Homework 9 – Due Monday, April 27, 2020 <u>before</u> 2:00PM

Reminder Collaboration is permitted, but you must write the solutions *by yourself without assistance*, and be ready to explain them orally to the course staff if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

Exercises Please practice on the following exercises and solved problems in Chapter 7 and Chapter 8: 7.23, 7.33, 8.1, 8.4, 8.6. The material they cover may appear on exams.

Problems There are 5 mandatory problems.

1. (**Hamiltonian Path**) Read the reduction from *3SAT* to *HAMPATH* on page 314 of Sipser.

    (a) Is this construction also a valid polynomial-time reduction from *2SAT* to *HAMPATH*?

    (b) Draw the graph G that the reduction outputs on input formula $\phi = (\bar{x} \vee y) \wedge (x \vee \bar{y})$. For both satisfying assignments of ϕ, give a corresponding Hamiltonian path in G.

    (c) Draw the graph G that the reduction outputs on input formula $\phi = (x \vee y) \wedge (\bar{x} \vee \bar{y}) \wedge (\bar{x} \vee y) \wedge (x \vee \bar{y})$. Argue that G does not have a Hamiltonian path (not relying on the fact that we already proved that the reduction is correct).

    (d) Why would a polynomial-time reduction from *HAMPATH* to *2SAT* have surprising implications, but a reduction in the other direction does not? Hint: You may use the result of Problem 7.51(b) in Sipser, without proof, to answer this question.

2. (**Inheritance**) Alice and Bob are siblings who have inherited a collection of marbles from an eccentric relative. There are k distinct kinds of marbles, but the collection may contain several of each kind of marble. The collection is partitioned into m boxes. Each box contains two bags of marbles such that within each bag, no marbles are duplicated. The will stipulates that Alice and Bob divide the collection as follows: For every box of marbles, Alice is to receive one bag from that box, and Bob is to receive the other bag. Their goal is to determine whether there is a way to partition the collection in this way such that both Alice and Bob obtain a complete set of all k kinds of marbles.

    For example, suppose $k = 4$ and the kinds of marbles are labeled $1, 2, 3, 4$. Suppose there are $m = 3$ boxes of marbles $B_1 = (\{1, 2\}, \{1\}), B_2 = (\{2\}, \{1, 4\}), B_3 = (\{3, 4\}, \{3\})$. Then if Alice takes the first bag from box B_1, the second bag from box B_2, and the second bag from box B_3 (and Bob takes the second bag from B_1, the first bag from B_2 and the first bag from B_3), then both Alice and Bob receive complete sets of marbles.

    Consider the problem $MARBLES = \{\langle k, B_1, \ldots, B_m \rangle \mid$ there exists a partition of the collection such that Alice and Bob both receive complete sets of marbles$\}$. Show that this problem is NP-complete.

3. (**EQ$_{\mathsf{NFA}}$**) Recall that EQ$_{\mathsf{NFA}} = \{\langle N_1, N_2 \rangle \mid N_1$ and N_2 are NFAs and $L(N_1) = L(N_2)\}$. Show that EQ$_{\mathsf{NFA}}$ is in PSPACE.

4. A *hypercube walk* is a sequence of strings $s_1, s_2, \ldots, s_k \in \{0,1\}^n$ such that each string differs from the previous string in exactly one bit. For example, the following is a hypercube walk starting with "0011" and ending with "1100" where every intermediate string is a member of the regular language $\{s \in \{0,1\}^4 \mid s$ contains at least one 0 and at least one 1$\}$:
0011, 0111, 0110, 1110, 1100.
Let $WALK_{\mathsf{DFA}} = \{\langle M, s, t \rangle \mid M$ is a DFA and $L(M)$ contains a hypercube walk of strings, starting with s and ending with $t\}$. Show that $WALK_{\mathsf{DFA}}$ is in PSPACE.

5. For each of the following, give a language B (if it exists) satisfying the stated property. Explain why your language B satisfies the given property, or explain why no such language B can exist.

   (a) $B \leq_{\mathrm{P}} SAT$ and B is not NP-complete.

   (b) $SAT \leq_{\mathrm{P}} B$ and B is not NP-complete.

   (c) $SAT \leq_{\mathrm{P}} B$ and B is not NP-hard.

   (d) B is regular and NP-complete.