BU CS 332 – Theory of Computation

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Lecture 11:

- More TM Variants and Closure Properties
- Church-Turing Thesis

Reading:

Sipser Ch 3.2

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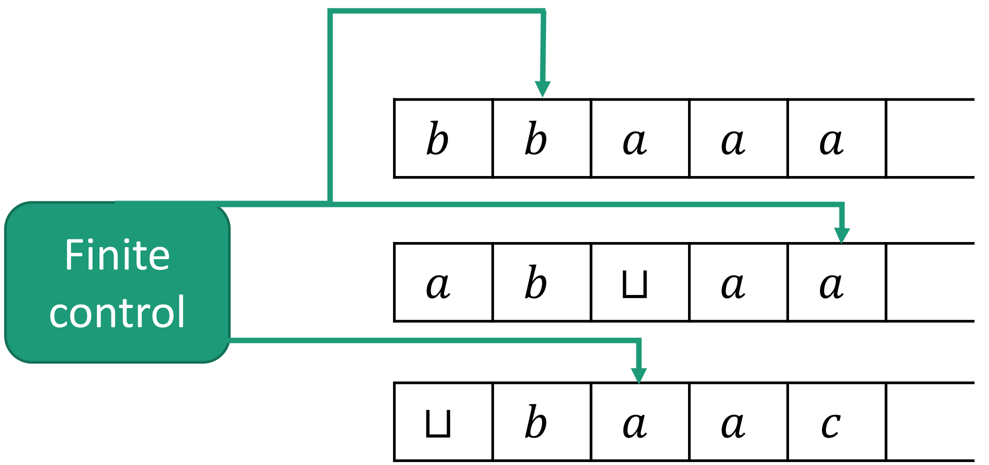
TM Variants

TMs are equivalent to…

- TMs with “stay put”
- TMs with 2-way infinite tapes
- Multi-tape TMs
- Nondeterministic TMs
- Random access TMs
- Enumerators
- Finite automata with access to an unbounded queue
- Primitive recursive functions
- Cellular automata

…

Multi-Tape TMs



Fixed number of tapes k

(k can't depend on input or change during computation)

Transition function $\delta : Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, R, S\}^k$

How to Simulate It

To show that a TM variant is no more powerful than the basic, single-tape TM:

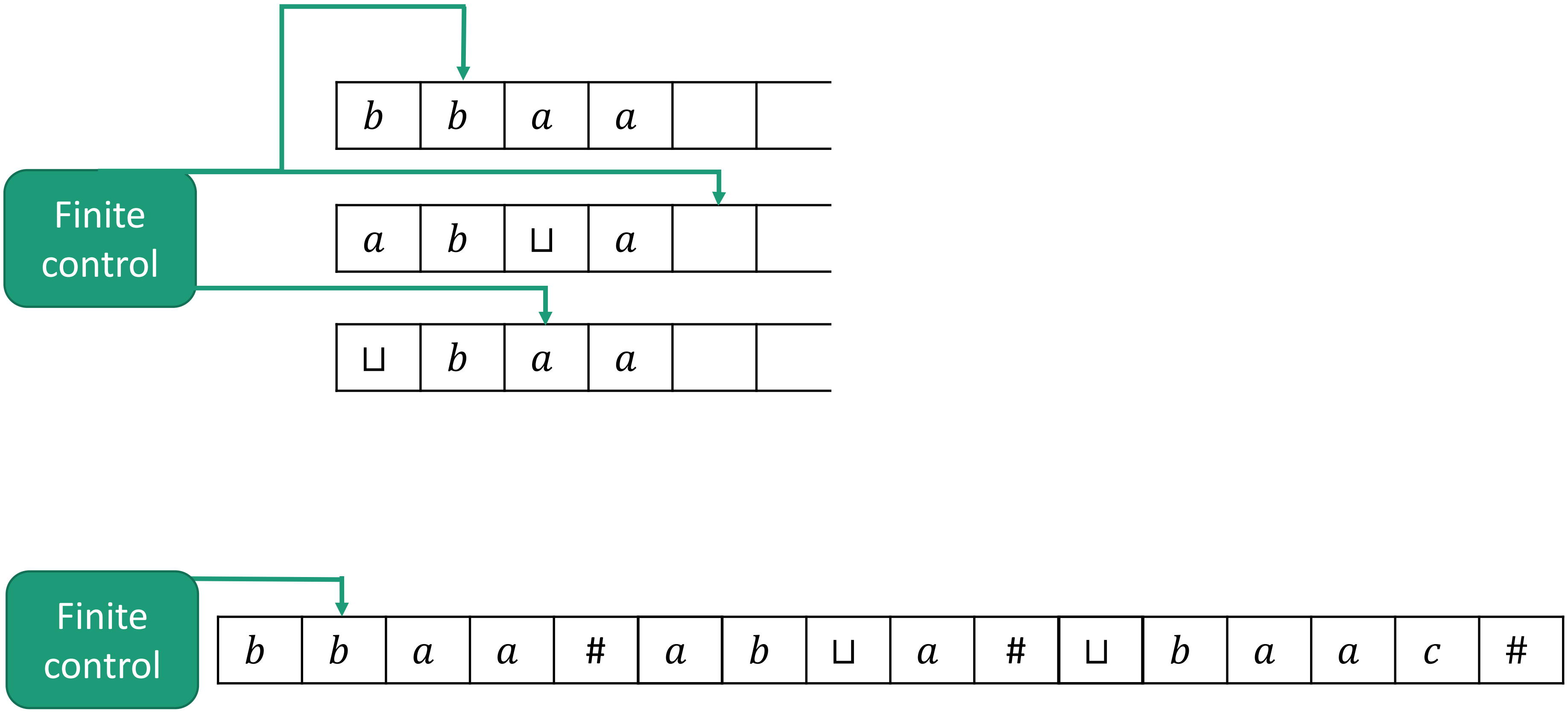
Show that if M is any variant machine, there exists a basic, single-tape TM M' that can simulate M

(Usual) parts of the simulation:

- Describe how to initialize the tapes of M' based on the input to M
- Describe how to simulate one step of M's computation using (possibly many steps of) M'

Multi-Tape TMs are Equivalent to Single-Tape TMs

Theorem: Every k-tape TM M with can be simulated by an equivalent single-tape TM M'



Simulating Multiple Tapes

Implementation-Level Description of M'

On input $w = w_1 w_2 \ldots w_n$

1. Format tape into $\# \dot{w}_1 w_2 \ldots w_n \# \dot{\sqcup} \# \dot{\sqcup} \# \ldots \#$
2. For each move of M:

   Scan left-to-right, finding current symbols

   Scan left-to-right, writing new symbols,

   Scan left-to-right, moving each tape head

   If a tape head goes off the right end, insert blank

   If a tape head goes off left end, move back right

Why are Multi-Tape TMs Helpful?

To show a language is Turing-recognizable or decidable, it's enough to construct a multi-tape TM

Often easier to construct multi-tape TMs

Ex. Decider for $\{a^i b^j \mid i > j\}$

On input w:

1) Scan tape 1 left-to-right to check that $w \in L(a^* b^*)$
2) Scan tape 2 left-to-right to copy all b's to tape 2
3) Starting from left ends of tapes 1 and 2, scan both tapes to check that every b on tape 2 has an accompanying a on tape 1. If not, reject.
4) Check that the first blank on tape 2 has an accompanying a on tape 1. If so, accept; otherwise, reject.

Why are Multi-Tape TMs Helpful?

To show a language is Turing-recognizable or decidable, it's enough to construct a multi-tape TM

Very helpful for proving closure properties

Ex. Closure of recognizable languages under union. Suppose M_1 is a single-tape TM recognizing L_1, M_2 is a single-tape TM recognizing L_2

On input w:

1) Scan tapes 1, 2, and 3 left-to-right to copy w to tapes 2 and 3
2) Repeat forever:
    a) Run M_1 for one step on tape 2
    b) Run M_2 for one step on tape 3
    c) If either machine accepts, accept

Closure Properties

The Turing-decidable languages are closed under:

- Union
- Concatenation
- Star
- Intersection
- Reverse
- Complement

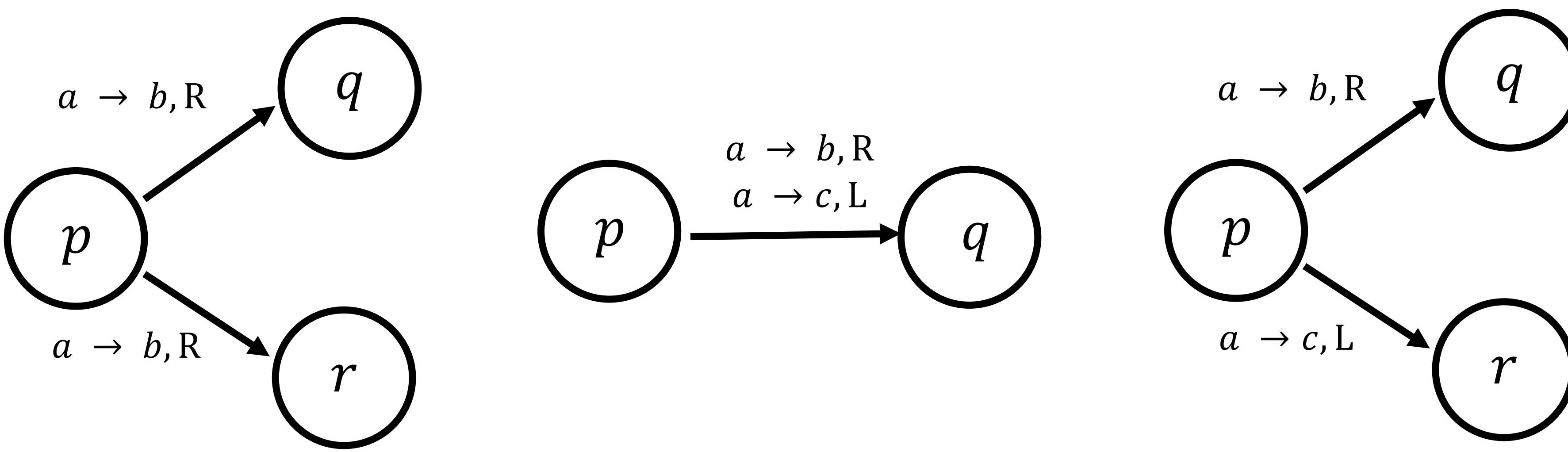
The Turing-recognizable languages are closed under:

- Union
- Concatenation
- Star
- Intersection
- Reverse

Nondeterministic TMs

At any point in computation, may nondeterministically branch. Accepts iff there exists an accepting branch.

Transition function $\delta : Q \times \Gamma \rightarrow P(Q \times \Gamma \times \{\mathrm{L}, \mathrm{R}, \mathrm{S}\})$



Nondeterministic TMs

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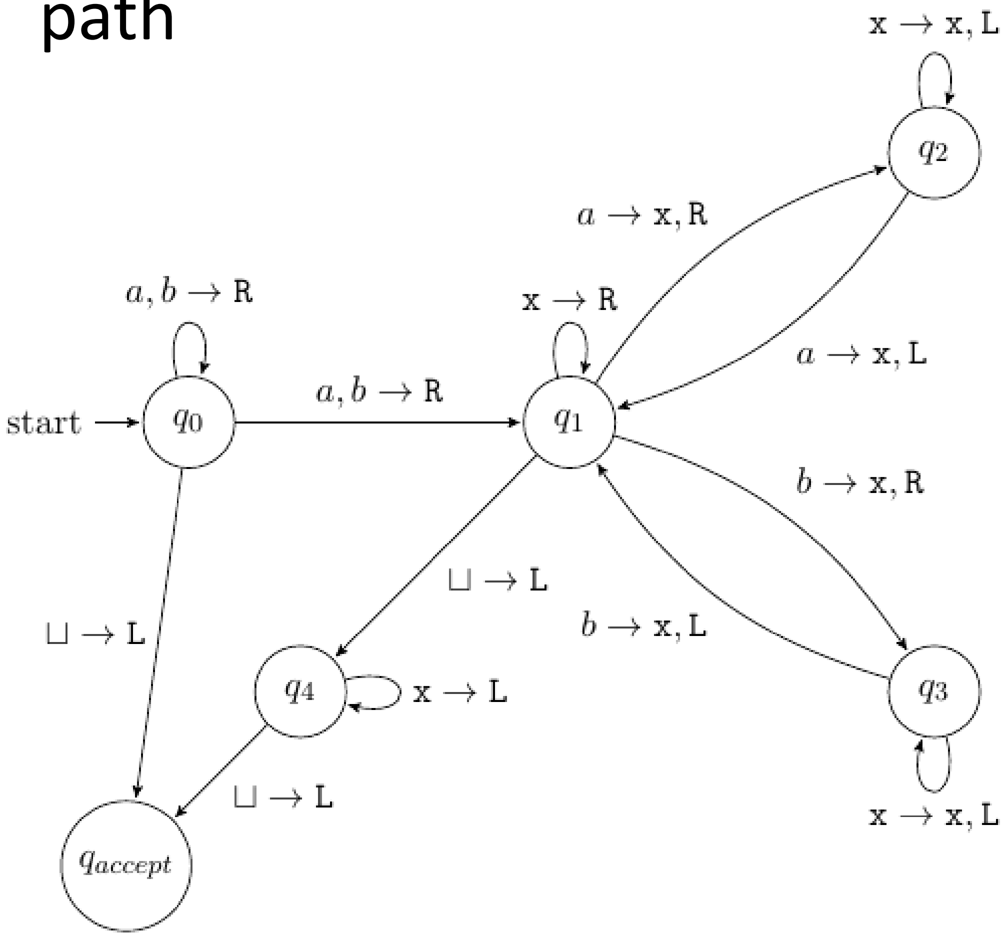

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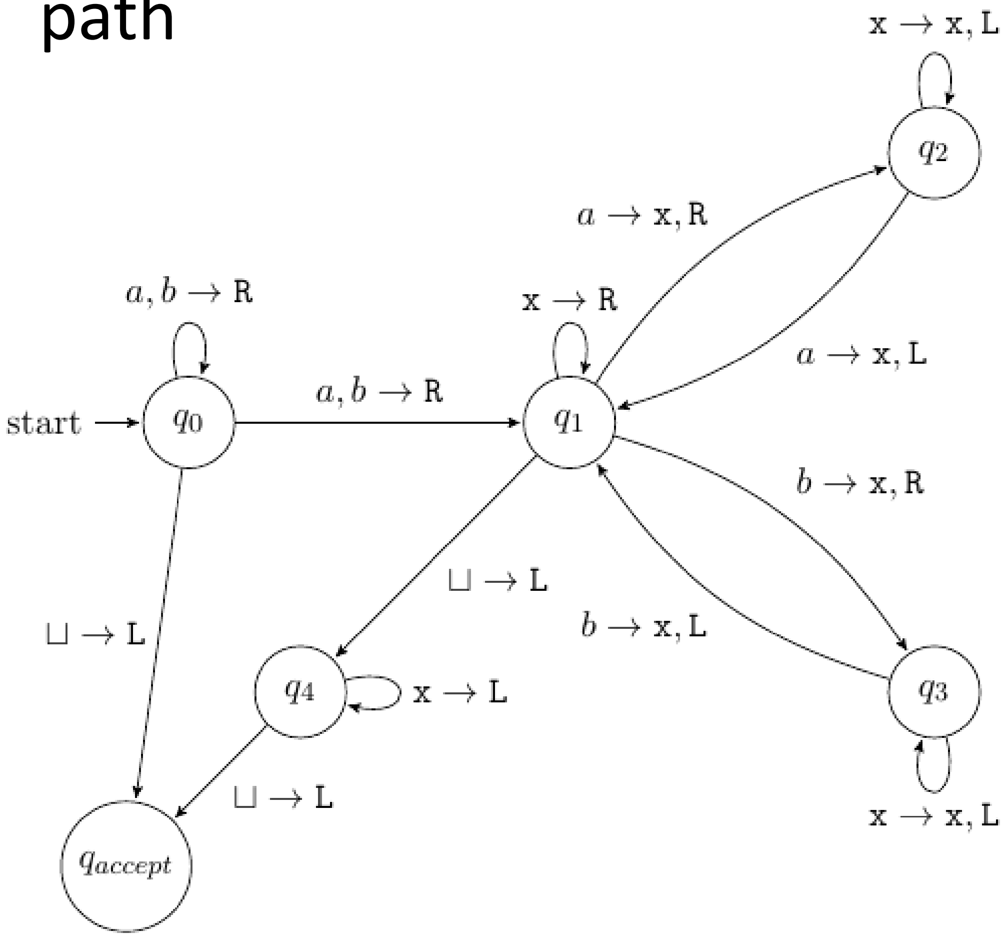
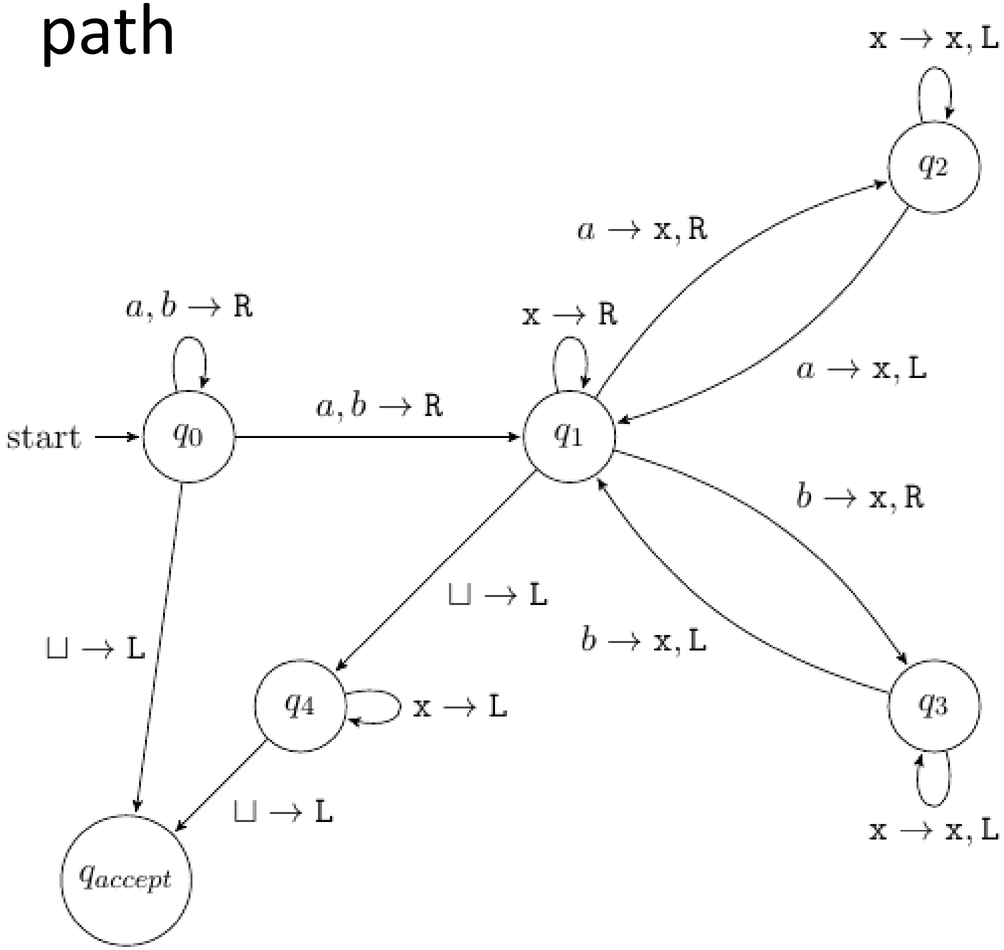


Implementation-Level Description:

Nondeterministic TMs

At any point in computation, may nondeterministically branch. Accepts iff there exists an accepting computation path



What is the language recognized by this NTM?

a) $\{ ww \mid w \in \{a, b\}^* \}$
b) $\{ ww^R \mid w \in \{a, b\}^* \}$
c) $\{ ww \mid w \in \{a, b, \mathrm{x}\}^* \}$
d) $\{ w\mathrm{x}^n w^R \mid w \in \{a, b\}^*, n \geq 0 \}$

Nondeterministic TMs

Ex. Given TMs M_1 and M_2, construct an NTM recognizing $L(M_1) \cup L(M_2)$

Nondeterministic TMs

Ex. NTM for $L = \{w \mid w$ is a binary number representing the product of two integers $a, b \geq 2\}$

<u>High-Level Description:</u>

Nondeterministic TMs

An NTM N accepts input w if when run on w it accepts on at least one computational branch

$$L(N) = \{w \mid N \text{ accepts input } w\}$$

An NTM N is a decider if on **every** input, it halts on **every** computational branch

Nondeterministic TMs

Theorem: Every nondeterministic TM can be simulated by an equivalent deterministic TM

Proof idea: Explore “tree of possible computations”

Simulating NTMs

Which of the following algorithms is always appropriate for searching the tree of possible computations for an accepting configuration?

a) Depth-first search: Explore as far as possible down each branch before backtracking

b) Breadth-first search: Explore all configurations at depth 1, then all configurations at depth 2, etc.

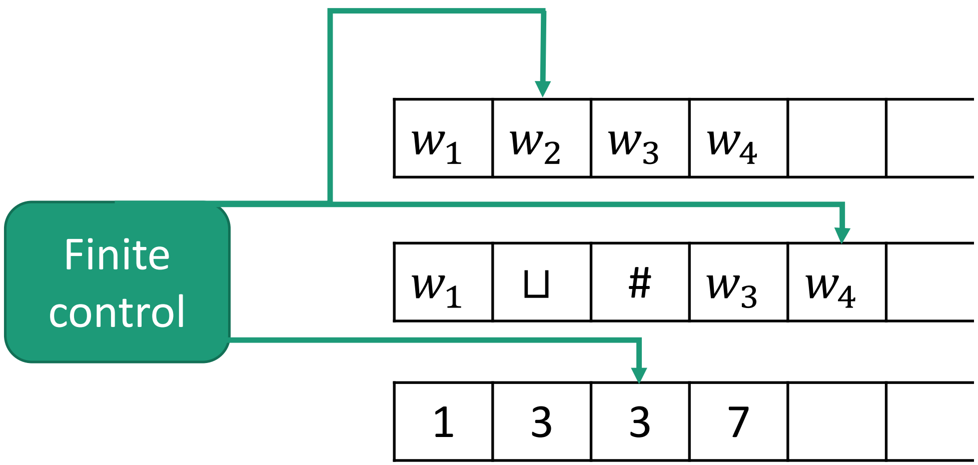
c) Both algorithms will always work

Nondeterministic TMs

Theorem: Every nondeterministic TM has an equivalent deterministic TM

Proof idea: Simulate an NTM N using a 3-tape TM

(See Sipser for full description)



Input w to N (read-only)

Simulation tape (run N on w using nondeterministic choices from tape 3)

Address in computation tree

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Church-Turing Thesis

The equivalence of these models is a mathematical theorem (you can prove that each can simulate another)

Church-Turing Thesis v1: The basic TM (hence all of these models) captures our intuitive notion of algorithms

Church-Turing Thesis v2: Any physically realizable model of computation can be simulated by the basic TM

The Church-Turing Thesis is not a mathematical statement! Can't be mathematically proved