BU CS 332 – Theory of Computation

- Test 1 Monday 2/24
- Practice Test on Piazza
- Look out for extra pre-test office hours

Lecture 9:

- Test 1 Review

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Test 1 Topics

Sets, Strings, Languages (0)

- Know the definition of a string and of a language (and the difference between them)
- Understand operations on strings: Concatenation, reverse
- Understand operations on languages: Union, intersection, concatenation, reverse, star, complement
- Know the difference between Ø and ε

How to prove two languages (L_1, L_2) (w/ different looking descriptions) are equal:

1) $L_1 \subseteq L_2 \Leftarrow \forall x \in L_1,\ x \in L_2$

2) $L_2 \subseteq L_1 \Leftarrow \forall x \in L_2,\ x \in L_1$

$$L_1 = L_1' = L_2' = L_2$$

Deterministic FAs (1.1)

- Given an English or formal description of a language L, draw the state diagram of a DFA recognizing L (and vice versa)
- Know the formal definition of a DFA (A DFA is a 5 tuple…) and convert between state diagram and formal description
- Know the formal definition of how a DFA computes
- Construction for closure of regular languages under complement

"class of languages recognized by DFAs

Nondeterministic FAs (1.2)

- Given an English or formal description of a language L, draw the state diagram of an NFA recognizing L (and vice versa)
- Know the formal definition of an NFA
- Know the subset construction for converting an NFA to a DFA
- Proving closure properties: Know the constructions for union, concatenation, star
- Know how to prove your own closure properties

$\delta : Q \times \Sigma_\varepsilon \to P(Q)$

$(q, a) \mapsto R$ (a set of states)

$\delta(q_1, a) = \{q_2, q_3\}$
$\delta(q_2, b) = \{q_1\}$
$\vdots$

$\boxed{\delta(q, \sigma) = \emptyset \ \forall (q, \sigma) \text{ not listed above}}$

Regular Expressions (1.3)

- Given an English or formal description of a language L, construct a regex generating L (and vice versa)
- Formal definition of a regex
- Know how to convert a regex to an NFA
- Know how to convert a DFA/NFA to a regex

constructions for closure under $\cup, \circ, *$

NFA → GNFA → regex

Syntax:

ε, $\emptyset$, a

$(R)^*$, $R_1 \cup R_2$, $R_1 \circ R_2$

Semantics:

$L(\varepsilon) = \{\varepsilon\}$

$L(a) = \{a\}$

$\vdots$

Limitations of DFAs (Myhill-Nerode Note)

- Understand the statements of the distinguishing set method for proving DFA size lower bounds / non-regularity
- Understand the proof of why the distinguishing set method works, and be able to use it to prove similar statements
- Know how to apply the method to specific languages
- Know how to use the distinguishing set method (Myhill-Nerode) to prove that languages are non-regular

Test format

Problem 1: "Check your type checker"

E.g., Is $\boxed{\text{aabba}}$ a $\boxed{\text{string}}$, language, or a $\boxed{\text{regex}}$?

How about {ab} U {aab}?

" {ab, aab} language

Problem 2: True/false with **justification**

Either provide a convincing explanation or a specific counterexample

Problems 3-5(?) Homework-style problems

Study tips

- Make sure you know how to solve the problems on the practice test and are familiar with the format. The format/length of the real test will be very similar.

- If you need more practice, there are lots of problems in the book. We're happy to talk about any of these problems in office hours.

A

1.26

8 1/2" x 11"

- You may bring a page of notes (writing on both sides ok) to the test. Preparing this note sheet is a great study aid.

Test tips

- You may cite without proof any result…
  - Stated in lecture
  - Stated and proved in the main body of the text (Ch. 0-1.3)
  - These include worked-out examples of state diagrams, regexes
- Not included above: homework problems, discussion problems, (solved) exercises/problems in the text

- Showing your work / explaining your answers will help us give you partial credit
- Make sure you're interpreting quantifiers (for all / there exists) correctly and in the correct order

Practice Problems

Name six operations under which the regular languages are closed

The regular languages are closed under operation op if

$\forall$ regular A, $op(A)$ is also regular

- Intersection $\cap$ $[\forall A, B$ regular, $A \cap B$ is regular$]$
- Concatenation $\circ$
- Union $\cup$
- Star $*$
- Complement $[\forall A$ regular, $\overline{A}$ is regular$]$
- Reverse $[\forall A$ regular, A^R is regular$]$

Prove or disprove: All finite languages are regular

A language L is finite if it consists of finitely many strings, i.e. $\boxed{L = \{w_1, w_2, \ldots, w_m\}}$

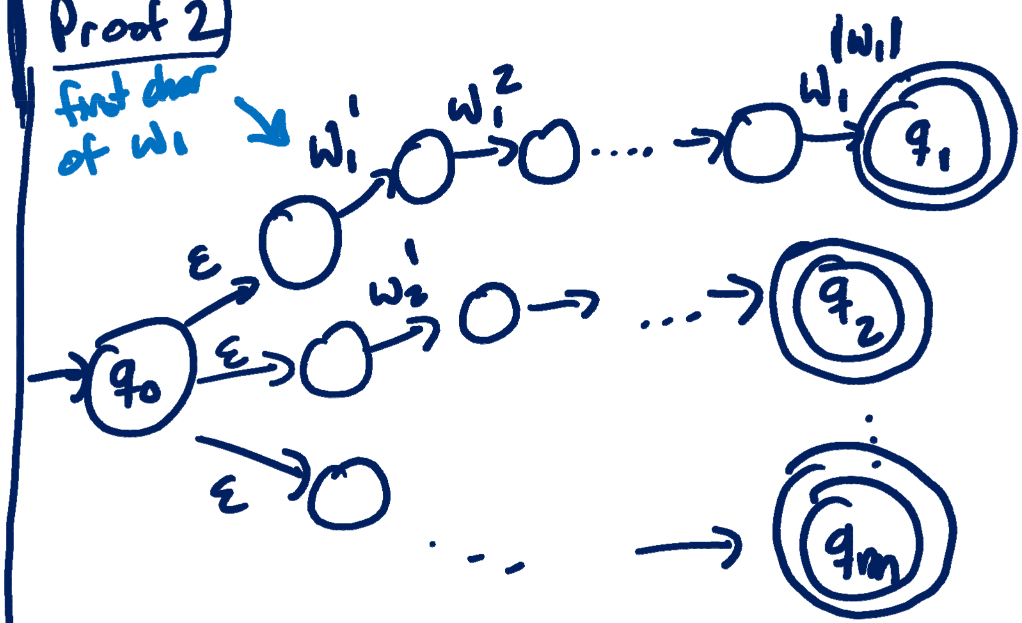
$m < \infty$ each a string

Proof 1

- For each $w_i \in L$, w_i is a regex generating $\{w_i\}$
- $\boxed{w_1 \cup w_2 \cup \ldots \cup w_m}$ is hence a regex generating L.

Proof 2

first char of w_1



$L = \{00, 01\}$

Myhill-Nerode converse: If a language does not have an infinite pairwise dist. set, it is regular

Claim: All finite languages are regular.

Proof. Let L be a finite language. Then $\exists\, m$ s.t.

$L = \{w_1, \ldots, w_m\}$ where each w_i is a string.

Observe that regex w_i generates $\{w_i\}$ for each i

$\Rightarrow$ $w_1 \cup \ldots \cup w_m$ is a regex generating $\{w_1\} \cup \ldots \cup \{w_m\}$
$= L$

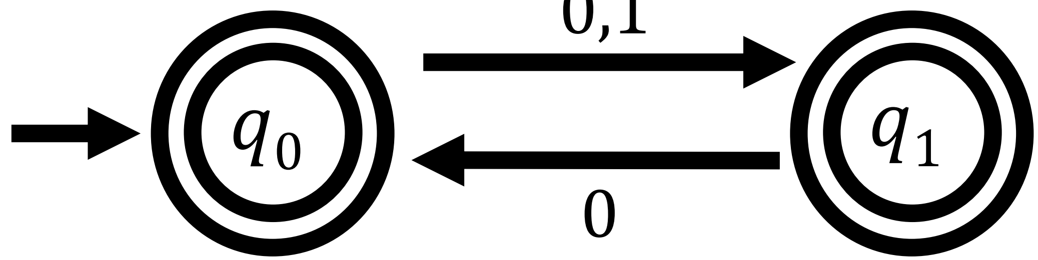
$\Rightarrow$ L is generated by a regex, hence regular

$\Sigma = \{a, b\}$ $\quad a \cup b$

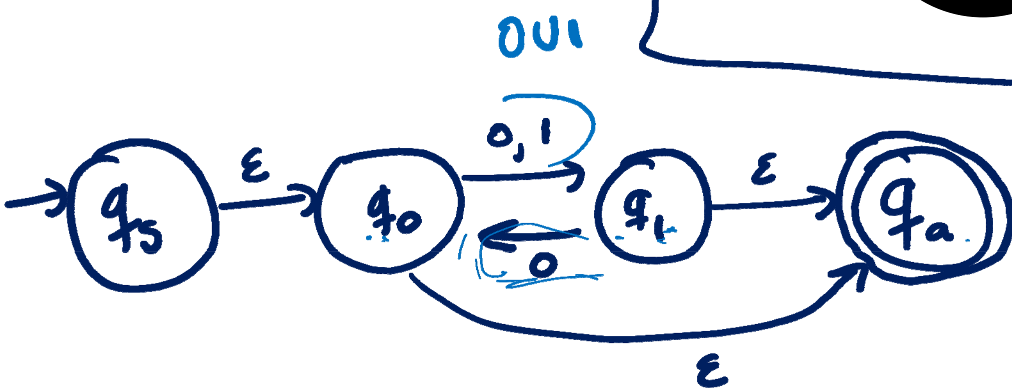
Prove or disprove: The **non-**regular languages are closed under intersection

Give the state diagram of an NFA recognizing the language $(01 \cup 10)^* \circ 1$

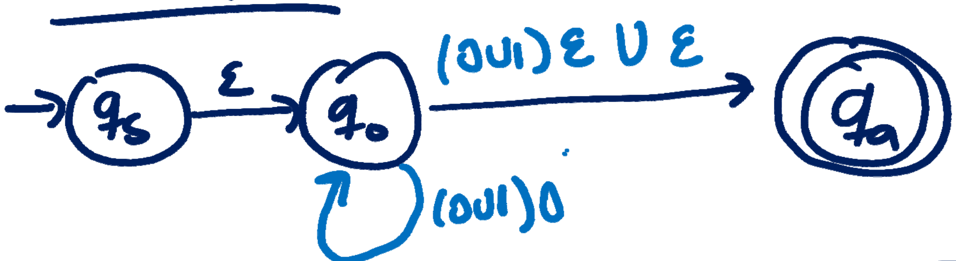
Give an equivalent regular expression for the following NFA



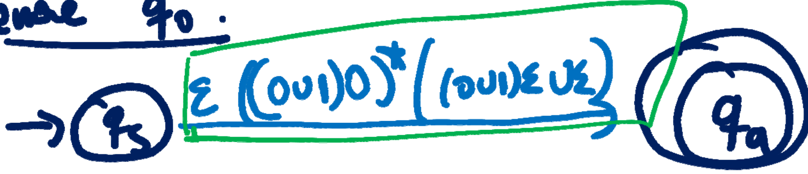
Convert to GNFA:



Remove q_1:

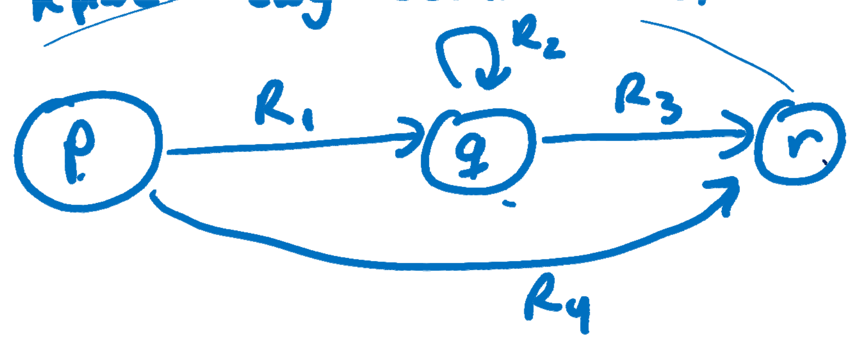


Remove q_0:

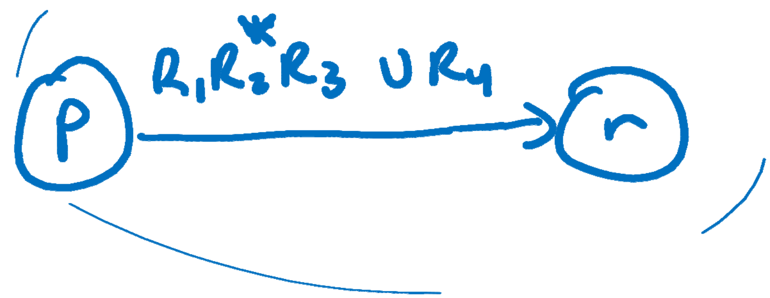


$= ((0\cup 1)0)^*(0\cup 1 \cup \varepsilon)$

Reminder: To rip out state q, replace every occurrence of



with



For a language L over $\{0, 1\}$, define the operation $\mathbf{split}(L) = \{x\#y \mid x, y \in L\}$. Show that the regular languages are closed under split

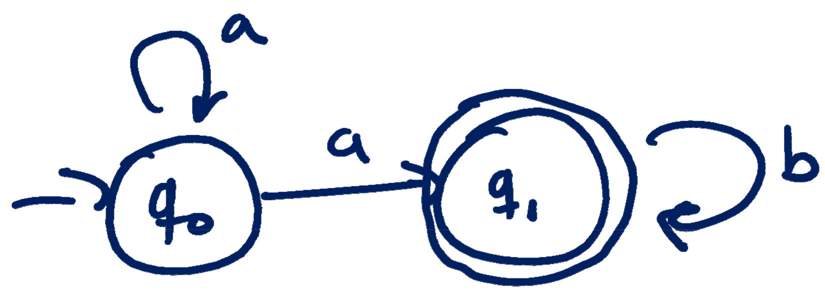
For a language L over alphabet Σ, define the operation $\mathbf{drop}(L) = \{xyz \mid xyz \in L, xy \in \Sigma^*, z \in \Sigma\}$. Show that the regular languages are closed under $\mathbf{drop}$.

Is the following language regular?
$\{0^n1^n \mid 0 \leq n \leq 2024\}$

Is the following language regular? $\{a^n a^n \mid n \geq 0\}$

How many states does a DFA recognizing $\{0^n1^n \mid 0 \leq n \leq 2024\}$ require?

State diagram → formal description for NFA



$Q = \{q_0, q_1\}$

$\Sigma = \{a, b\}$ $\quad \Sigma_\varepsilon = \{a, b, \varepsilon\}$

δ :

$\delta(q, \sigma)$	a	b	ε
q_0	$\{q_0, q_1\}$	ϕ	ϕ
q_1	ϕ	$\{q_1\}$	ϕ

Start state q_0

$F = \{q_1\}$