

# BU CS 332 – Theory of Computation

<https://forms.gle/L9BMDsJ48ezNaEW76>



## Lecture 8:

- More on non-regularity
- Test 1 review

Reading:

“Myhill-Nerode” note

Alexander Poremba & Mark Bun

February 12, 2026

# Last Time: Distinguishing Set Method

**Definition:** Strings  $x$  and  $y$  are **distinguishable** by  $L$  if there exists a “distinguishing extension”  $z \in \Sigma^*$  such that exactly one of  $xz$  or  $yz$  is in  $L$ .

**Definition:** A set of strings  $S$  is **pairwise distinguishable** by  $L$  if every pair of distinct strings  $x, y \in S$  is distinguishable by  $L$ .

**Theorem:** If  $S$  is pairwise distinguishable by  $L$ , then every DFA recognizing  $L$  needs at least  $|S|$  states.

**Corollary:** If language  $L$  has an infinite pairwise distinguishable set, then  $L$  is not regular.

# Palindromes

**Theorem:**  $L = \{w \in \{0,1\}^* \mid w = w^R\}$  is not regular

**Proof:** We construct an infinite pairwise distinguishable set

# Now you try!



Use the distinguishing set method to show that the following language is not regular

$$L_1 = \{0^i 1^j \mid i > j \geq 0\}$$

Your job: Build an infinite set  $S$  such that for all  $x \neq y \in S$ , there exists a  $z$  such that exactly one of  $xz$  and  $yz$  is in  $L$

# Perfect Squares

Use the distinguishing set method to show that the following language is not regular

$$L_2 = \{1^{n^2} \mid n \geq 0\}$$

# Reusing a Proof



Finding a distinguishing set can take some work...

Let's try to reuse that work!

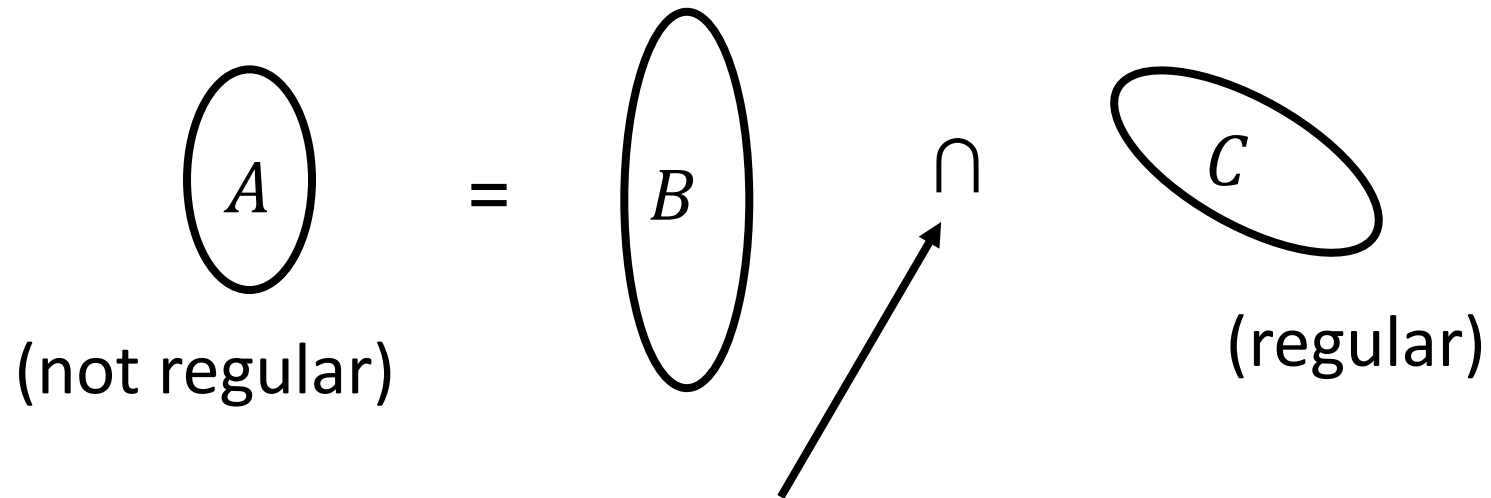
How might we show that

$BALANCED = \{w \mid w \text{ has an equal \# of 0s and 1s}\}$   
is not regular?

$\{0^n 1^n \mid n \geq 0\} = BALANCED \cap \{w \mid \text{all 0s in } w \text{ appear before all 1s}\}$

# Using Closure Properties

If  $A$  is not regular, we can show a related language  $B$  is not regular



any of  $\{\circ, \cup, \cap\}$  or, for one language,  $\{\neg, ^R, *\}$

By contradiction: If  $B$  is regular, then  $B \cap C (= A)$  is regular.

But  $A$  is not regular so neither is  $B$ !

# Example



Prove  $B = \{0^i 1^j \mid i \neq j\}$  is not regular using

- Nonregular language

$$A = \{0^n 1^n \mid n \geq 0\} \text{ and}$$

- Regular language

$$C = \{w \mid \text{all 0s in } w \text{ appear before all 1s}\}$$

Which of the following expresses  $A$  in terms of  $B$  and  $C$ ?

a)  $A = B \cap C$

c)  $A = B \cup C$

b)  $A = \bar{B} \cap C$

d)  $A = \bar{B} \cup C$

# Proof that $B$ is nonregular

Assume for the sake of contradiction that  $B$  is regular

We know:  $A = \bar{B} \cap C$

# !DANGER!



Let  $B = \{0^i 1^j \mid i \neq j\}$  and write  $B = A \cup C$  where

- Nonregular language

$$A = \{0^i 1^j \mid i > j \geq 0\} \text{ and}$$

- Nonregular language

$$C = \{0^i 1^j \mid j > i \geq 0\} \text{ and}$$

Does this let us conclude  $B$  is nonregular?

# Test 1 Topics

# Sets, Strings, Languages (0)

- Know the definition of a string and of a language (and the difference between them)
- Understand operations on strings: Concatenation, reverse
- Understand operations on languages: Union, intersection, concatenation, reverse, star, complement
- Know the difference between  $\emptyset$  and  $\varepsilon$

# Deterministic FAs (1.1)

- Given an English or formal description of a language  $L$ , draw the state diagram of a DFA recognizing  $L$  (and vice versa)
- Know the formal definition of a DFA (A DFA is a 5 tuple...) and convert between state diagram and formal description
- Know the formal definition of how a DFA computes
- Construction for closure of regular languages under complement

# Nondeterministic FAs (1.2)

- Given an English or formal description of a language  $L$ , draw the state diagram of an NFA recognizing  $L$  (and vice versa)
- Know the formal definition of an NFA
- Know the subset construction for converting an NFA to a DFA
- Proving closure properties: Know the constructions for union, concatenation, star
- Know how to prove your own closure properties

# Regular Expressions (1.3)

- Given an English or formal description of a language  $L$ , construct a regex generating  $L$  (and vice versa)
- Formal definition of a regex
- Know how to convert a regex to an NFA
- Know how to convert a DFA/NFA to a regex

# Limitations of DFAs (Myhill-Nerode Note)

- Understand the statements of the distinguishing set method for proving DFA size lower bounds / non-regularity
- Understand the proof of why the distinguishing set method works, and be able to use it to prove similar statements
- Know how to apply the method to specific languages
- We won't ask you to prove that some language is non-regular, since you didn't have any homework problems on this yet

# Test format

Problem 1: “Check your type checker”

E.g., Is aabba a string, language, or a regex?

How about  $\{ab\} \cup \{aab\}$ ?

Problem 2: True/false with **justification**

Either provide a convincing explanation or a specific counterexample

Problems 3-5(?) Homework-style problems

# Study tips

- Make sure you know how to solve the problems on the practice test and are familiar with the format. The format/length of the real test will be very similar.
- If you need more practice, there are lots of problems in the book. We're happy to talk about any of these problems in office hours.
- You may bring a page of notes (writing on both sides ok) to the test. Preparing this note sheet is a great study aid.

# Test tips

- You may cite without proof any result...
  - Stated in lecture
  - Stated and proved in the main body of the text (Ch. 0-1.3 + Myhill-Nerode note)
  - These include worked-out examples of state diagrams, regexes
- **Not included above:** homework problems, discussion problems, (solved) exercises/problems in the text
- Showing your work / explaining your answers will help us give you partial credit
- Make sure you're interpreting quantifiers (for all / there exists) correctly and in the correct order

# Practice Problems

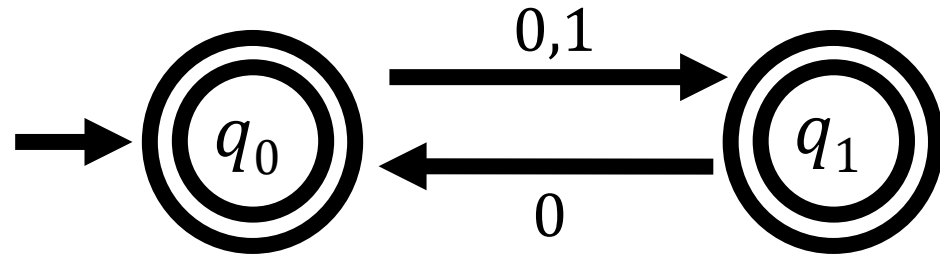
Name six operations under which the regular languages are closed

Prove or disprove: All finite languages are regular

Prove or disprove: The **non-regular** languages are closed under intersection

Give the state diagram of an NFA recognizing the language  $(01 \cup 10)^* \circ 1$

Give an equivalent regular expression for the following NFA



For a language  $L$  over  $\{0, 1\}$ , define the operation  $\text{split}(L) = \{x\#y \mid x, y \in L\}$ . Show that the regular languages are closed under split

For a language  $L$  over alphabet  $\Sigma$ , define the operation  $\text{drop}(L) = \{xz \mid xyz \in L \text{ where } x, z \in \Sigma^*, y \in \Sigma\}$ . Show that the regular languages are closed under  $\text{drop}$ .

Is the following language regular?

$\{0^n 1^n \mid 0 \leq n \leq 2026\}$

Is the following language regular?  $\{a^n a^n \mid n \geq 0\}$

How many states does a DFA recognizing  $\{0^n 1^n \mid 0 \leq n \leq 2026\}$  require?