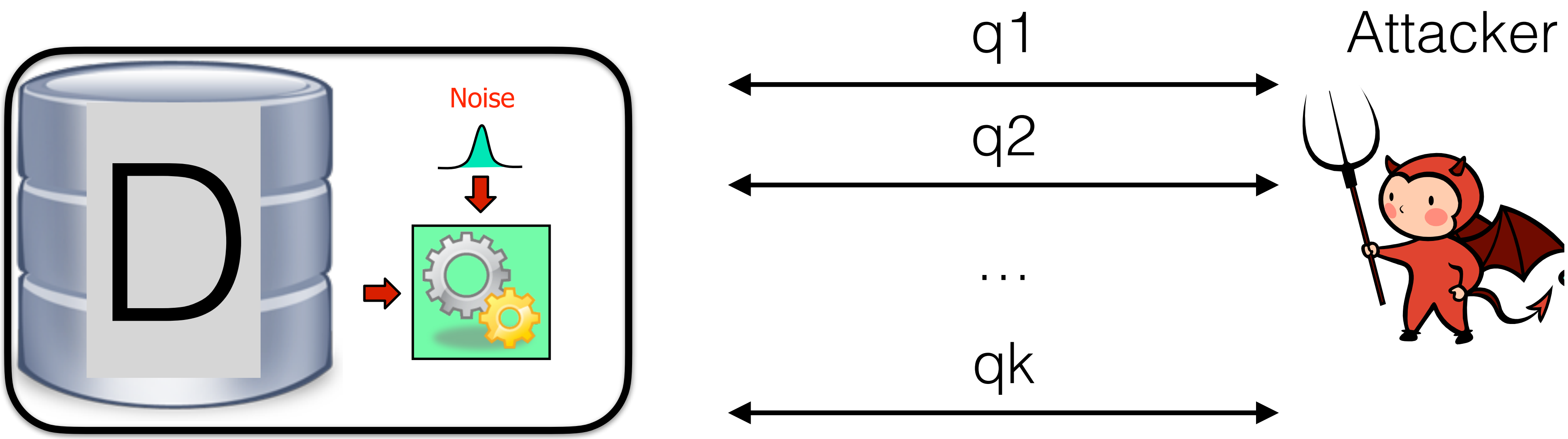

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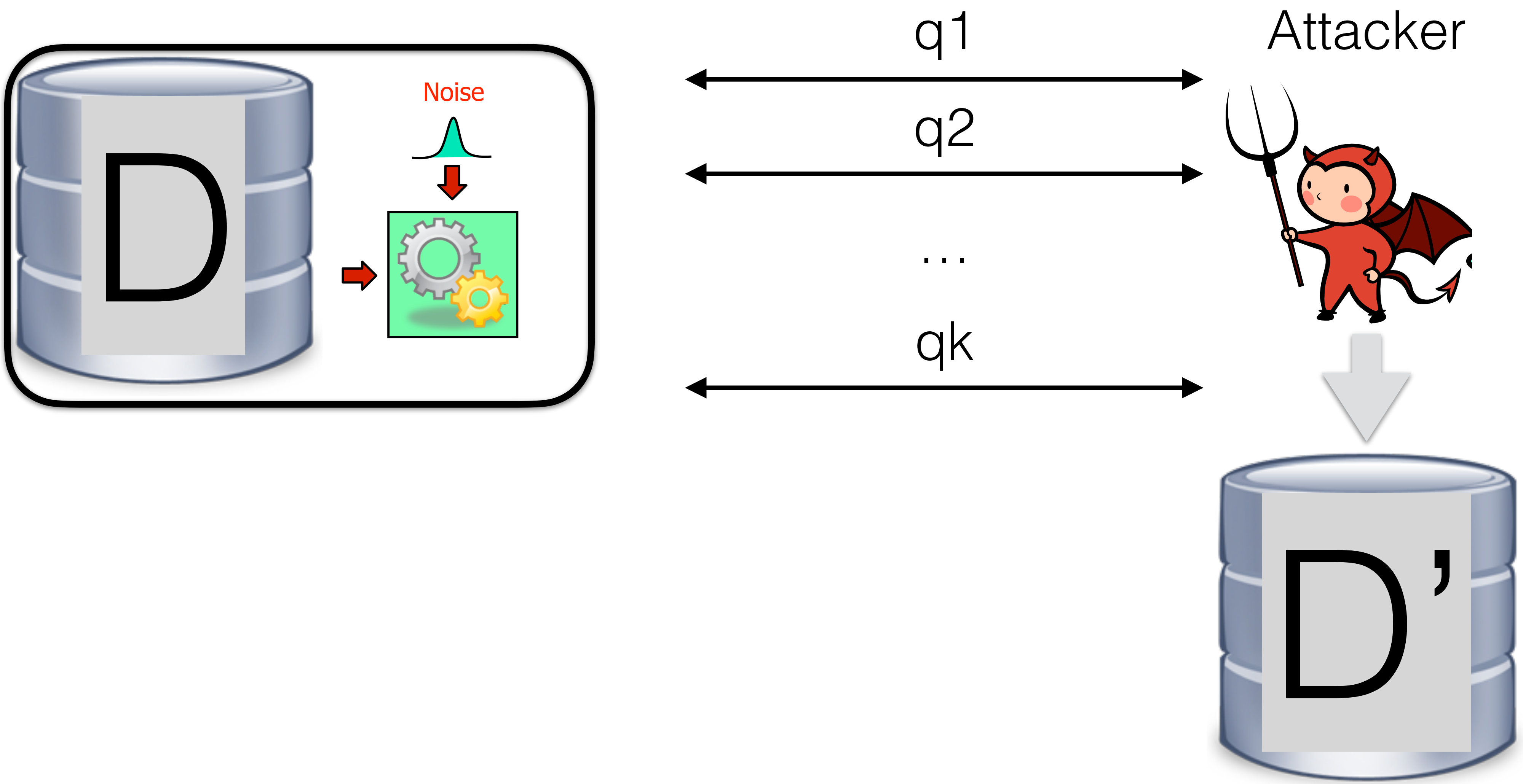

Reconstruction attack

We say that the attacker wins if

$$d(D, D') \sim 0$$

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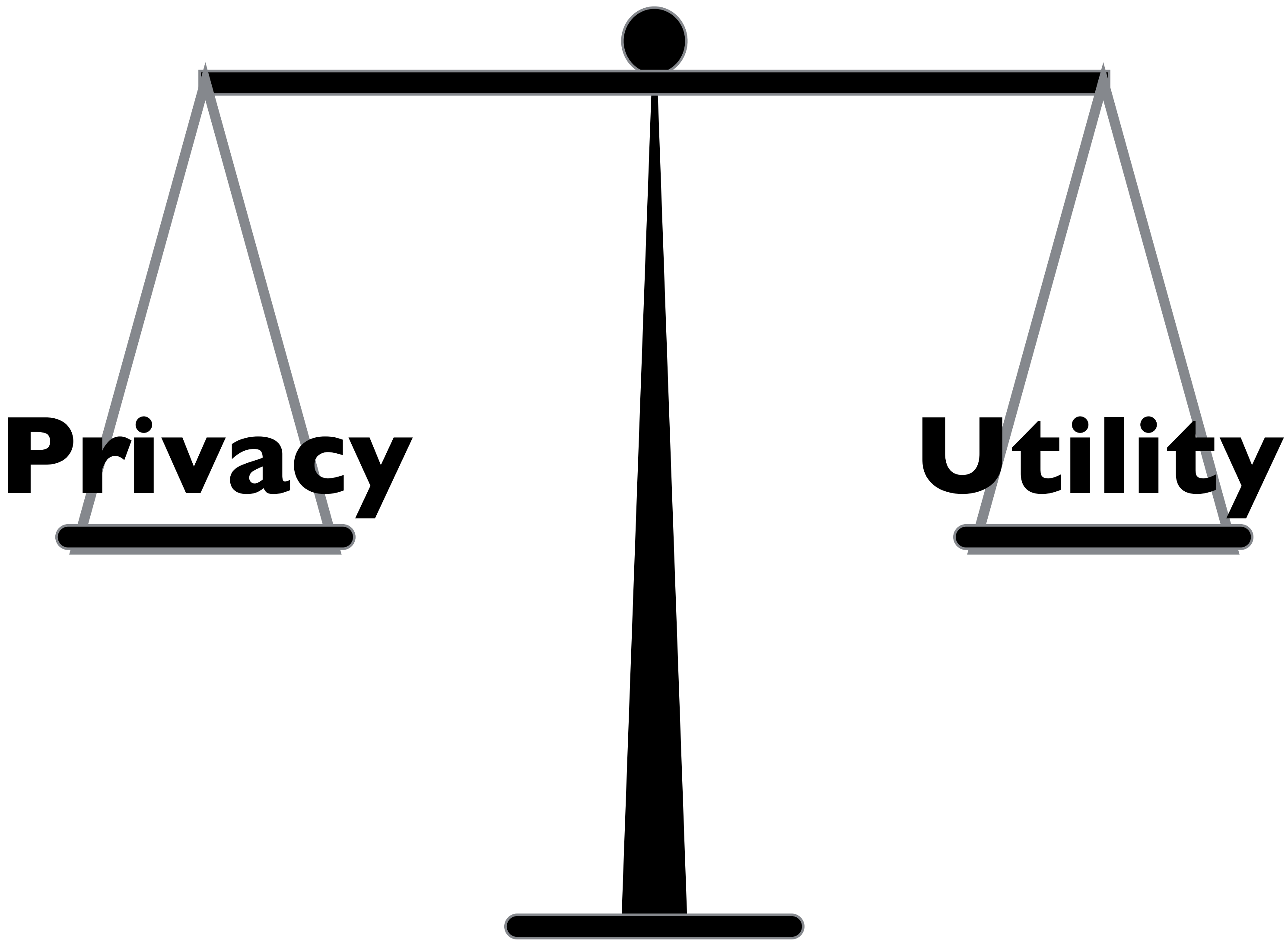
In this class case we can use Hamming distance

Privacy vs Utility



Quantitative notions of Privacy

- The impossibility results discussed above suggest a quantitative notion of privacy,
- a notion where the privacy loss depends on the number of queries that are allowed,
- and on the accuracy with which we answer them.

Differential privacy:
understanding the <u>mathematical</u> and <u>computational</u> meaning of this trade-off.

[Dwork, McSherry, Nissim, Smith, TCC06]

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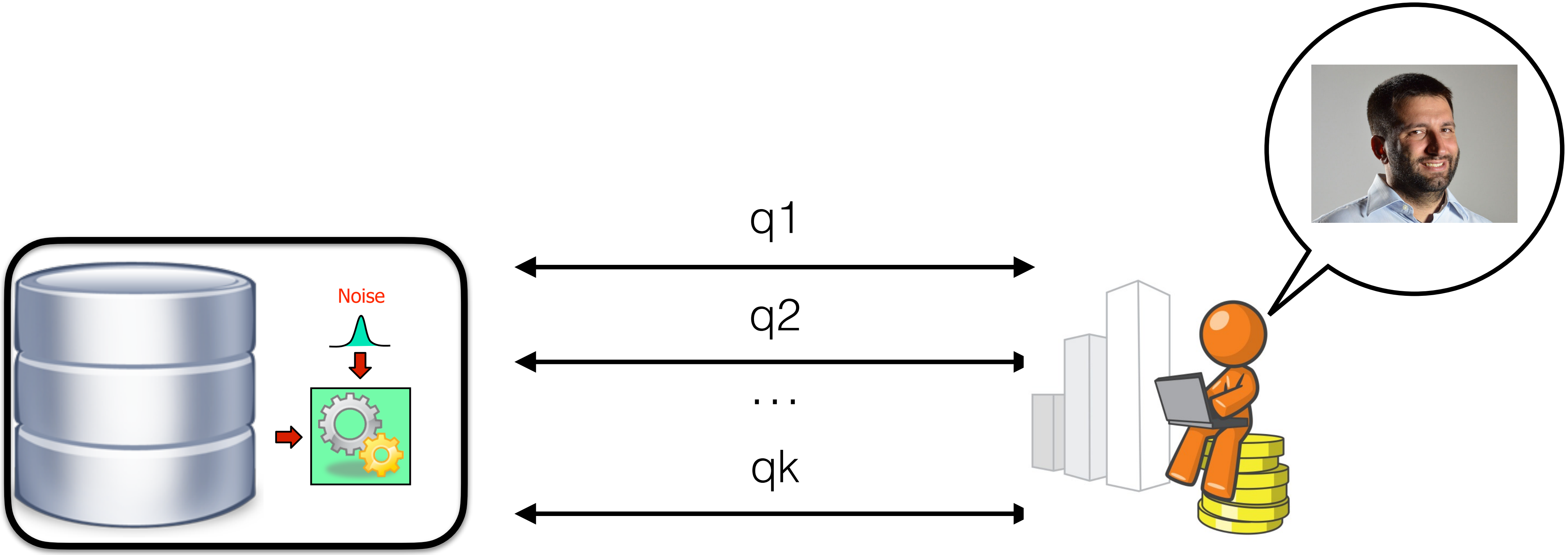

Privacy-preserving data analysis?

Prior Knowledge

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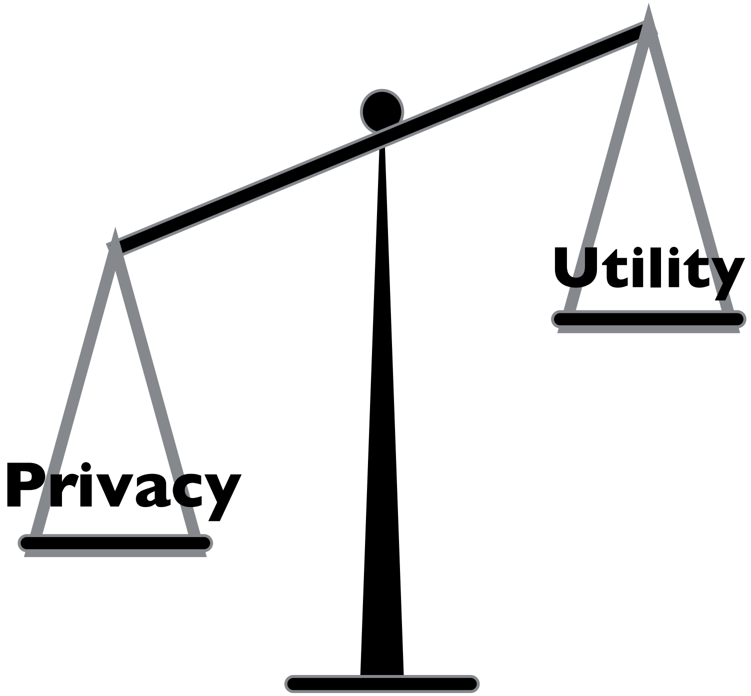
Posterior Knowledge

Privacy-preserving data analysis?

Privacy-preserving data analysis?

Question: What is the problem with this requirement?

Privacy-preserving data analysis?



If nothing can be learned about an individual, then nothing at all can be learned at all!

[DworkNaor10]

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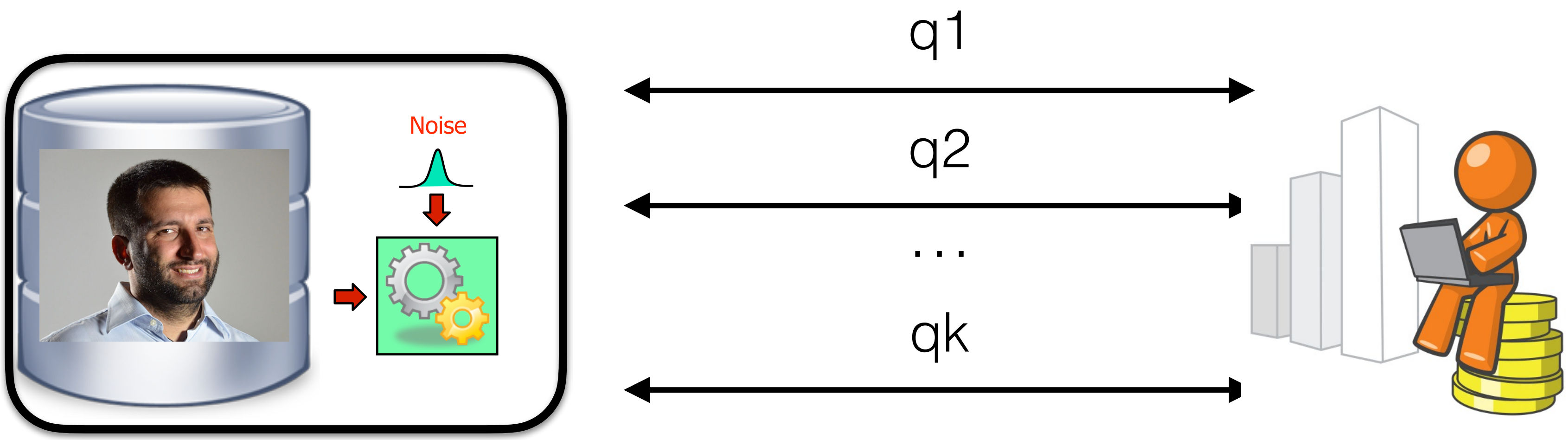

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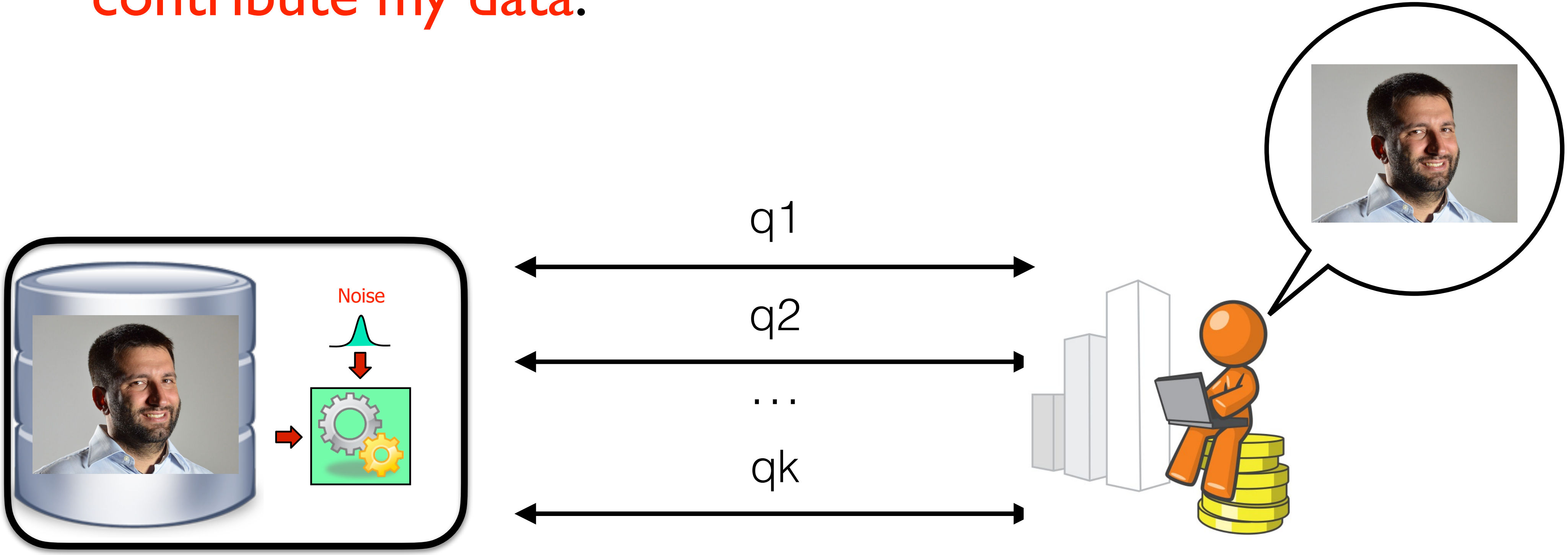

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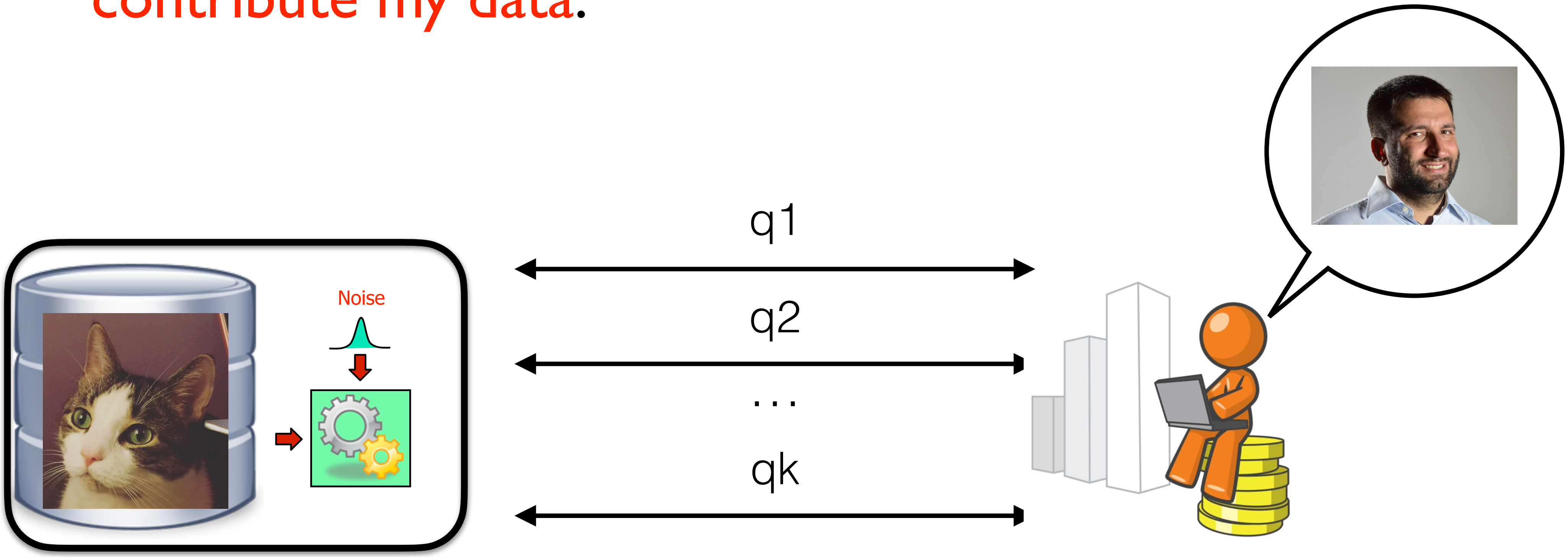


Adjacent databases

- We can formalize the concept of contributing my data or not in terms of a notion of distance between datasets.
- Given two datasets D, D'$\in$DB, their distance is defined as:

$$D\Delta D' = |\{k \leq n \mid D(k) \neq D'(k)\}|$$

- We will call two datasets adjacent when $D\Delta D'=1$ and we will write D~D'.

Privacy Loss

In general we can think about the following quantity as the privacy loss incurred by observing r on the databases b and b'.

$$L_{b,b'}(r) = \log \frac{\Pr[Q(b)=r]}{\Pr[Q(b')=r]}$$

(ε,δ)-Differential Privacy

Definition

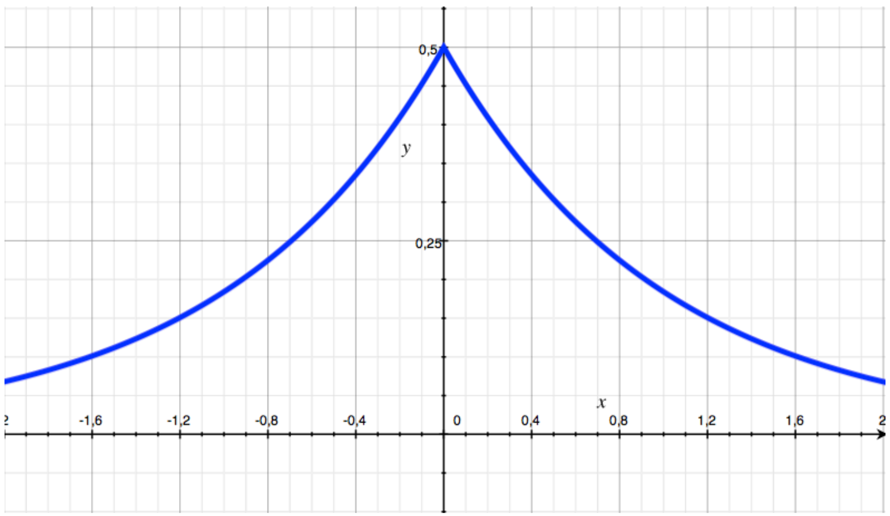
Given $\varepsilon,\delta \geq 0$, a probabilistic query $Q: X^n \rightarrow R$ is (ε,δ)-differentially private iff for all adjacent database b_1, b_2 and for every $S \subseteq R$:

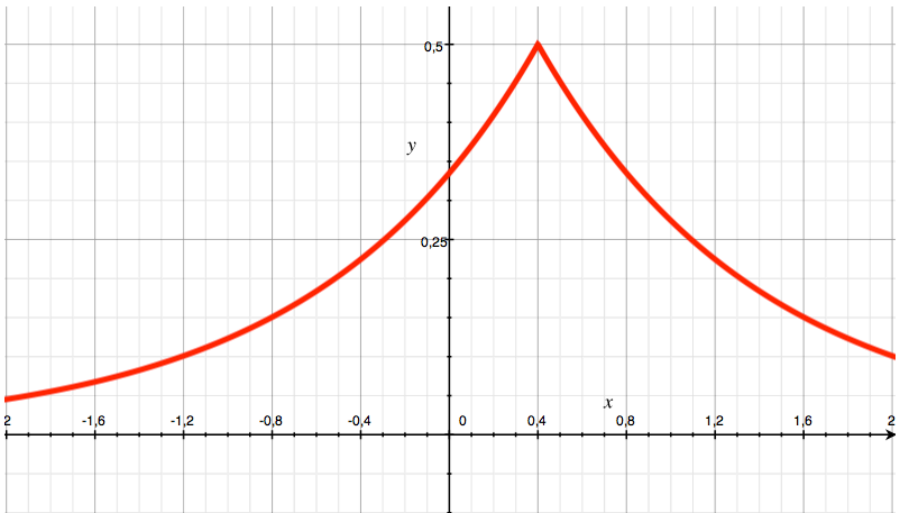
$$\Pr[Q(b_1) \in S] \leq \exp(\varepsilon)\Pr[Q(b_2) \in S] + \delta$$

Differential Privacy

Q : db => R   probabilistic

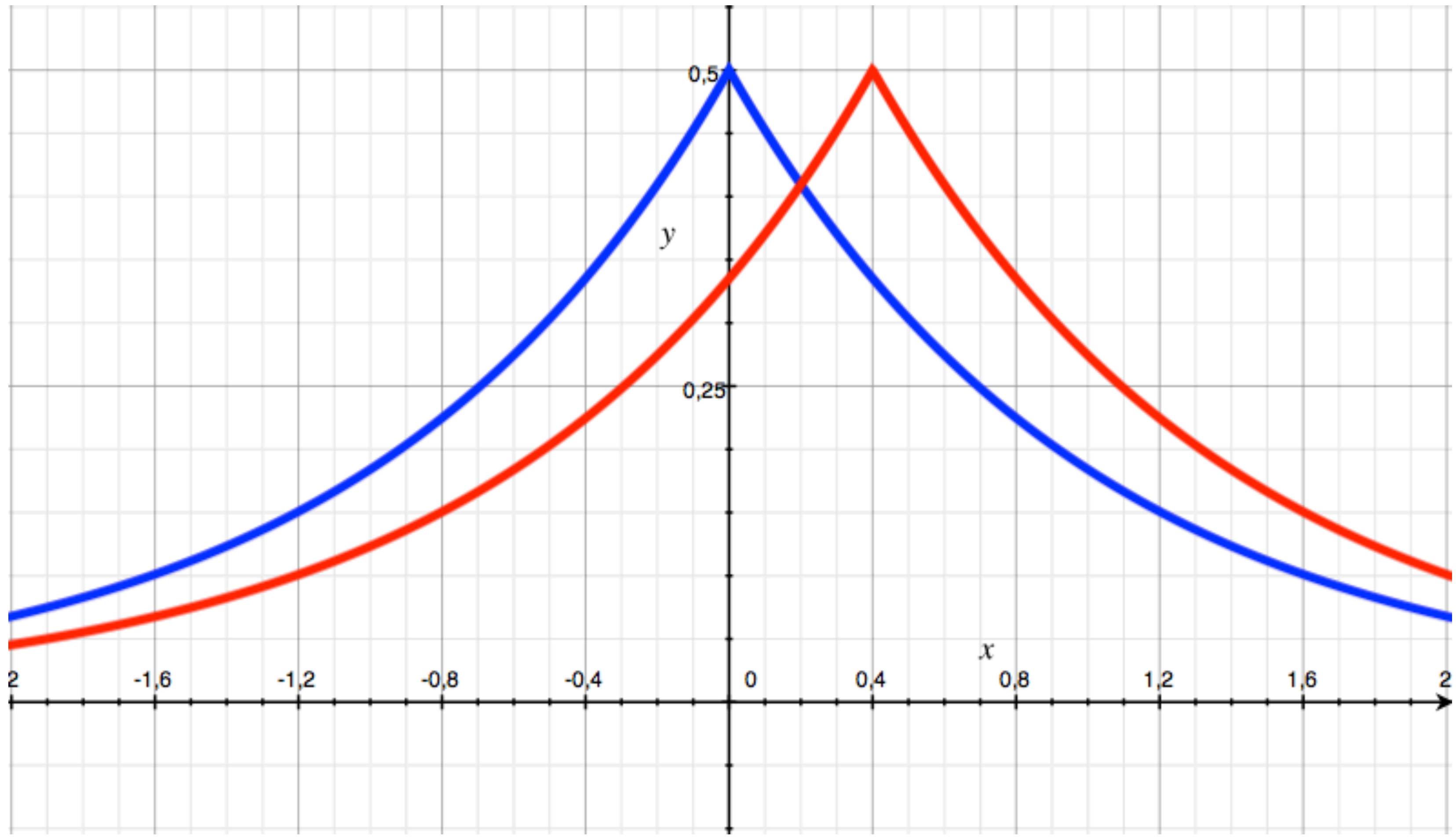
$Q(b \cup \{x\})$ $Q(b \cup \{y\})$





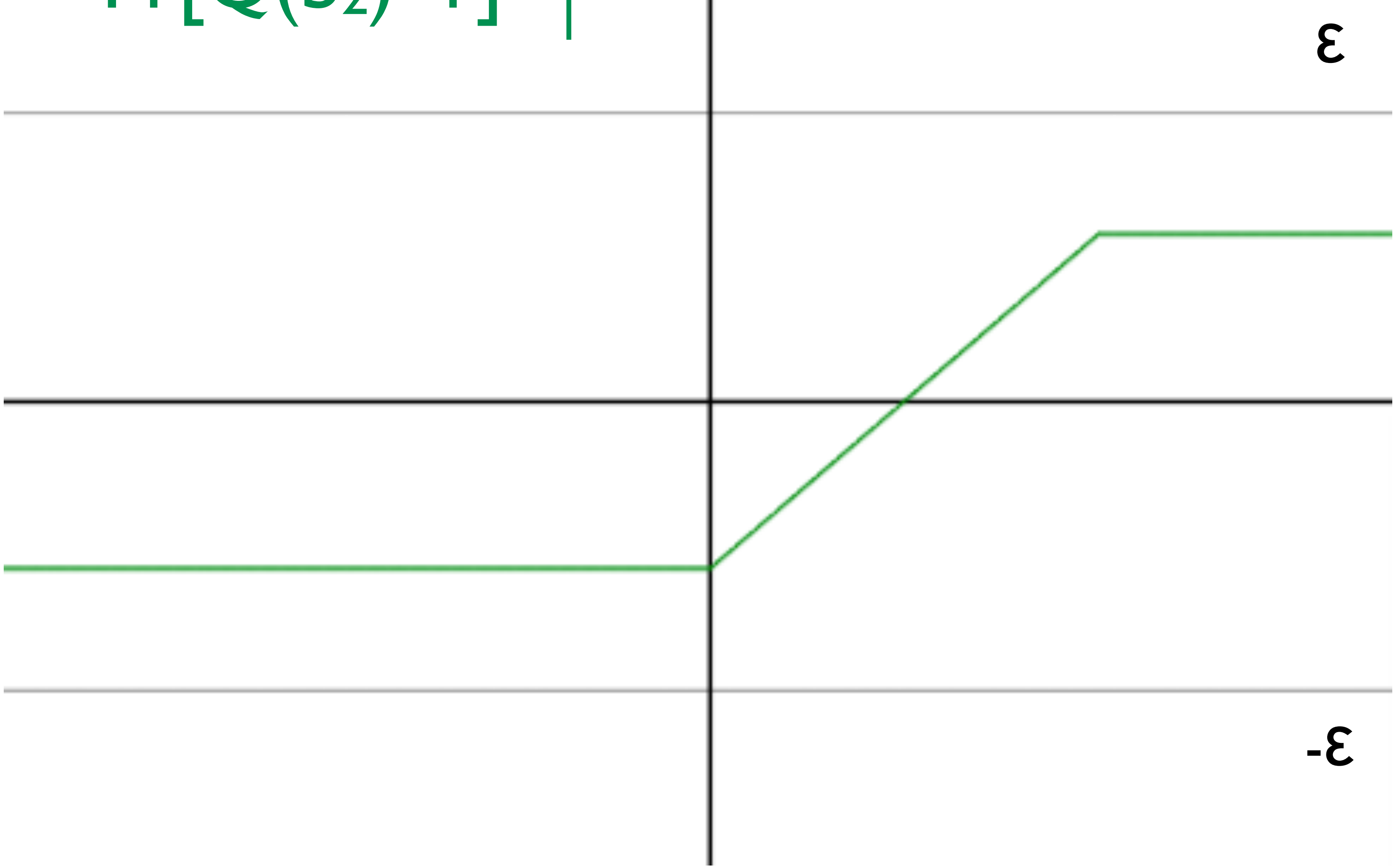
Differential Privacy

$d(Q(b \cup \{x\}), Q(b \cup \{y\})) \leq \varepsilon$ with probability $1-\delta$



(ε,δ)-Differential Privacy

$$\left| \log \frac{\Pr[Q(b_1)=r]}{\Pr[Q(b_2)=r]} \right| \leq \varepsilon \quad \text{with probability } 1-\delta$$



(ε,δ)-indistinguishability

Statistical distance:

$$\Delta(\mu_1,\mu_2)=\max_{E\subseteq A} \mid \mu_1(E)-\mu_2(E) \mid = \delta$$

can be seen as a notion of δ-indistinguishability.

We say that two distributions $\mu_1, \mu_2 \in D(A)$, are at δ-indistinguishable if:

$$\Delta(\mu 1,\mu 2)\leq \delta$$

(ε,δ)-indistinguishability

We can define a ε-skewed version of statistical distance. We call this notion ε-distance.

$$\Delta_\varepsilon(\mu 1,\mu 2)=\sup_{E\subseteq A} \max(\mu_1(E)-e^{\varepsilon}\mu_2(E),\ \mu_2(E)-e^{\varepsilon}\mu_1(E),0)$$

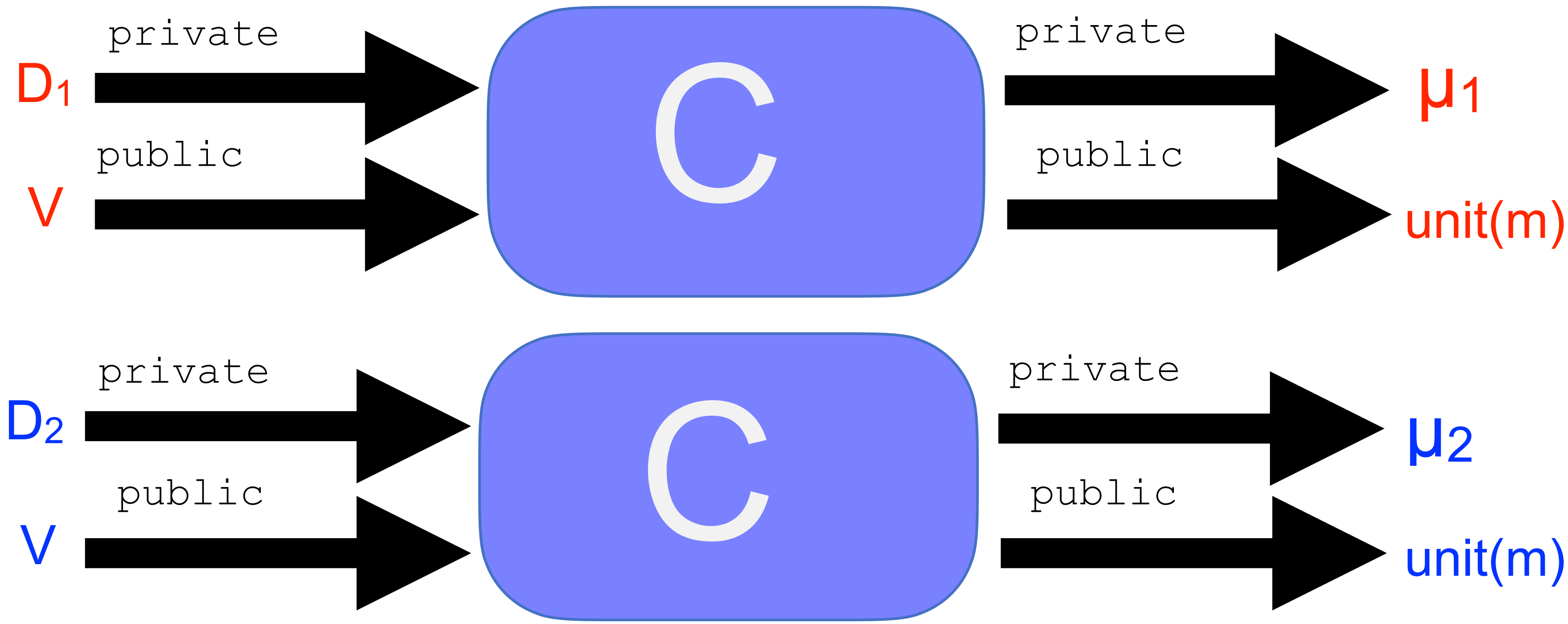
We say that two distributions $\mu_1, \mu_2 \in D(A)$, are at (ε,δ)-indistinguishable if:

$$\Delta_\varepsilon(\mu 1,\mu 2) \leq \delta$$

Differential Privacy as a Relational Property

c is differentially private if and only if for every $m_1 \sim m_2$ (extending the notion of adjacency to memories):

$\{c\}_{m1}=\mu_1$ and $\{c\}_{m2}=\mu_2$ implies $\Delta_\varepsilon(\mu_1,\mu_2) \leq \delta$



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   i:=0;
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Adding Noise

Question: What is a good way to add noise to the output of a statistical query to achieve (ε,0)-DP?

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Intuitive answer: it should depend on ε or the accuracy we want to achieve, and on the scale that a change of an individual can have on the output.

Global Sensitivity

$$GS_q = \max\{\, |q(D) - q(D')| \text{ s.t. } D \sim D' \}$$

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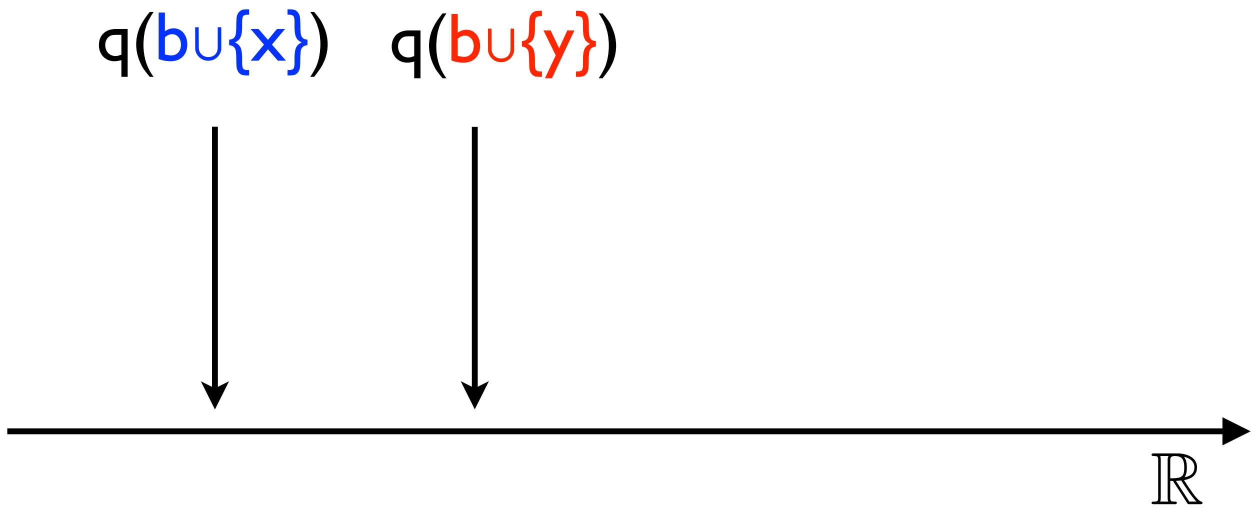

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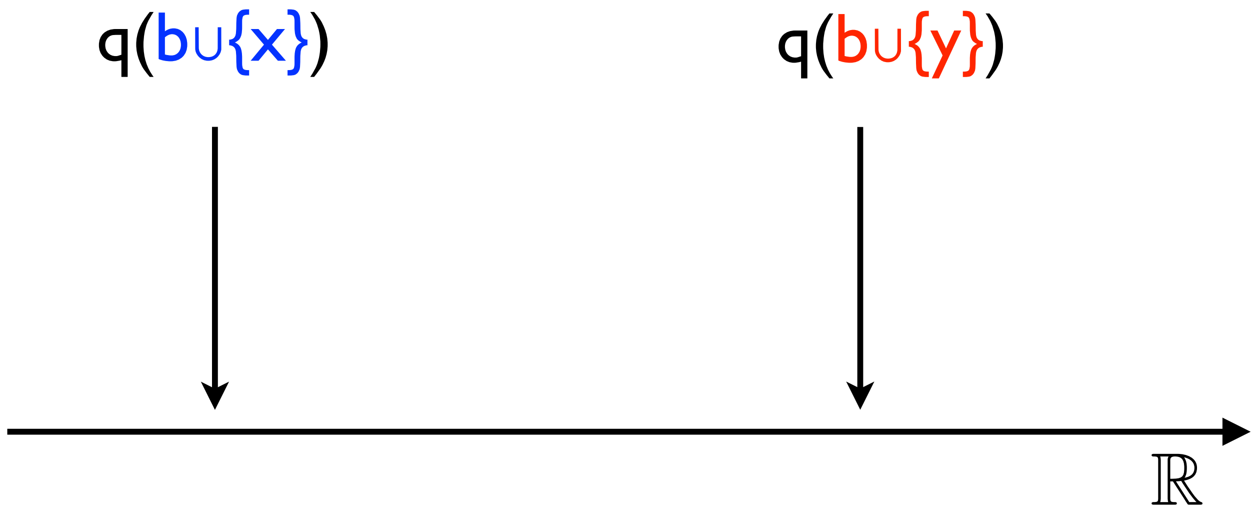
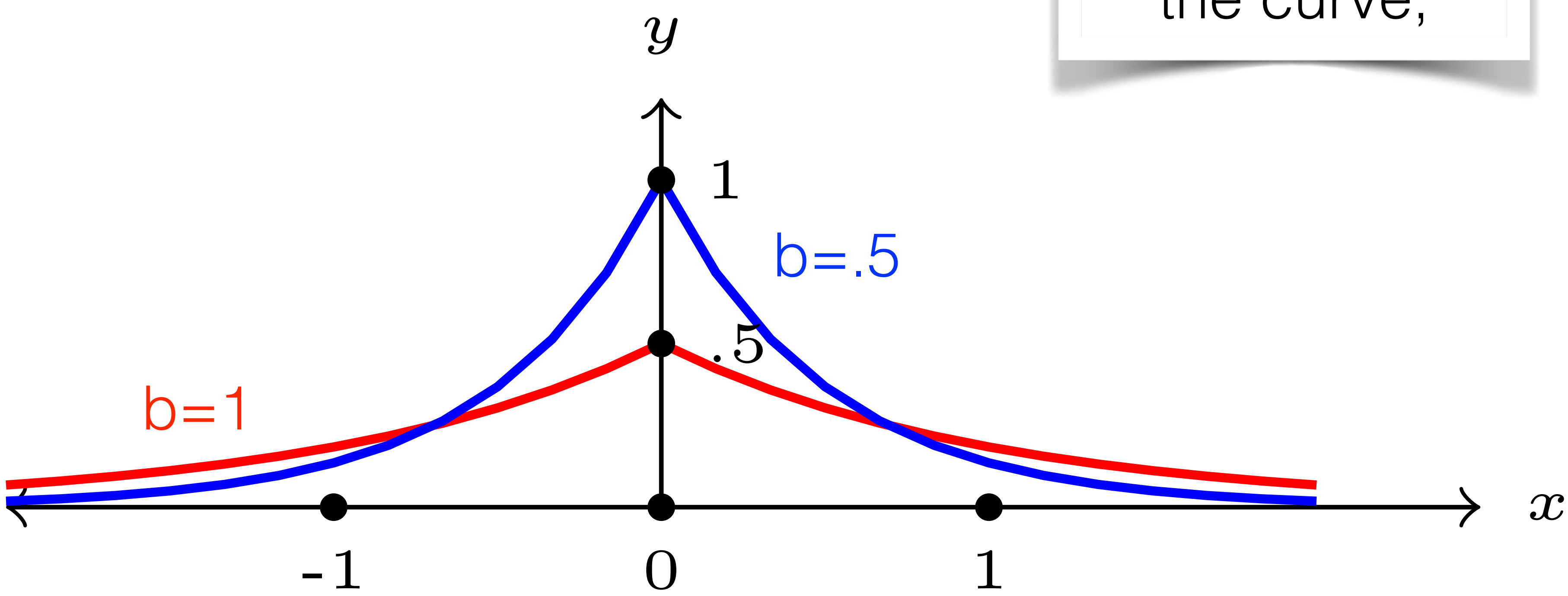


Laplace Distribution

$$\mathsf{Lap}(b,\mu)(X) = \frac{1}{2b}\exp\left(-\frac{|\mu - X|}{b}\right)$$

b regulates the skewness of the curve,



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      i:=i+1;
   z:=$ Laplace(sens/eps,0)
   z:= (s/i)+z
   return z
```

Laplace Mechanism

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Lap(d : priv data)(f: data -> real)
    (e:real) : pub real
   z:=$ Laplace(GSf/e,0)
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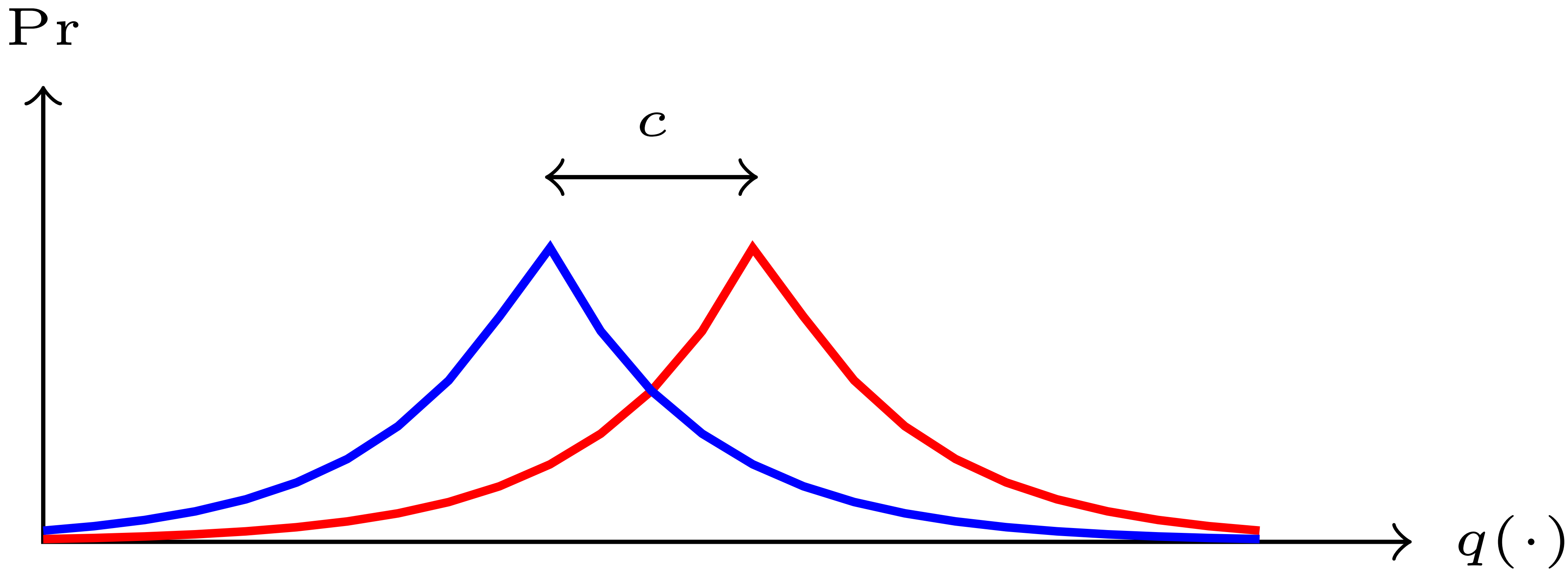
It turns out that we could also write it as:

```
Lap(d : priv data)(f: data -> real)
    (e:real) : pub real
   z:=$ Laplace(GSf/e,f(d))
   return z
```

Laplace Mechanism

Theorem (Privacy of the Laplace Mechanism)
The Laplace mechanism is (ε,0)-differentially private.

Proof: Intuitively



Laplace Mechanism

Question: How accurate is the answer that we get from the Laplace Mechanism?