

BU CS 332 – Theory of Computation

Please point a browser to

<https://forms.gle/cnUbzqTNibyoqt8w9> for in-class polls

- Lecture 1:

- Course information
- Overview

Reading:

Sipser Ch 0

Mark Bun

September 2, 2021

Course Information

Course Staff

- Me: **Mark Bun** (he/him/his)

- At BU since Sept. 2019
- Office hours: Mon 5-6PM, Fri 5-6PM
- Research interests: Theory of computation (!)

More specifically: Computational complexity, data privacy, cryptography, foundations of machine learning



- TF: **Islam Faisal**

- Office hours: TBD



- ...hopefully others

Course Webpage

https://cs-people.bu.edu/mbun/courses/332_F21/

Serves as the syllabus
and schedule

Check back frequently
for updates!

CS 332: Elements of the Theory of Computation, Fall 2021

Course Overview

This course is an introduction to the theory of computation. This is the branch of computer science that aims to understand which problems can be solved using computational devices and how efficiently those problems can be solved. To be able to make precise statements and rigorous arguments, computational devices are modeled using abstract mathematical "models of computation." The learning objectives of the course are to:

- Foremost, understand how to rigorously reason about computation through the use of abstract, formal models.
- Learn the definitions of several specific models of computation including finite automata, context-free grammars, and Turing machines, learn tools for analyzing their power and limitations, and understand how they are used in other areas of computer science.
- Learn how fundamental philosophical questions about the nature of computation (Are there problems which cannot be solved by computers? Can every problem for which we can quickly verify a solution also be solved efficiently?) can be formalized as precise mathematical problems.
- Gain experience with creative mathematical problem solving and develop the ability to write correct, clear, and concise mathematical proofs.

Instructor: [Mark Bun](mailto:mbun@bu.edu), mbun [at] bu [dot] edu
Instr. Office Hours: Mon 5:00-6:00 PM (MCS 114)
Fri 5:00-6:00 PM (MCS 114)

Teaching Fellow: [Islam Faisal](mailto:islam.faisal@bu.edu), islam [at] bu [dot] edu
TF Office Hours: TBD

Class Times: Tue, Thu 2:00-3:15 PM (EPC 209)
Discussion Sections: Wed 11:15-12:05 AM (MCS B31)
Wed 12:20-1:10 PM (KCB 104)

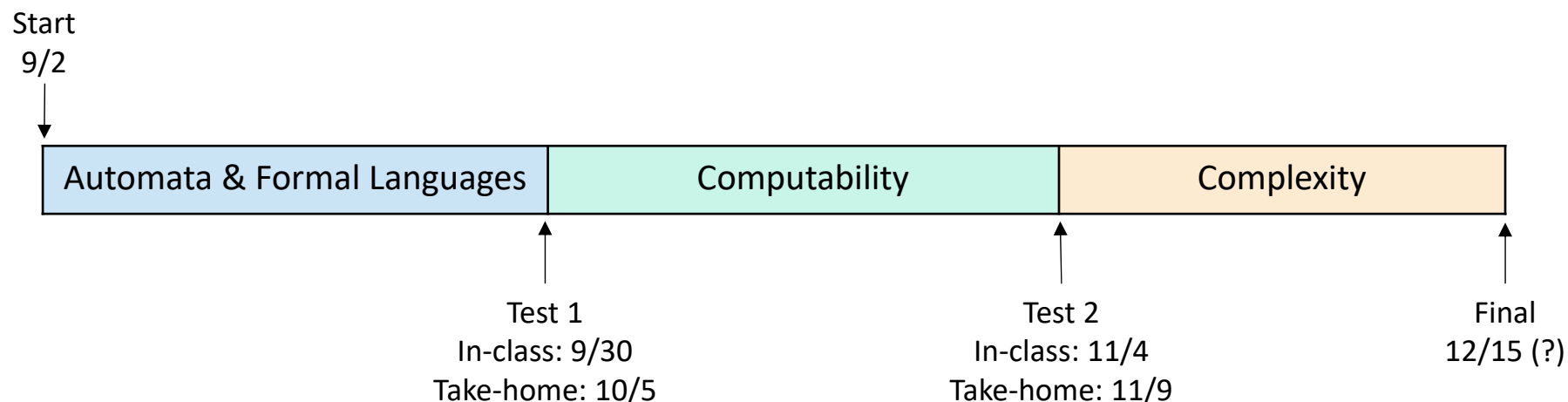
Important Links

Course Website: https://cs-people.bu.edu/mbun/courses/332_F21/. The website contains the course syllabus, schedule with assigned readings, homework assignments, and other course materials.

Piazza: <https://piazza.com/bu/fall2021/cs332>. All class announcements will be made through Piazza, so please set your notifications appropriately. Please post questions about the course material to Piazza. Instead of emailing the course staff directly, it is likely that other students will have the same questions as you and may be able to provide answers in a more timely fashion. Active participation on Piazza may add extra points to your participation grade.

Gradescope: <https://gradescope.com>. Sign up for a student account on Gradescope using your BU email address. The entry code for the course is 2RYZ3P. Homework assignments are to be submitted to Gradescope in PDF format.

Course Structure



Grading

- Homework (45%): Roughly 10 of these
- Tests – each half in-class, half take-home (42%):
 - Test 1 (12%)
 - Test 2 (12%)
 - Final (18%)
- Participation (13%): In-class polls, HW0, etc.

Homework Policies

- Weekly assignments due Tuesday @ 11:59PM
- No late days, no extensions
- Lowest homework score will be dropped
- Homework to be submitted via Gradescope
 - Entry code: 2RYZ3P
- You are encouraged to typeset your solutions in LaTeX (resources available on course webpage)
- HW0 out, due Tu 9/7 (just some housekeeping)
- HW1 to be released on Tu 9/7, due Tu 9/14

Homework Policies: Collaboration

- You are encouraged to work with your classmates to discuss homework problems
- **HOWEVER:**
 - You may collaborate with at most 3 other students
 - You must acknowledge your collaborators and write “Collaborators: none” if you worked alone
 - You must write your solutions by yourself
 - You **may not** share written solutions
 - You **may not** search for solutions using the web or other outside resources
 - You **may not** receive help from anyone outside the course (including students from previous years)

Collaboration Dilemma

<https://forms.gle/cnUbzqTNibyoqt8w9>



If you worked alone on a homework assignment...

- (a) You don't need to include a collaboration statement
- (b) You should invent a fake collaborator and acknowledge them appropriately
- (c) You should include the collaboration statement "Collaborators: none"
- (d) You should hurriedly call up three of your friends in the class at 11:55PM, briefly discuss the problems, and acknowledge them

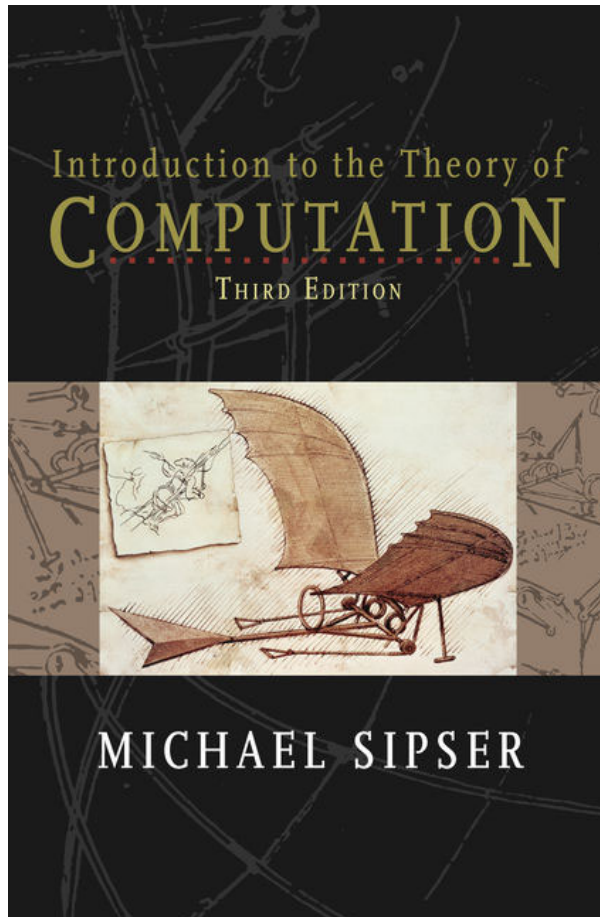
Homework Policies: Collaboration

Details of the collaboration policy may be found here:

https://cs-people.bu.edu/mbun/courses/332_F21/handouts/collaboration.pdf

Important: Sign this document to affirm you understand it, and turn it in via Gradescope by 11:59PM, Tu 9/7

Textbook



Introduction to the Theory of Computation
(Third Edition)
by Michael Sipser

- It's fine if you want to use an older edition, but section numbers may not be the same
- Other resources available on course webpage

Participation

- Your class participation score (13% of the course grade) will be determined by
 - Answering in-class polls on Google Forms
 - Handing in completed worksheets at the end of each discussion section
- You can also increase your participation score by participating thoughtfully in lecture, discussion, office hours, and on Piazza

Piazza

- We will use Piazza for announcements and discussions
 - Ask questions here and help your classmates
 - Please use private messages / email sparingly

<https://piazza.com/bu/fall2021/cs332>



Expectations and Advice for Succeeding in CS 332

Our (the Course Staff's) Responsibilities

- Guide you through difficult parts of the material in lecture
- Encourage active participation in lectures / section
- Assign practice problems and homework that will give you a deep understanding of the material
- Give detailed (formative) feedback on assignments
- Be available outside of class (office hours, Piazza)
- Regularly solicit feedback to improve the course

Your Responsibilities

- Concepts in this course take some time to sink in. Keep at it, and be careful not to fall behind.
- Do the assigned reading on each topic **before** the corresponding lecture.
- Take advantage of office hours.
- Participate actively in lectures/sections and on Piazza.
- Allocate lots of time for the course: comparable to a project-based course, but spread more evenly.

Prerequisites

This class is fast-paced and assumes experience with mathematical reasoning and algorithmic thinking

You must have passed CS 330 – Intro to Algorithms

This means you should be comfortable with:

- Set theory
- Functions and relations
- Graphs
- Pigeonhole principle
- Propositional logic
- Asymptotic notation
- Graph algorithms (BFS, DFS)
- Dynamic programming
- NP-completeness

Come talk to me if you have questions about your preparation for the course

Advice on Homework

- Start working on homework early! You can get started as soon as it's assigned.
- Spread your homework time over multiple days.
- You may work in groups (of up to 4 people), but think about each problem for at least 30 minutes before your group meeting.
- To learn problem solving, you have to do it: ✓
 - Try to think about how you would solve **any** presented problem before you read/hear the answer
 - Do exercises in the textbook in addition to assigned homework problems

Advice on Reading

- Not like reading a novel
- The goal is not to find out the answers, but to learn and understand the techniques
- Always try to predict what's coming next
- Always think about how you would approach a problem before reading the solution
- This applies to things that are not explicitly labeled as exercises or problems!

Academic Integrity

Extremely important: Read and understand the Collaboration and Honesty policy before you sign it

Violations of the collaboration policy...will result in an automatic failing grade and will be reported to the Academic Conduct Committee (ACC). The ACC often suspends or expels students deemed guilty of plagiarism or other forms of cheating.

If you find yourself in a desperate situation:

- Hand in as much of the assignment as you're able to complete
- Remember the lowest HW grade is dropped
- Talk to us! We want to help

...cheating is seriously not worth it

Course Overview

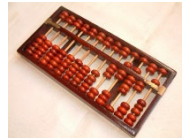
Objective

Build a *theory* out of the idea of *computation*

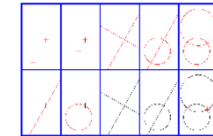
What is “computation”

- Examples:

- Paper + pencil arithmetic
- Abacus
- Mechanical calculator
- Ruler and compass geometry constructions
- Java/C programs on a digital computer



$$\begin{array}{r} \overset{1}{2} 3 7 \\ + 3 7 1 \\ \hline 6 0 8 \end{array}$$



- **For us:** Computation is the processing of information by the unlimited application of a finite set of operations or rules

Other examples of computation?

Finding averages (descriptive statistics)

Computation in the brain

Search, retrieval

Sorting

"self-driving" cars

Internet / distributed systems

Equilibria in games/markets

DNA / protein folding

Crypto / security protocols

Quantum computation

Slime molds

What do we want in a “theory”?

- General ideas that apply to many different systems
- Expressed simply, abstractly, and precisely
- **Generality**
 - Independence from Technology: Applies to the future as well as the present
 - Abstraction: Suppresses inessential details
- **Precision:** Can prove formal mathematical theorems
 - Positive results (what *can* be computed): correctness of algorithms and system designs
 - Negative results (what *cannot* be computed): proof that there is no algorithm to solve some problem in some setting (with certain cost)

Parts of a Theory of Computation

- Models for **machines** (computational devices)
- Models for the **problems** machines can be used to solve
- **Theorems** about what kinds of machines can solve what kinds of problems, and at what cost

This course: Sequential, single-processor computing

Not covered:

- Parallel machines
- Real-time systems
- Distributed systems
- Mobile computing
- Quantum computation
- Embedded systems

What is a (Computational) Problem?

A single question with infinitely many instances

Examples:

abba *aaabba*

Parity: Given a string consisting of a 's and b 's, does it contain an even number of a 's?

Primality: Given a natural number x (represented in binary), is x prime?

Halting Problem: Given a C program, can it ever get stuck in an infinite loop?

For us: Focus on *decision* problems (yes/no answers) on *discrete* inputs

What is a (Computational) Problem?

For us: A problem will be the task of **recognizing whether a string is in a language**

Parity: $\Sigma = \{a, b\}$



Strings over $\{a, b\}$
 $\Sigma^* = \{a, b\}^*$
 $= \{\epsilon, a, b, aa, ab, \dots\}$

"Parity language" $L = \{ w \mid w \text{ has an even \# of } a\text{'s} \}$

What is a (Computational) Problem?

For us: A problem will be the task of **recognizing whether a string is in a language**

- **Alphabet:** A finite set Σ

Ex. $\Sigma = \{a, b, \dots, z\}$

Ex: $\Sigma = \{a, b\}$

Ex: $\Sigma = \{0, 1\}$

- **String:** A finite concatenation of alphabet symbols

Ex. $bqr, ababb$

ε denotes empty string, length 0

Σ^* = set of all strings using symbols from Σ

Ex: $\Sigma = \{a, b\}$
 $\Sigma^* = \{\varepsilon, a, b, aa, \dots, ab, ba, bb, \dots\}$

- **Language:** A set L of strings

$L \subseteq \Sigma^*$

Examples of Languages

Parity: Given a string consisting of a 's and b 's, does it contain an even number of a 's?

$$\Sigma = \{a, b\} \quad L = \{w \mid w \text{ has an even \# of } a\text{'s}\}$$

Primality: Given a natural number x (represented in binary), is x prime?

$$\Sigma = \{0, 1\} \quad L = \{x \mid \text{natural number corresponding to } x \text{ is prime}\}$$

$\Sigma^* = \{\text{binary strings}\}$

Halting Problem: Given a C program, can it ever get stuck in an infinite loop?

$$\Sigma = \text{ASCII (?) } \quad L = \{w \mid \text{C program } w \text{ gets stuck in a loop (on some input)}\}$$

Primality language

<https://forms.gle/cnUbzqTNibyoqt8w9>



Which best represents the language corresponding to the Primality problem? (Strings over the alphabet $\{0, 1\}$ that are binary representations of prime numbers.) Let's say the most significant bit is on the left, so "100" is the binary representation of 4.

(a) $\{2, 3, 5, 7, \dots\}$

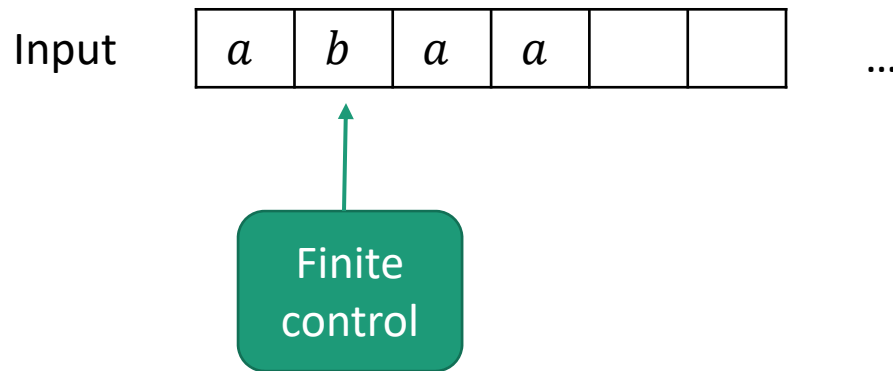
(b) $\{11, 111, 11111, 1111111, \dots\}$

(c) $\{10, 11, 101, 111, \dots\}$

(d) $\{11, 011, 101, 110, 111, 0111, \dots\}$

Machine Models

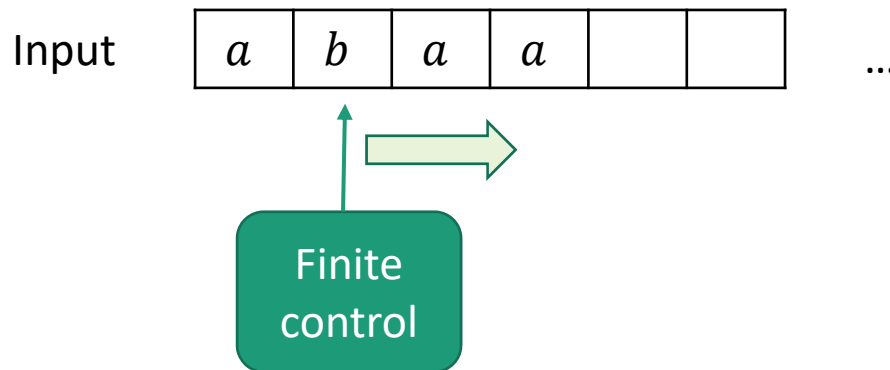
Computation is the processing of information by the **unlimited application** of a **finite set** of operations or rules



Abstraction: We don't care how the control is implemented. We just require it to have a finite number of states, and to transition between states using fixed rules.

Machine Models

- Finite Automata (FAs): Machine with a finite amount of unstructured memory

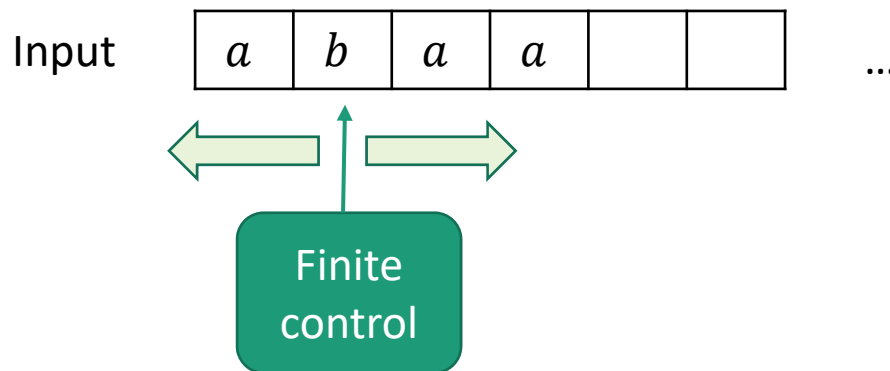


Control scans left-to-right
Can check simple patterns
Can't perform unlimited counting

Useful for modeling chips, simple control systems, choose-your-own adventure games...

Machine Models

- Turing Machines (TMs): Machine with unbounded, unstructured memory



Control can scan in both directions
Control can both read and write

Model for general sequential computation

Church-Turing Thesis: Everything we intuitively think of as “computable” is computable by a Turing Machine

What theorems would we like to prove?

We will define classes of languages based on which machines can recognize them

Inclusion: Every language recognizable by a FA is also recognizable by a TM

Non-inclusion: There exist languages recognizable by TMs which are not recognizable by FAs

Completeness: Identify a “hardest” language in a class

Robustness: Alternative definitions of the same class

Ex. Languages recognizable by FAs = regular expressions