BU CS 332 – Theory of Computation

Lecture 8:

Test 1 Review

Reading:
"Myhill-Nerode" note
Sipser Ch 1.4 (optional)
Sipser Ch 2.1

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Mea Culpa

What I wrote:

Let $L = \{ww \mid w = w^R\}$ and consider the distinguishing set $S = \{0^n \mid n \geq 0\}$. For $x = 0^n$ and $y = 0^m$, $m \neq n$, which of the following is a distinguishing extension for x and y?

a) $z = 0^n$

b) $z = 1^n$

c) $z = 10^n$

d) $z = 01^n$

Mea Culpa

What I meant to write:

Let $L = \{w \mid w = w^R\}$ and consider the distinguishing set $S = \{0^n \mid n \geq 0\}$. For $x = 0^n$ and $y = 0^m$, $m \neq n$, which of the following is a distinguishing extension for x and y?

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Reusing a Proof

Finding a distinguishing set can take some work…

Let's try to reuse that work!

How might we show that

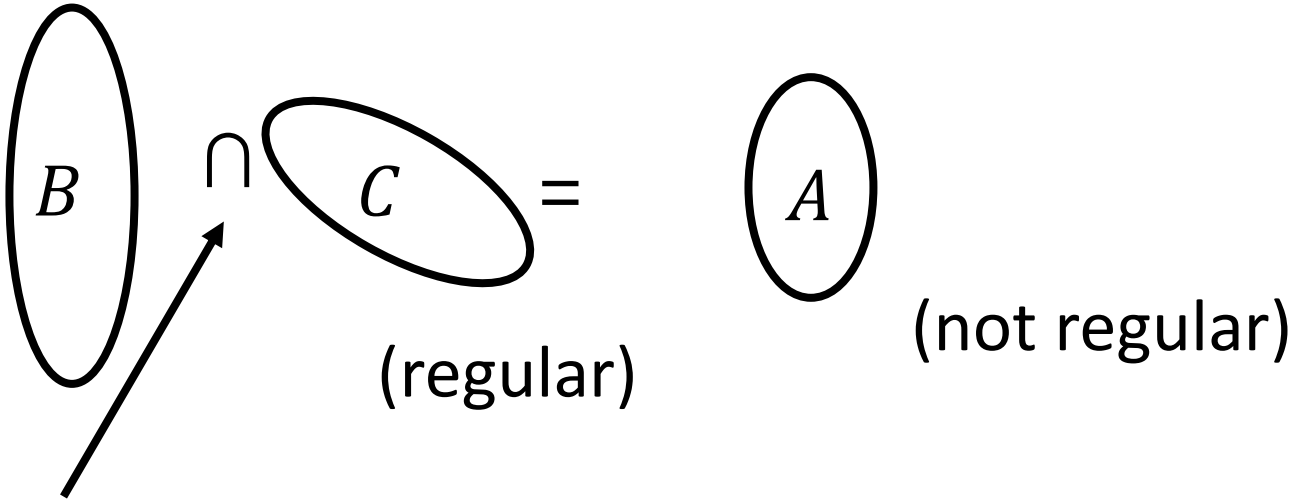
$BALANCED = \{w \mid w$ has an equal # of 0s and 1s$\}$

is not regular?

$\{0^n 1^n \mid n \geq 0\} = BALANCED \cap \{w \mid$ all 0s in w appear before all 1s$\}$

Using Closure Properties

If A is not regular, we can show a related language B is not regular



any of $\{\circ, \cup, \cap\}$ or, for one language, $\{\neg, ^R, *\}$

<u>By contradiction:</u> If B is regular, then $B \cap C\ (= A)$ is regular.

But A is not regular so neither is B!

Example

Prove $B = \{0^i 1^j | i \neq j\}$ is not regular using

- nonregular language
  $A = \{0^n 1^n | n \geq 0\}$ and
- regular language
  $C = \{w \mid \text{all } 0\text{s in } w \text{ appear before all } 1\text{s}\}$

Which of the following expresses A in terms of B and C?

a) $A = B \cap C$

b) $A = \bar{B} \cap C$

c) $A = B \cup C$

d) $A = \bar{B} \cup C$

!DANGER!

Let $B = \{0^i 1^j | i \neq j\}$ and write $B = A \cup C$ where

- nonregular language

$$A = \{0^i 1^j | i > j \geq 0\} \text{ and}$$

- nonregular language

$$C = \{0^i 1^j | j > i \geq 0\} \text{ and}$$

Does this let us conclude B is nonregular?

Test 1 Topics

Sets, Strings, Languages (0)

- Know the definition of a string and of a language (and the difference between them)
- Understand operations on strings: Concatenation, reverse
- Understand operations on languages: Union, intersection, concatenation, reverse, star, complement
- Know the difference between $\emptyset$ and ε

Deterministic FAs (1.1)

- Given an English or formal description of a language L, draw the state diagram of a DFA recognizing L (and vice versa)
- Know the formal definition of a DFA (A DFA is a 5 tuple…) and convert between state diagram and formal description
- Know the formal definition of how a DFA computes
- Construction for closure of regular languages under complement

Nondeterministic FAs (1.2)

- Given an English or formal description of a language L, draw the state diagram of an NFA recognizing L (and vice versa)
- Know the formal definition of an NFA
- Know the power set construction for converting an NFA to a DFA
- Proving closure properties: Know the constructions for union, concatenation, star
- Know how to prove your own closure properties

Regular Expressions (1.3)

- Given an English or formal description of a language L, construct a regex generating L (and vice versa)
- Formal definition of a regex
- Know how to convert a regex to an NFA
- Know how to convert a DFA/NFA to a regex

Non-regular Languages (Myhill-Nerode Note)

- Understand the statements of the distinguishing set method for proving DFA size lower bounds / non-regularity
- Understand the proof of why the distinguishing set method works, and be able to use it to prove similar statements
- Know how to apply the method to specific languages
- Know how to show languages are non-regular by combining distinguishing set method with closure properties

Test tips

- You may cite without proof any result…
  - Stated in lecture
  - Stated and proved in the main body of the text (Ch. 0-1.4)
  - These include worked-out examples of state diagrams, regexes
- Not included above: homework problems, discussion problems, (solved) exercises/problems in the text

- Showing your work / explaining your answers will help us give you partial credit
- Make sure you're interpreting quantifiers (for all / there exists) correctly and in the correct order

Practice Problems

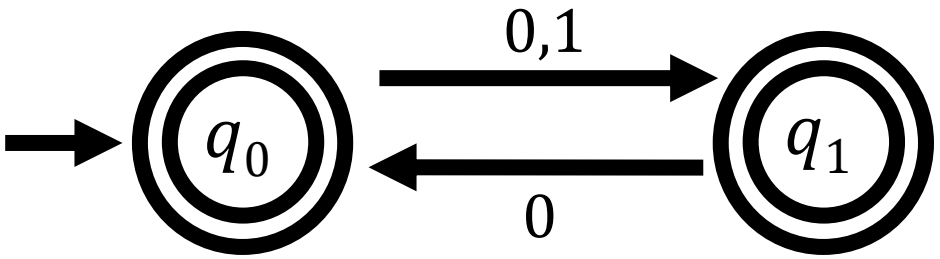
Name six operations under which the regular languages are closed

Prove or disprove: All finite languages are regular

Prove or disprove: The **non-**regular languages are closed under union

Give the state diagram of an NFA recognizing the language (01 U 10)*

Give an equivalent regular expression for the following NFA



Is the following language regular?

$$\{a^n a^n | n \geq 0\}$$

Is the following language regular?

$$\{0^n 1^n | 0 \leq n \leq 2021\}$$

How many states does a DFA recognizing $\{0^n 1^n | 0 \leq n \leq 2021\}$ require?