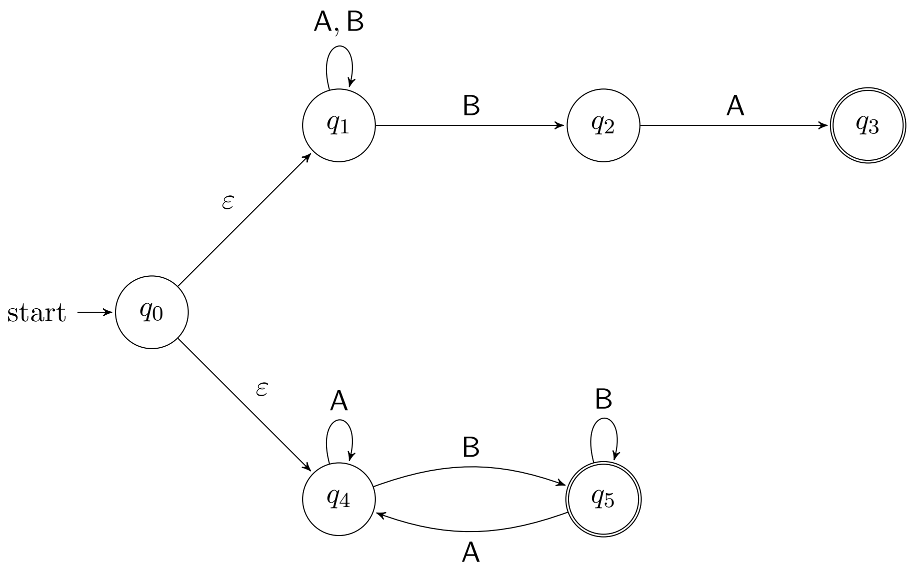
Homework 2 – Due Thursday, February 11, 2021 at 11:59 PM

Reminder Collaboration is permitted, but you must write the solutions *by yourself without assistance*, and be ready to explain them orally to the course staff if asked. You must also identify your collaborators and write "Collaborators: none" if you worked by yourself. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

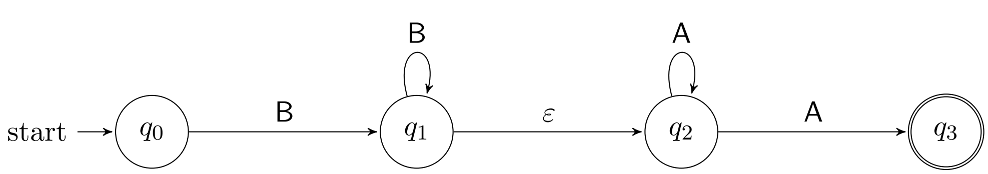
Problems There are 5 required problems and two bonus problems.

1. (The problem formerly known as HW1 Problem 5.) Consider the following state diagram of an NFA N over alphabet $\{\mathsf{A}, \mathsf{B}\}$.



   (a) Give a formal description of the machine N as a 5-tuple.
   (b) Does the machine accept the string AAB?
   (c) Does the machine accept the string BBBA?
   (d) Does the machine accept the string AAA?
   (e) What is the language recognized by N?

2. Consider the following state diagram of an NFA N over alphabet $\{\mathsf{A}, \mathsf{B}\}$.



   (a) Consider running N on input BBA. Give examples (i) of a computation path that leads N to an accept state when run on this input, (ii) a computation path that leads N to a reject state, and (iii) a computation path that leads N to fail before it's read the entire input.

(b) What is the language recognized by N?

(c) Use the subset construction to convert N into a DFA recognizing the same language. Give the state diagram of this DFA – only include states that are reachable from the start state.

3. *Think about, but do not hand in:* A DFA or NFA can, in general, have zero, one, or many accept states. Show that every NFA can be converted into another NFA recognizing the same language, but which has exactly one accept state. (This is solved exercise 1.11 in Sipser if you'd like to check your solution.)

   To hand in: Prove that this is not true for DFAs. That is, show that there is a regular language L such that every DFA recognizing L requires at least two accept states. *Hint:* If L contains the empty string, what can you say about the start state of any DFA recognizing L?

4. On Monday, we'll show that the class of regular languages is closed under the star operation. This problem will help you investigate this property.

   (a) Let $A = \{w \in \{0,1\}^* \mid w \text{ ends with } 1\}$. Give the state diagram of a 2-state NFA N recognizing A.

   (b) Give a simple description of the language A^*.

   (c) Consider the following **failed** attempt to construct an NFA recognizing A^*: Add an ε transition from every accept state of N to the start state, and make the start state an accept state. Draw the state diagram of this NFA, and call it N'.

   (d) What is $L(N')$? Give an example of a string w such that $w \in L(N')$, but $w \notin A^*$.

   (e) Give the state diagram of an NFA that *does* recognize A^*.

5. (a) Given languages A, B, define the language $MIX(A, B)$ by

   $$MIX(A,B) = \{x_1y_1x_2y_2 \dots x_ny_n \mid n \geq 0, x_i \in A, y_i \in B\}.$$

   Note that each x_i, y_i is a *string*. Show that the class of regular languages is closed under MIX. Hint: You don't need to construct an NFA recognizing $MIX(A,B)$ if you can find a way to express it in terms of other operations.

   (b) Given a language A over alphabet Σ, define the language $TAIL(A) = \{y \in \Sigma^* \mid xy \in A \text{ for some } x \in \Sigma^*\}$. Show that the regular languages are closed under $TAIL$.

Bonus Problems

1. In this problem, you will show that in the worst case, the subset construction uses a number of states that is optimal up to a factor of 2.

   (a) For a natural number k, let R_k be the language over alphabet $\{0,1\}$ consisting of strings w such that the kth symbol from the right of w is a 0. Show that R_k is recognized by an NFA with $k+1$ states.

   (b) Show that every DFA recognizing R_k requires at least 2^k states.

2. A coNFA is like an NFA, except it accepts an input w if and only if *every* possible state it could end up in when reading w is an accept state. (By contrast, an NFA accepts w iff *there exists* an accept state it could end up in when reading w.) Show that the class of languages recognized by coNFAs is exactly the regular languages.