BU CS 332 – Theory of Computation

Lecture 3:

- Deterministic Finite Automata
- Non-deterministic FAs

Reading:

Sipser Ch 1.1-1.2

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Last Time

- Parts of a theory of computation: Model for machines, model for problems, theorems relating machines and problems

- Strings: Finite concatenations of symbols
- Languages: Sets L of strings
- Computational (decision) problem: Given a string x, is it in the language L?

Deterministic Finite Automata

A (Real-Life?) Example

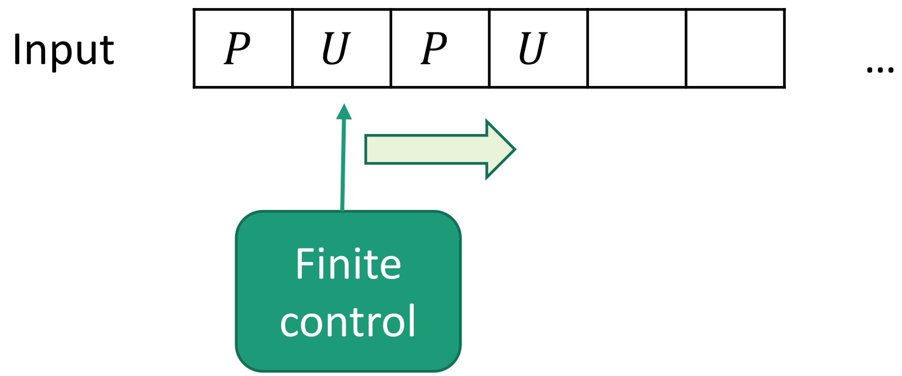
- Example: Kitchen scale
- P = Power button (ON / OFF)
- U = Units button (cycles through g / oz / lb)

  Only works when scale is ON, but units remembered when scale is OFF
- Starts OFF in g mode

- A computational problem: Does a sequence of button presses in $\{P, U\}^*$ leave the scale ON in oz mode?

Machine Models

- Finite Automata (FAs): Machine with a finite amount of unstructured memory



Control scans left-to-right

A DFA for the Kitchen Scale Problem

A DFA Recognizing Parity

The language recognized by a DFA is the set of inputs on which it ends in an "accept" state

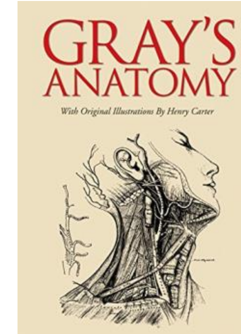
Parity: Given a string consisting of a's and b's, does it contain an even number of a's?

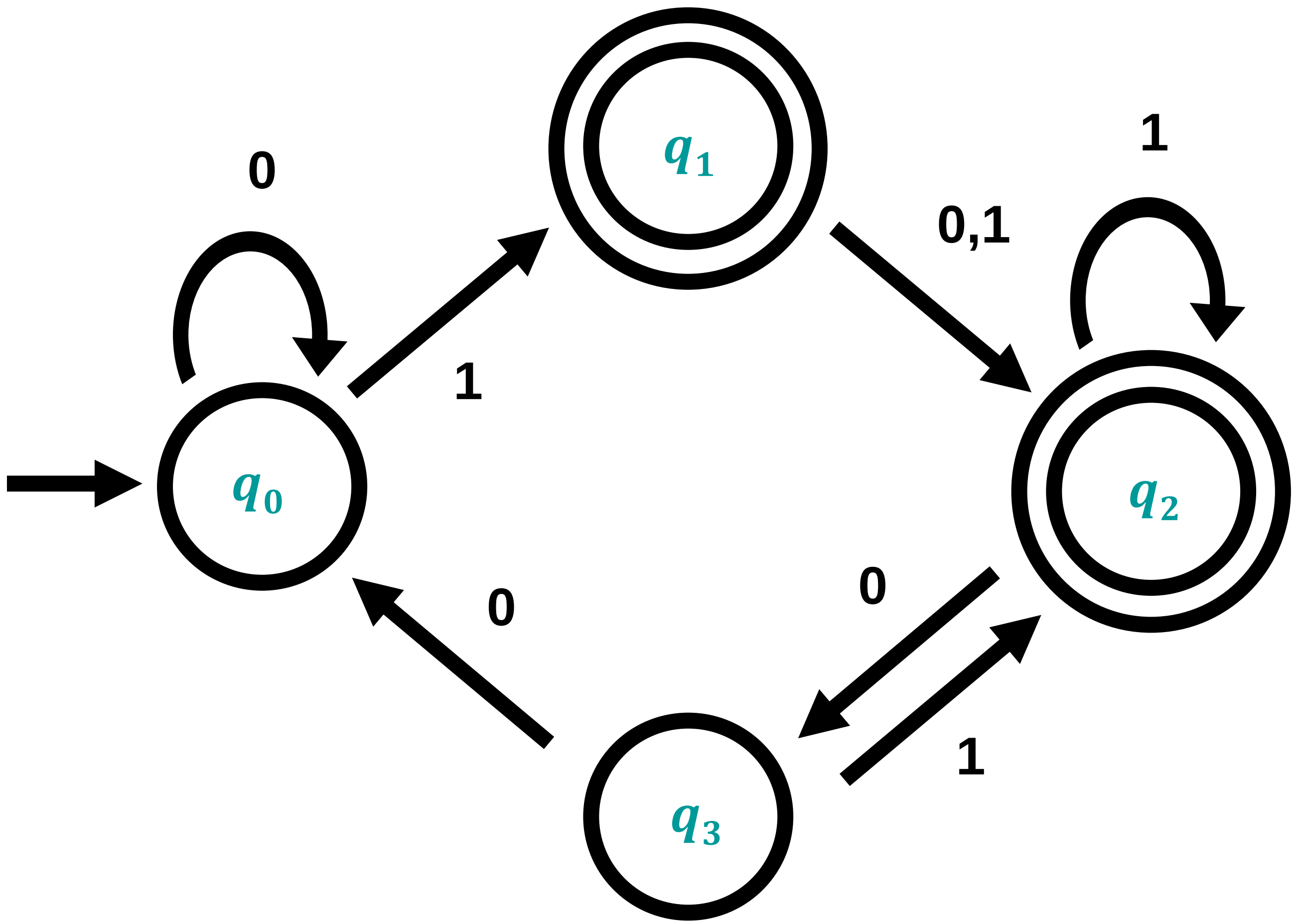
$\Sigma = \{a, b\}$ $\quad$ $L = \{w \mid w \text{ contains an even number of } a\text{'s}\}$

Which state is reached by the parity DFA on input aabab?

a) "even"
b) "odd"

Anatomy of a DFA





Some Tips for Thinking about DFAs

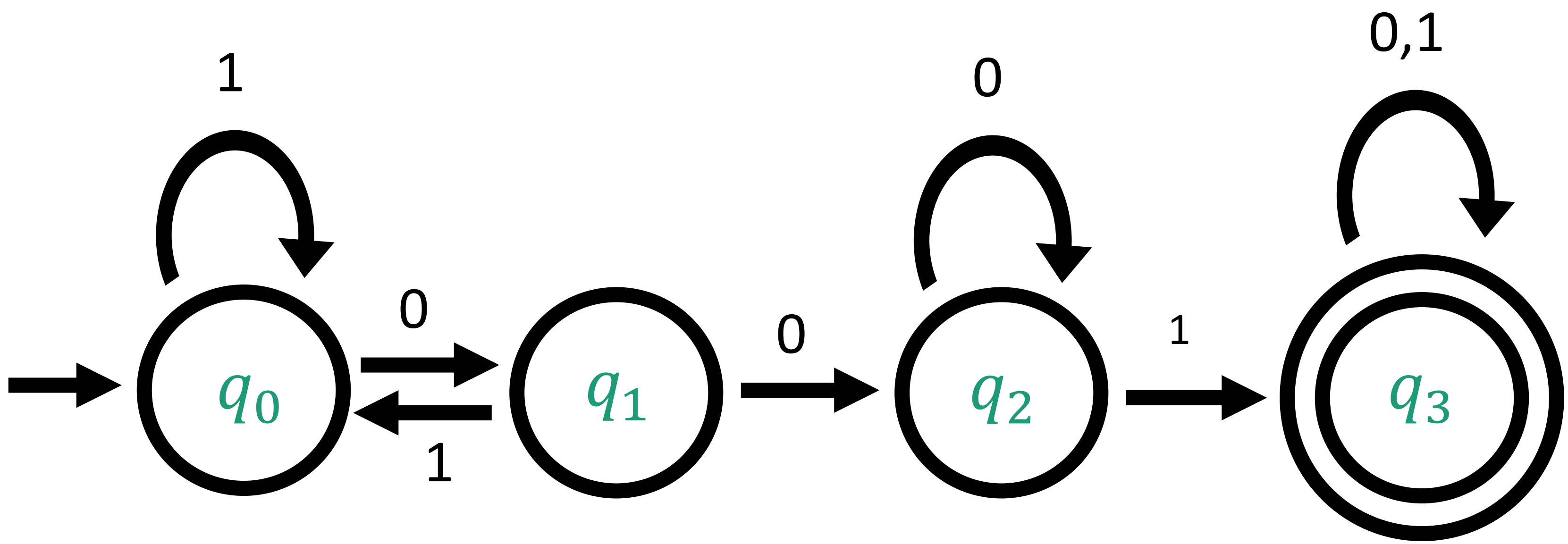
Given a DFA, what language does it recognize?

- Try experimenting with it on short strings. Do you notice any patterns?
- What kinds of inputs cause the DFA to get trapped in a state?

Given a language, construct a DFA recognizing it

- Imagine you are a machine, reading one symbol at a time, always prepared with an answer
- What is the essential information that you need to remember? Determines set of states.

What language does this DFA recognize?



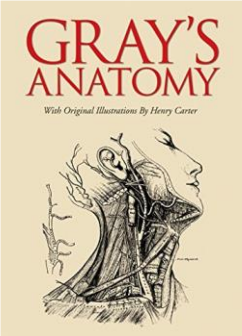
Practice!

- Lots of worked out examples in Sipser
- Tomorrow's discussion section
- Automata Tutor: https://automata-tutor.model.in.tum.de/

Formal Definition of a DFA

A finite automaton is a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$

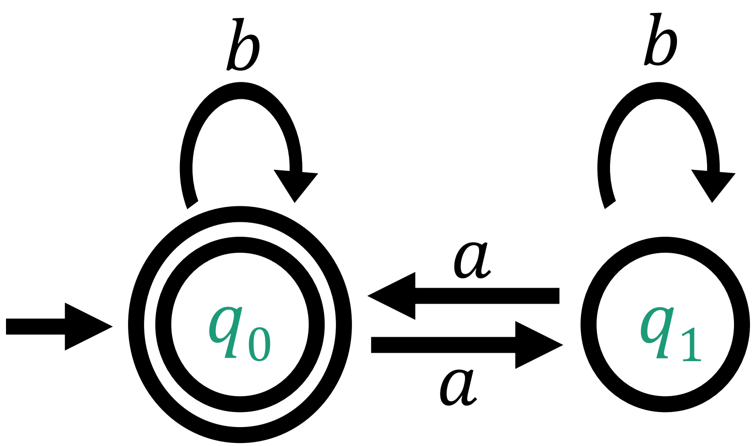
- Q is the set of states
- Σ is the alphabet
- $\delta: Q \times \Sigma \rightarrow Q$ is the transition function
- $q_0 \in Q$ is the start state
- $F \subseteq Q$ is the set of accept states



A DFA for Parity

Parity: Given a string consisting of a's and b's, does it contain an even number of a's?

$\Sigma = \{a, b\}$ $\quad$ $L = \{w \mid w$ contains an even number of a's$\}$



State set Q =

Alphabet Σ =

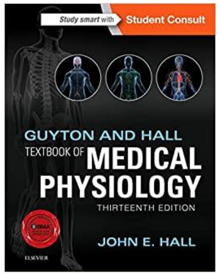
Transition function δ

δ	a	b
q_0		
q_1		

Start state q_0

Set of accept states F =

Formal Definition of DFA Computation



A DFA $M = (Q, \Sigma, \delta, q_0, F)$ accepts a string $w = w_1 w_2 \cdots w_n \in \Sigma^*$ (where each $w_i \in \Sigma$) if there exist $r_0, \ldots, r_n \in Q$ such that

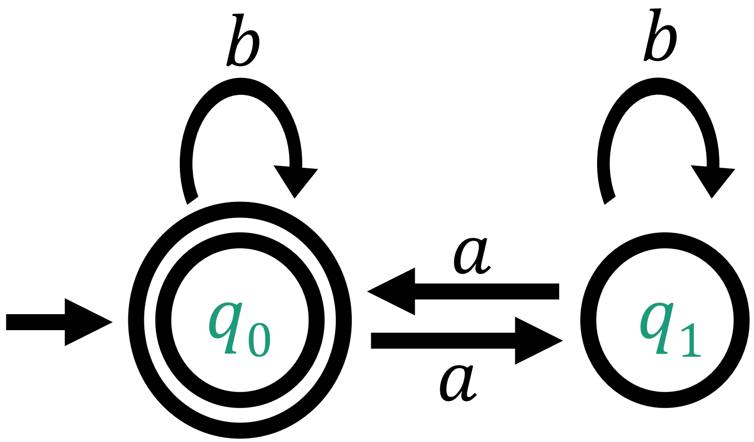
1. $r_0 = q_0$
2. $\delta(r_i, w_{i+1}) = r_{i+1}$ for each $i = 0, \ldots, n-1$, and
3. $r_n \in F$

$L(M)$ = the language of machine M

= set of all strings machine M accepts

M recognizes the language $L(M)$

Example: Computing with the Parity DFA



Let $w = abba$

Does M accept w?

What is $\delta(r_2, w_3)$?

a) q_0

b) q_1

A DFA $M = (Q, \Sigma, \delta, q_0, F)$ accepts a string $w = w_1 w_2 \cdots w_n \in \Sigma^*$ (where each $w_i \in \Sigma$) if there exist $r_0, \ldots, r_n \in Q$ such that

1. $r_0 = q_0$
2. $\delta(r_i, w_{i+1}) = r_{i+1}$ for each $i = 0, \ldots, n-1$, and
3. $r_n \in F$

Regular Languages

Definition: **A language is regular if it is recognized by a DFA**

$L = \{ w \in \{a, b\}^* \mid w \text{ has an even number of } a\text{'s} \}$ is regular

$L = \{ w \in \{0, 1\}^* \mid w \text{ contains } 001 \}$ is regular

Many interesting programs recognize regular languages

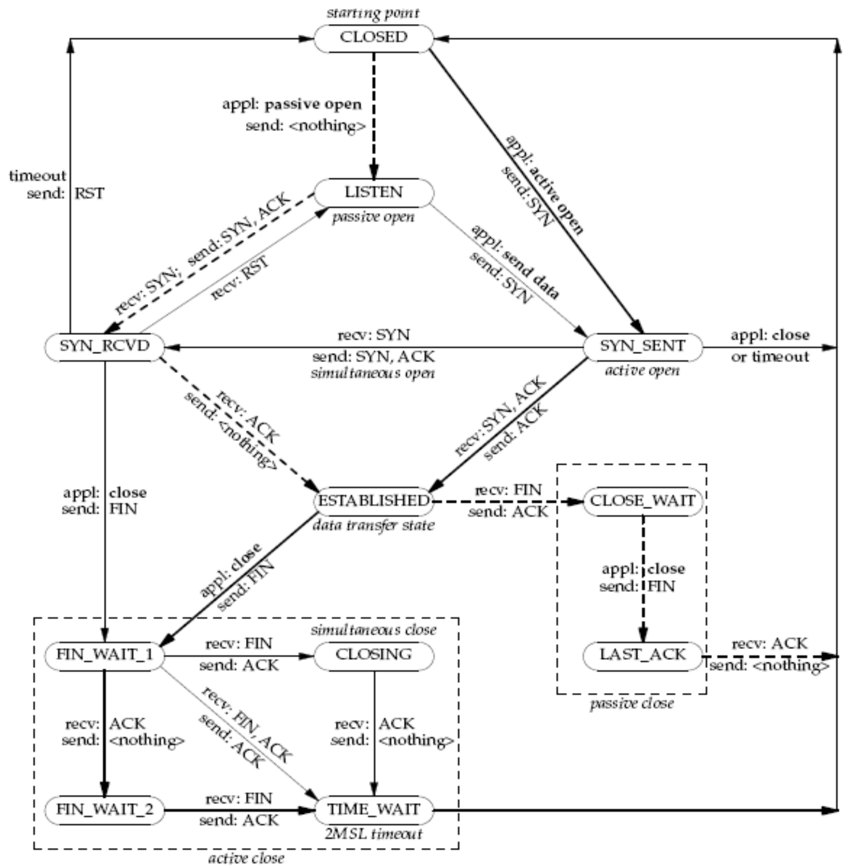
NETWORK PROTOCOLS

COMPILERS

GENETIC TESTING

ARITHMETIC

Internet Transmission Control Protocol



Let TCPS = { w | w is a complete TCP Session}

Theorem. TCPS is regular

Compilers

Comments :

Are delimited by /* */

Cannot have nested /* */

Must be closed by */

***/ is illegal outside a comment**

COMMENTS = {strings over {0,1, /, *} with legal comments}

Theorem. COMMENTS is regular.

Genetic Testing

DNA sequences are strings over the alphabet $\{A, C, G, T\}$.

A gene g is a special substring over this alphabet.

A genetic test searches a DNA sequence for a gene.

GENETICTEST$_g$ = {strings over $\{A, C, G, T\}$ containing g as a substring}

Theorem. GENETICTEST$_g$ is regular for every gene g.

Arithmetic

LET Σ = $\left\{ \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix} \right\}$

- **A string over Σ has three ROWS (ROW$_1$, ROW$_2$, ROW$_3$)**
- **Each ROW $b_0 b_1 b_2 \ldots b_N$ represents the integer**

$$b_0 + 2b_1 + \ldots + 2^N b_N.$$

- **Let ADD = {$S \in \Sigma^*$ | ROW$_1$ + ROW$_2$ = ROW$_3$ }**

Theorem. ADD is regular.

Nondeterministic Finite Automata

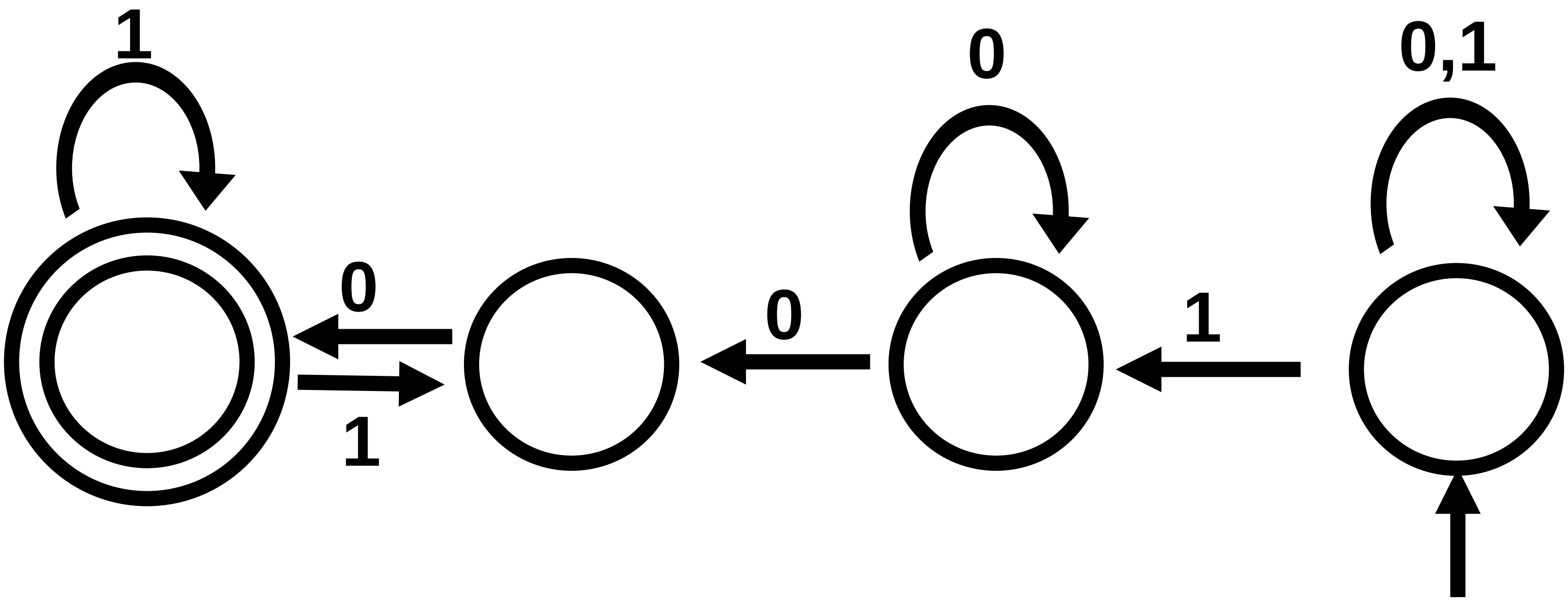
Nondeterminism

In a DFA, the machine is always in exactly one state upon reading each input symbol

In a nondeterministic FA, the machine can try out many different ways of reading the same string

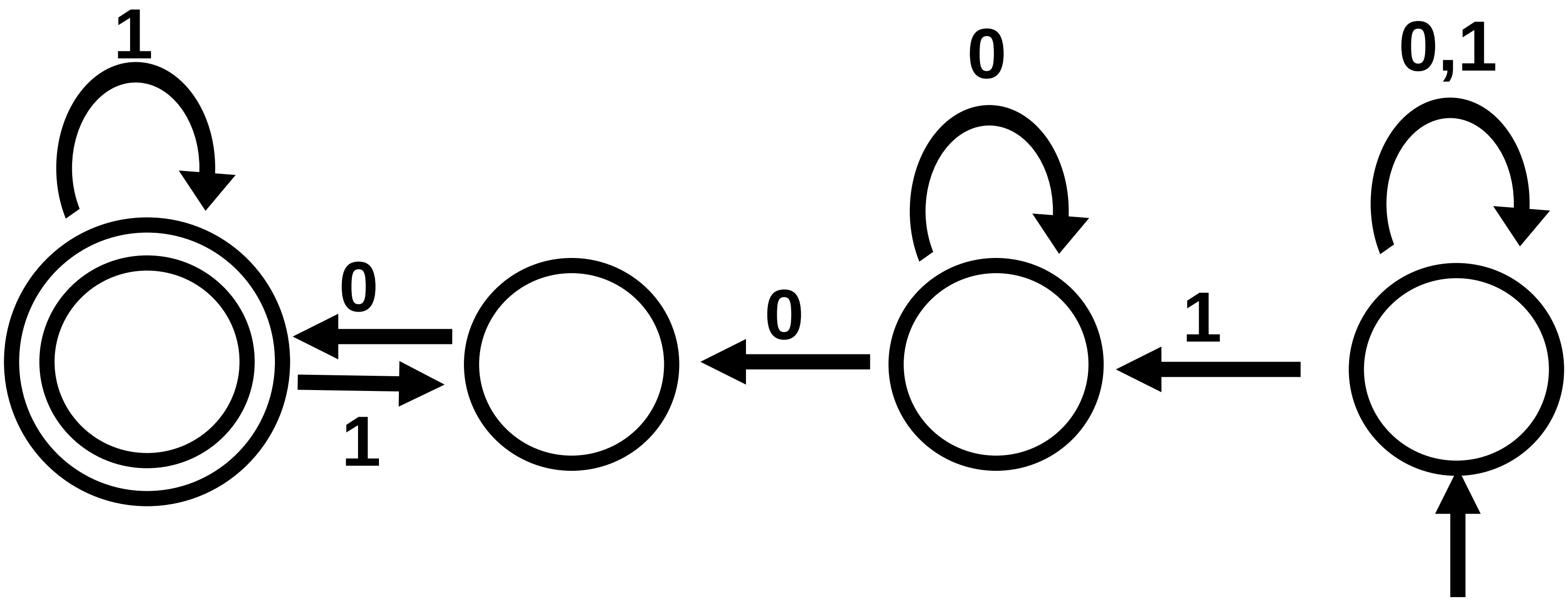
- Next symbol may cause an NFA to “branch” into multiple possible computations
- Next symbol may cause NFA’s computation to fail to enter any state at all

Nondeterminism



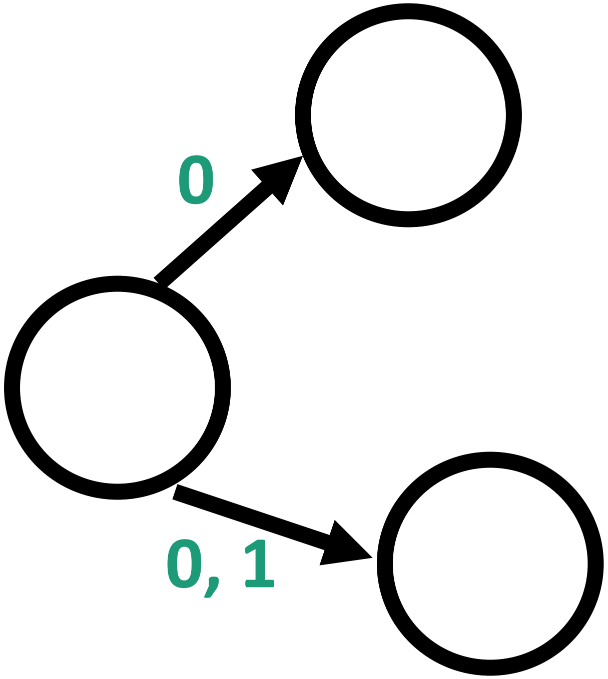
A Nondeterministic Finite Automaton (NFA) accepts if there *exists* a way to make it reach an accept state.

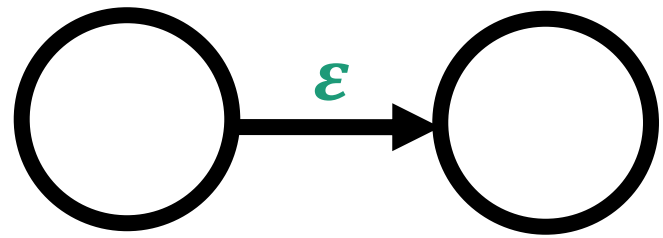
Nondeterminism

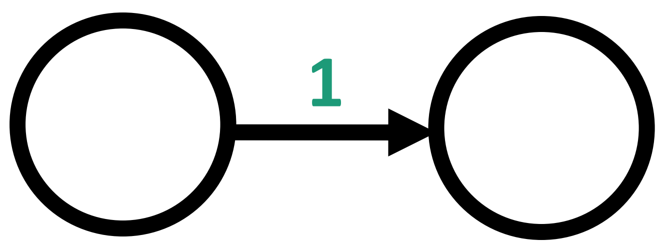


Example: Does this NFA accept the string 1100?

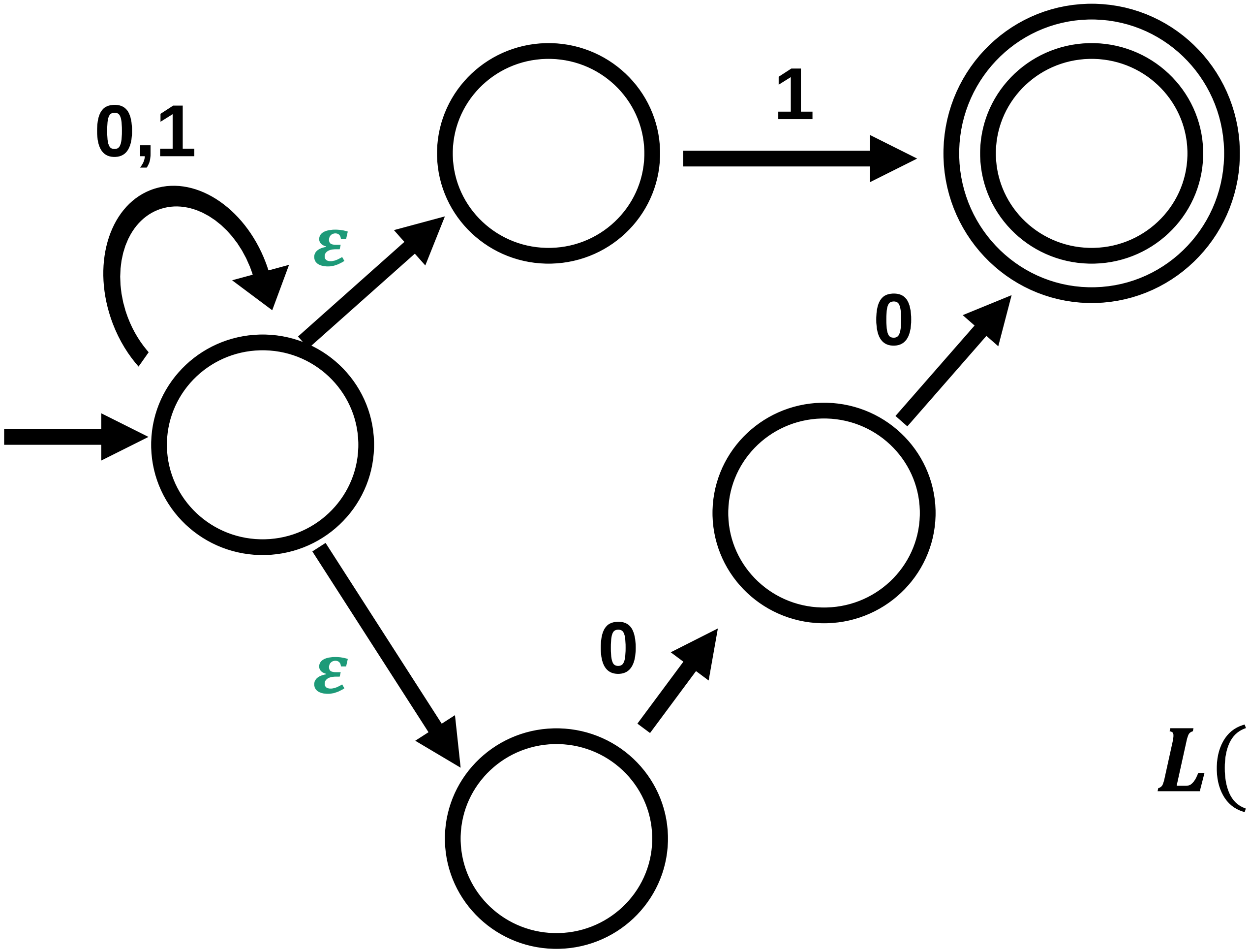
Some special transitions





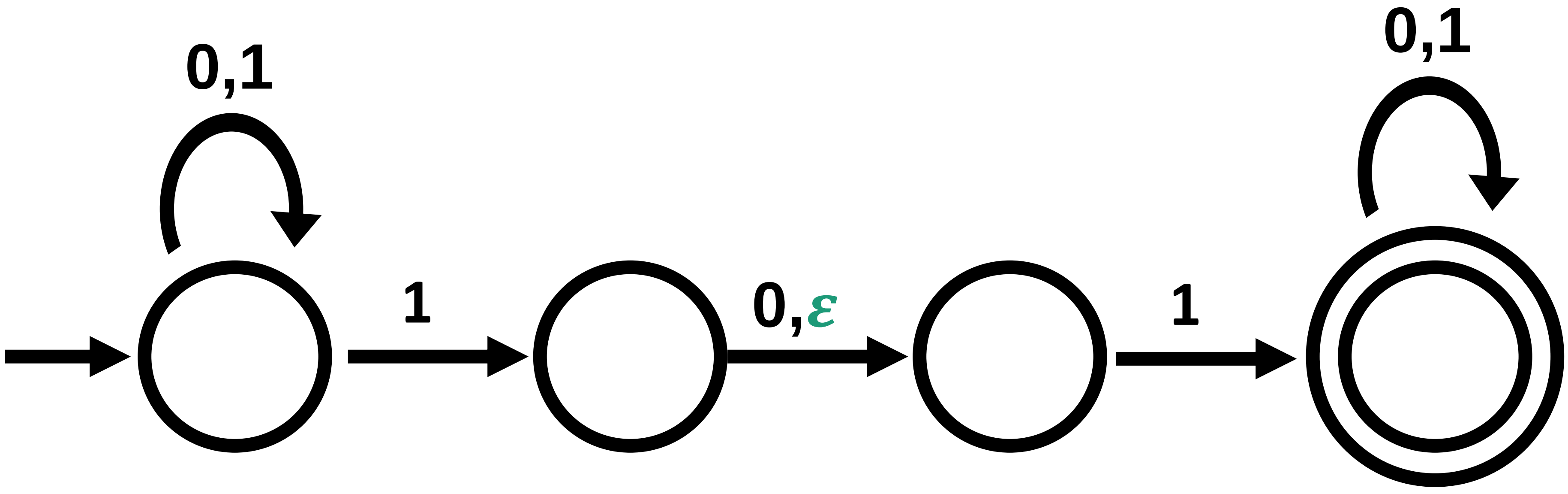


Example

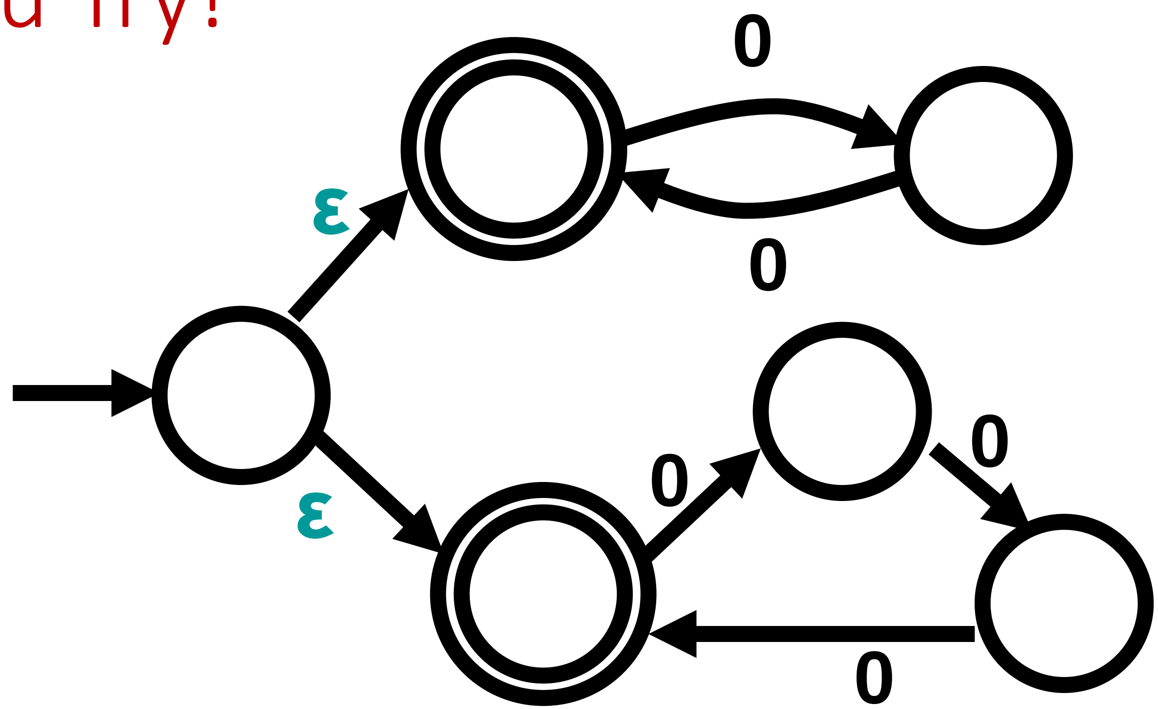


$$L(M) =$$

Example



Now You Try!



What is the language of this NFA? (over alphabet $\{0\}$)

a) $\{0^k \mid k \text{ is a multiple of } 2\}$
b) $\{0^k \mid k \text{ is a multiple of } 3\}$
c) $\{0^k \mid k \text{ is a multiple of } 6\}$
d) $\{0^k \mid k \text{ is a multiple of } 2 \text{ or a multiple of } 3\}$

Formal Definition of a NFA

An NFA is a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$

Q is the set of states

Σ is the alphabet

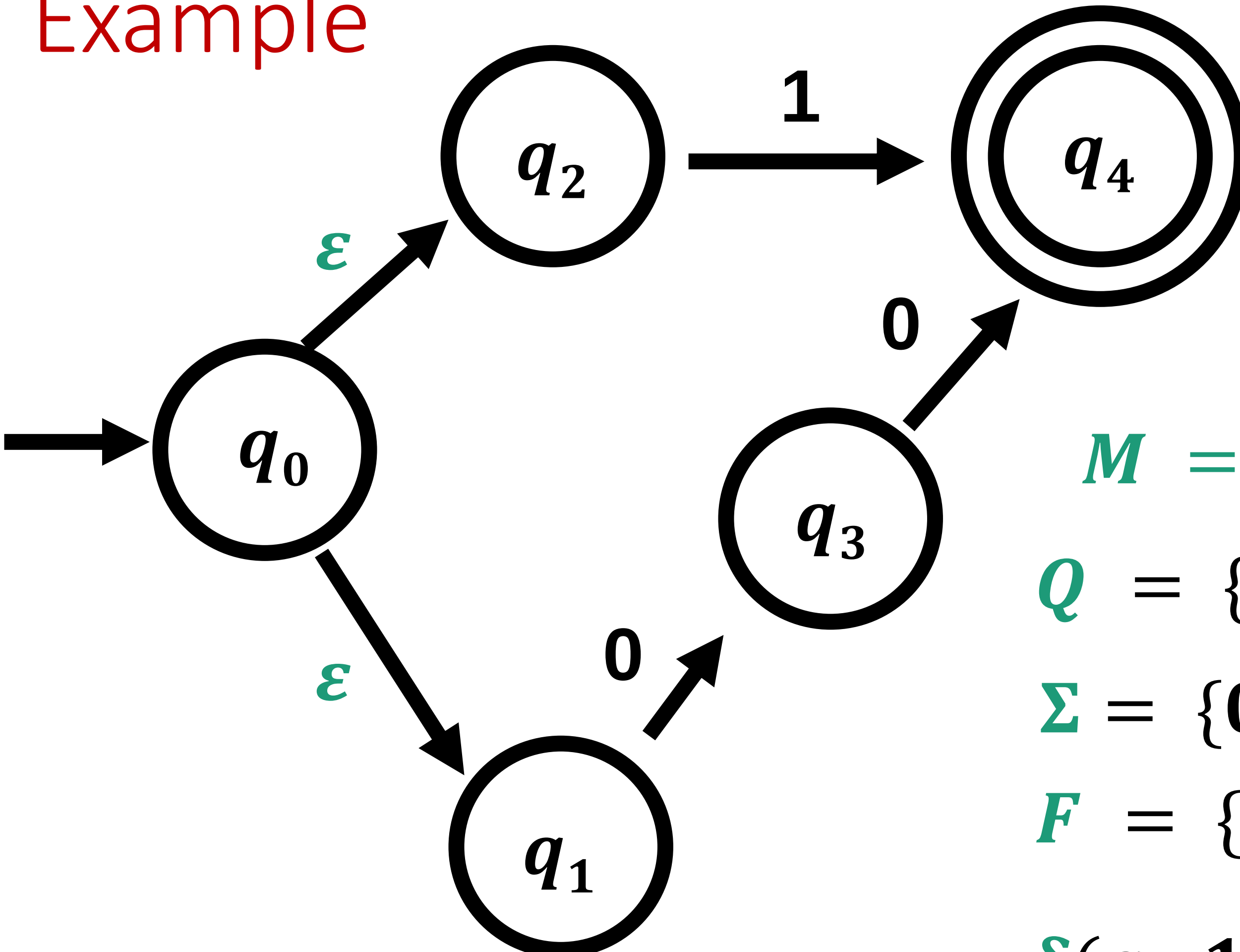
$\delta: Q \times \Sigma_\varepsilon \rightarrow P(Q)$ is the transition function

$q_0 \in Q$ is the start state

$F \subseteq Q$ is the set of accept states

M ***accepts*** a string w if **there exists** a path from q_0 to an accept state that can be followed by reading w.

Example



$$M = (Q, \Sigma, \delta, Q_0, F)$$

$$Q = \{q_0, q_1, q_2, q_3, q_4\}$$

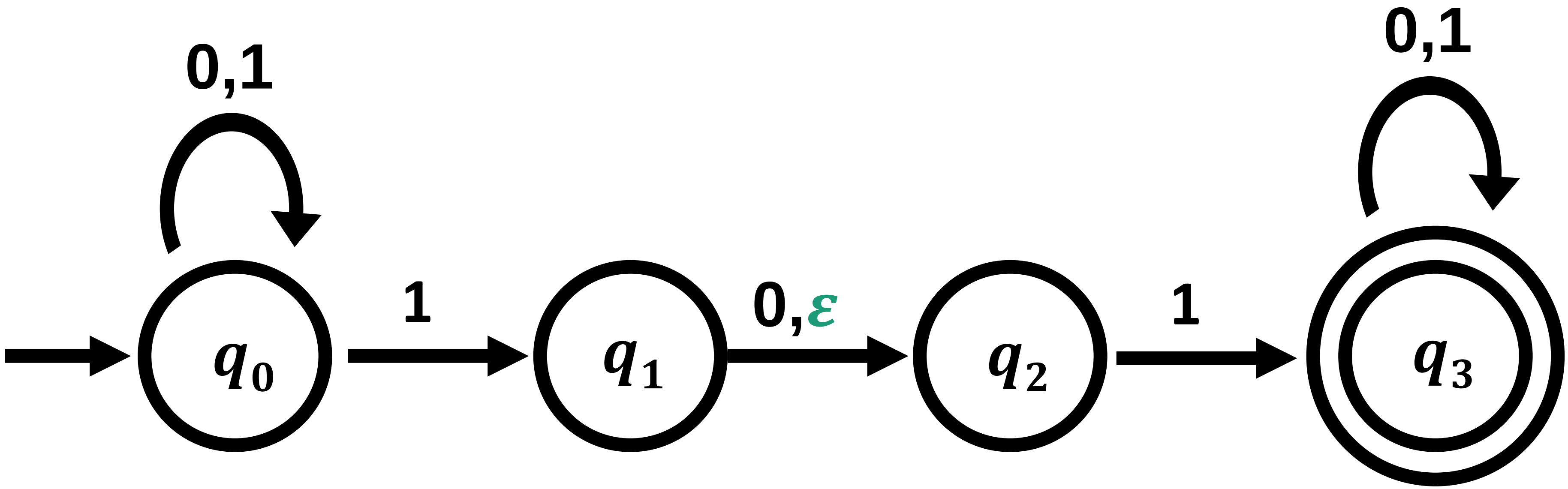
$$\Sigma = \{0, 1\}$$

$$F = \{q_4\}$$

$$\delta(q_2, 1) =$$

$$\delta(q_3, 1) =$$

Example



$N = (Q, \Sigma, \delta, q_0, F)$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0, 1\}$

$F = \{q_3\}$

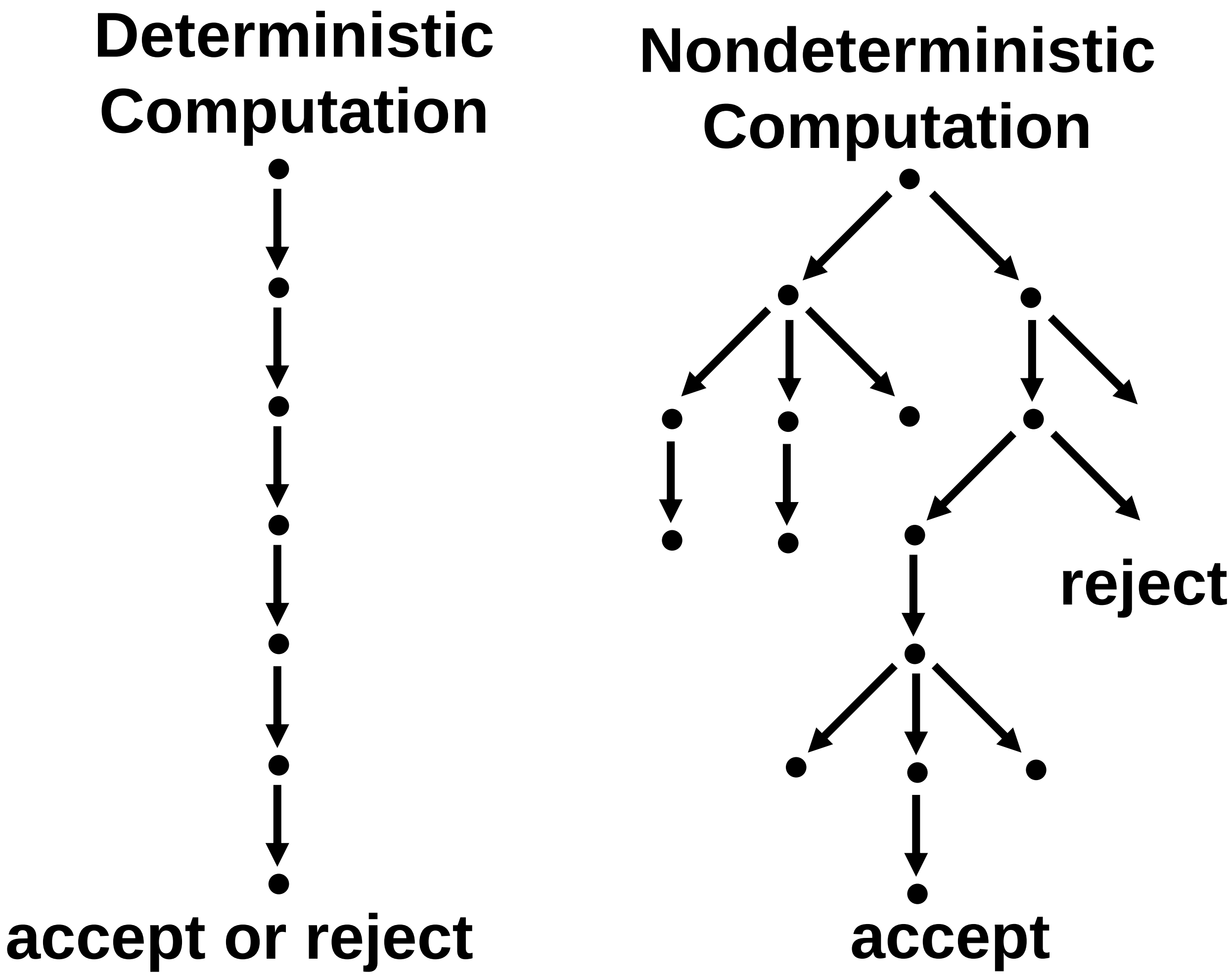
$\delta(q_0, 0) =$

$\delta(q_0, 1) =$

$\delta(q_1, \varepsilon) =$

$\delta(q_2, 0) =$

Nondeterminism



Ways to think about nondeterminism

- **(restricted) parallel computation**
- **tree of possible computations**
- **guessing and verifying the "right" choice**

Why study NFAs?

- Not really a realistic model of computation: Real computing devices can't actually try many possibilities in parallel

But:

- Useful tool for understanding power of DFAs/regular languages
- NFAs can be simpler than DFAs
- Lets us study "nondeterminism" as a resource

  (cf. P vs. NP)