# CS 535: Complexity Theory, Fall 2020

## Homework 8

Due: 2:00AM, Saturday, November 14, 2020.

**Reminder.** Homework must be typeset with LaTeX preferred. Make sure you understand the course collaboration and honesty policy before beginning this assignment. Collaboration is permitted, but you must write the solutions *by yourself without assistance*. You must also identify your collaborators. Assignments missing a collaboration statement will not be accepted. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

**Problem 0** (Term Paper)**.** Give the paper you are reviewing a careful reading and start thinking about the structure and content of your review. (A draft of your review is due on Nov. 21, so don't delay!)

**Problem 1** (**NP**, **BPP**, and **RP**)**.**

(a) Suppose $\mathbf{NP} \subseteq \mathbf{BPP}$. Show that SearchSAT can be solved in randomized polynomial-time. That is, show that there is a probabilistic poly-time algorithm $M$ such that for all satisfiable CNF formulas $\varphi$, we have that $M(\varphi)$ outputs a satisfying assignment to $\varphi$ with probability at least 2/3. (7 points)

(b) Use part (a) to conclude that if $\mathbf{NP} \subseteq \mathbf{BPP}$, then $\mathbf{NP} = \mathbf{RP}$. (5 points)

**Problem 2** (Counting Cycles)**.** A Hamiltonian cycle in a directed graph $G$ is a cycle that visits every vertex in $G$ exactly once. Define the problem #HAM[1] as follows: Given a directed graph $G$, count the number of Hamiltonian cycles in $G$. It is known that #HAM is #**P**-complete. Use this fact to prove that #CYCLE is also #**P**-complete. (8 points)

[1] I'm not so sure about sharp ham, but I like my ham with sharp cheddar.