Homework 1 – Due Tuesday, January 28, 2025 at 11:59 PM

Problems

1. Sign up on piazza at `www.piazza.com/bu/spring2025/cs332`.

2. Sign up as a student on Gradescope at `www.gradescope.com` with course entry code **NY66PW**. Use your **BU email ID** to sign up on Gradescope.

3. Read and sign the Collaboration and Honesty Policy `https://cs-people.bu.edu/mbun/courses/332_S25/handouts/collaboration-policy.pdf` and submit it on Gradescope. We will be able to grade your homework only after you hand this in.

4. Check out the following links and resources:

   (a) Course webpage: `https://cs-people.bu.edu/mbun/courses/332_S25/`;

   (b) Math & algorithms review: `https://cs-people.bu.edu/mbun/courses/332_S25/handouts/mathreview.pdf`;

   (c) Supplementary textbook to review proof techniques:
       Richard Hammack. *Book of Proof*: `https://richardhammack.github.io/BookOfProof/Main.pdf/`

Reminder Collaboration is permitted, but you must write the solutions *by yourself without assistance*, and be ready to explain them orally to the course staff if asked. You must also identify your collaborators and write "Collaborators: none" if you worked by yourself. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden. Collaboration is not allowed on problems marked "INDIVIDUAL."

5. (**Review of Sets and Functions**) For a natural number n, let $[n]$ denote the set $\{1, 2, 3, \ldots, n - 1, n\}$.

   (a) What is $[n] \cap [2n]$?

   (b) What is $[n] \cup [2n]$?

   (c) Is there an surjective (onto) function from $[2n]$ to $[n]$? Prove your answer.

   (d) Is there an injective (one-to-one) function from $[2n]$ to $[n]$? Prove your answer.

6. (**Review of Logic**) Consider the following statement: "Elephants can jump only if pigs can fly."

   (a) What is the negation of this statement?

   (b) What is the converse of this statement?

   (c) What is the contrapositive of this statement?

7. (**Review of Graphs**) Determine whether each of the following statements is true or false. If it is true, give a proof. If it is false, give a counterexample.

(a) Every undirected graph with two or more vertices has two vertices with the same degree.

(b) Every undirected graph with three or more vertices has three vertices with the same degree.

8. Consider the alphabet $\Sigma = \{0, 1, \#\}$. Consider the following computational problem: Given a string $z \in \Sigma^*$, determine whether z takes the form $z = x\#y$ where $x \in \{0,1\}^*$ is a substring of $y \in \{0,1\}^*$. Here, x is a substring of y if it appears contiguously within y. For example, 010 is a substring of 00101. However, 011 is not a substring of 00101 because, while its symbols appear as a subsequence, they are not contiguous.

(a) Formulate this problem as the task of determining whether a given string z is in some language L, and describe the language L using set-builder notation.

(b) Give pseudocode for a program that recognizes the language L. That is, your program should take as input a string z and return "accept" if and only if z is in the language L. Your program does not need to be efficient, but we encourage you to think about its runtime and whether it's reasonable.