CS 332: *Theory of Computation* — Profs. Alexander Poremba & Mark Bun
Boston University — February 20, 2026

Homework 4 – Due Thursday, February 26, 2026 at 11:59 PM

Reminder Collaboration is permitted, but you must write the solutions *by yourself without assistance*, and be ready to explain them orally to the course staff if asked. You must also identify your collaborators and write "Collaborators: none" if you worked by yourself. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden. Collaboration is not allowed on problems marked "INDIVIDUAL."

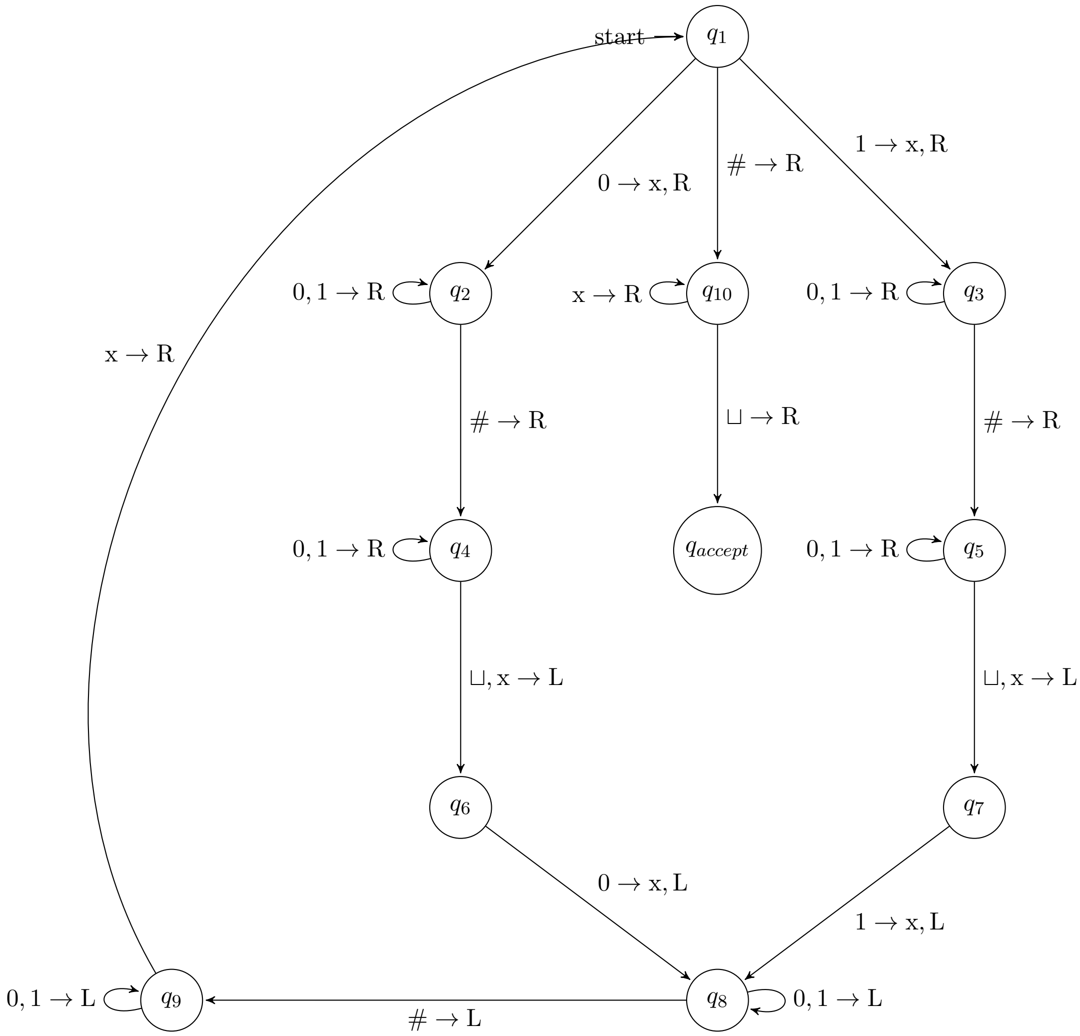
Problems There are three required problems and 1 bonus problem.

1. (**INDIVIDUAL: Complement and Star**) For each of the following statements, give a proof or provide a counterexample.
   (a) For every language $L \subseteq \{0,1\}^*$, we have $(\overline{L})^* \neq \overline{L^*}$.
   (b) For every natural number k, if $L \subseteq \{0,1\}^*$ is recognized by a DFA with k states, then there also exists a DFA with k states recognizing $(\overline{L})^*$.
   Hint: Think about the language A from Homework 2, Problem 6.

2. (**Non-regular languages**) Prove that the following languages are not regular. <u>You may only use the distinguishing set method and the closure of the class of regular languages under union, intersection, complement, and reverse.</u>
   (a) $L_1 = \{0^n 1^m \mid n, m \geq 0 \text{ and } n = m^2\}$.
   (b) $L_2 = \{0^n 1^m 0^n \mid m, n \geq 0\}$.
   (c) $L_3 = \left\{1^k y \mid y \in \{0,1\}^* \text{ and } |y| = k\right\}$.
   (d) $L_4 = \left\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\right\}$.
   (e) Let $\Sigma = \{0, 1, \#\}$. Let $L_5 = \{x\#y\#z \mid x, y, z \text{ are nonnegative integers written in binary where } x + y = z\}$.

3. (**Low-Level to Implementation-Level**) The following page illustrates the state diagram of a Turing machine using input alphabet $\Sigma = \{0, 1, \#\}$ and tape alphabet $\Gamma = \{0, 1, \#, \mathrm{x}, \sqcup\}$.

   The notation "$a \to \mathrm{R}$" is shorthand for "$a \to a, \mathrm{R}$." The reject state and transitions to the reject state are not shown. Whenever the TM tries to read a character for which there is no explicit transition that means that the TM goes to the reject state. Use the convention that the head moves right in each of these transitions to the reject state.



(a) Give the sequences of configurations that this TM M enters when given as input strings (i) 010#100, (ii) 10#01, and (iii) 0##0. Use the same representation for your configurations as we did in lecture 9.

(b) Give an implementation-level description of the Turing machine described by this state diagram. Hint: The machine is similar to Example 3.9 in Sipser.

(c) What is the language decided by M?

4. (**Bonus Problem**) The distinguishing set method doesn't tell us directly about the minimum number of states an NFA needs to recognize a language. And in general, we don't know anything quite as powerful as the Myhill-Nerode theorem for characterizing the minimum size of an NFA for language. Nevertheless, some techniques for proving NFA state size lower bounds are available, as this problem illustrates.

A *fooling set* for a language L is a set of pairs of strings $\{(x_i, y_i) \mid i = 1, \ldots, k\}$ such that:

(a) For every $i = 1, \ldots, k$, we have $x_i y_i \in L$, and

(b) For every $i \neq j$, at least one of the strings $x_i y_j$ or $x_j y_i$ is not in L.

Show that if L has a fooling set of size k as described above, then every NFA recognizing L requires at least k states.