Probability in Computing

LECTURE 4

CS 237

Last time

- Axioms of probability
- How to compute probabilities
- Probability rules

Today

- Common fallacies
- Tree diagrams
- Monty Hall problem

Reminders

- HW2 due Thursday
- The collaboration policy you submitted should be signed and dated

Top Hat question (Join Code: 033357)

A family has 2 children. Assume that each child is equally likely to be a boy or a girl. Which of the following is more likely?

A. They have 2 boys.
B. They have 2 girls.
C. They have 2 kids of different gender.
D. All the three possibilities above are equally likely.

Top Hat question (Join Code: 033357)

Toss a coin twice. Which outcomes did you get?

A. HH
B. TT
C. Two different outcomes.

Top Hat question (Join Code: 033357)

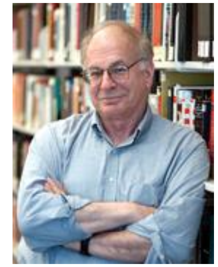
``Linda is 31, single, outspoken, and very bright. She majored in philosophy in college. As a student, she was deeply concerned with racial discrimination and other social issues, and participated in anti-nuclear demonstrations."

Rank the likelihood of the following alternatives:

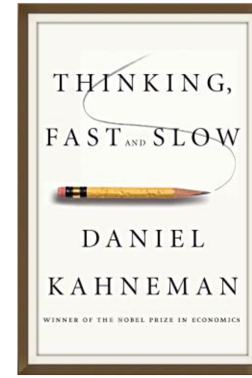
1) Linda is active in the feminist movement
2) Linda is a bank teller
3) Linda is a bank teller and active in the feminist movement

Conjunction Fallacy

Amos Tversky
[1937-1996]



Daniel Kahneman
[1934-]



Between 80 and 95 percent of the subjects ranked: (1) > (3) > (2)

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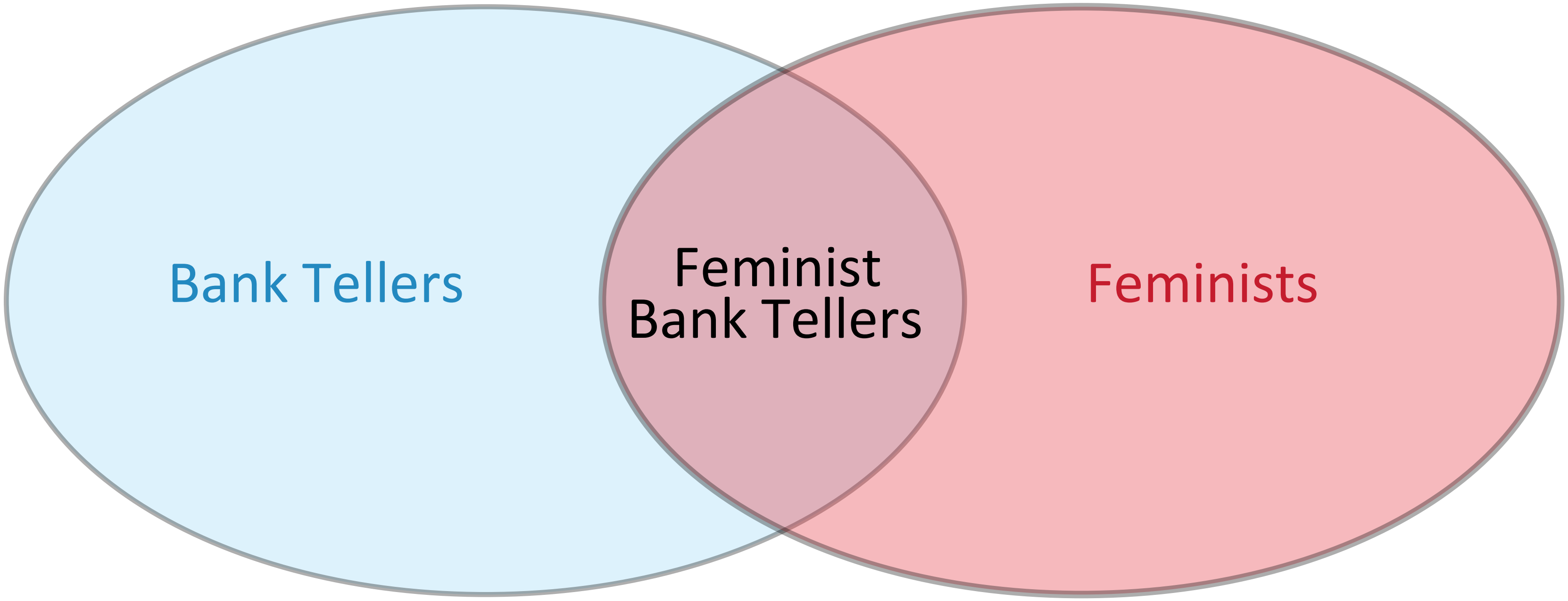
Conjunction Fallacy

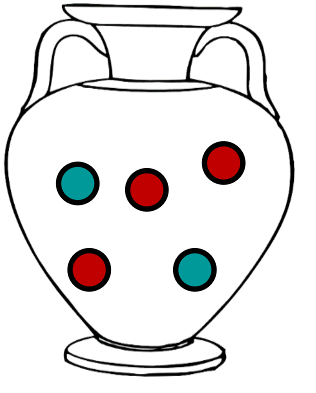


Rank the likelihood of the following alternatives:

1) Linda is active in the feminist movement
2) Linda is a bank teller
3) Linda is a bank teller and active in the feminist movement

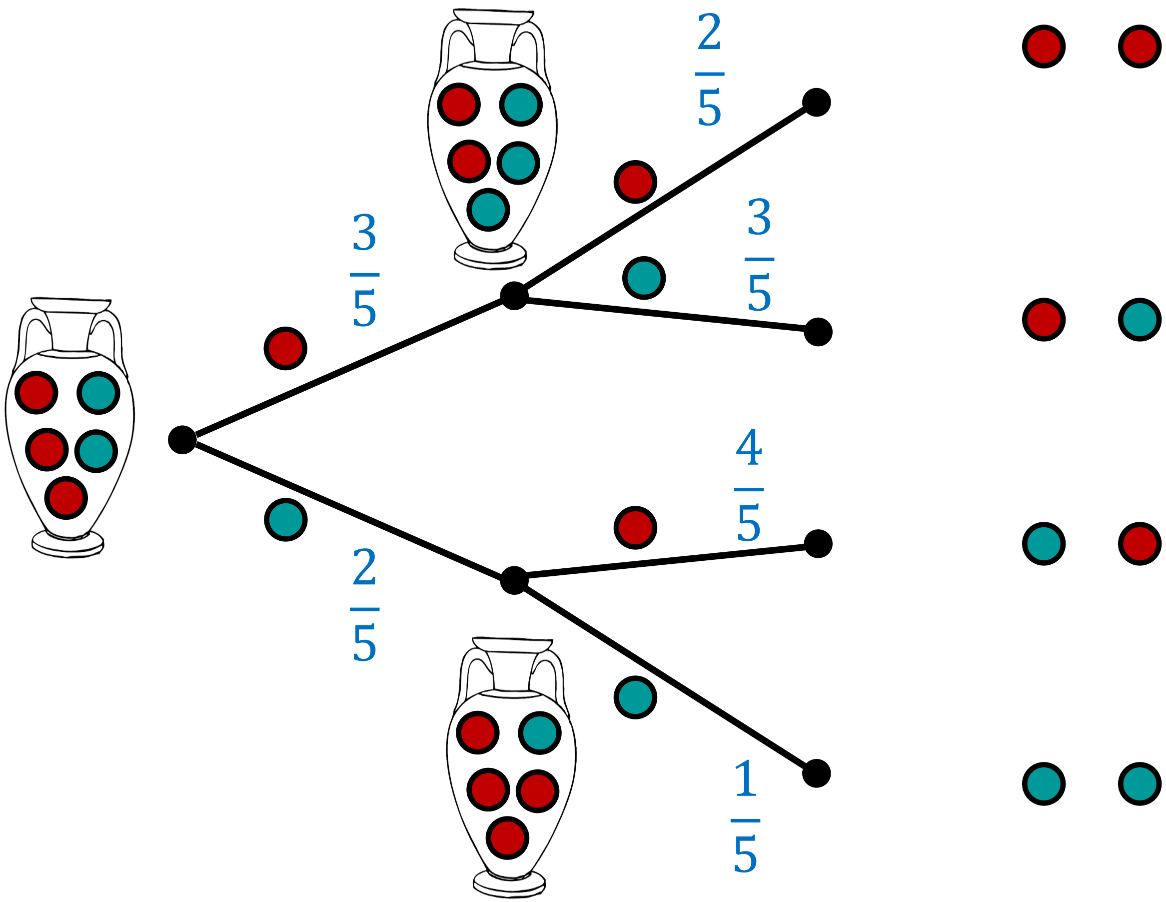
Tree Diagram Method: Example

- An urn contains 3 red balls and 2 green balls
- Draw a ball uniformly at random
  - If the ball drawn is red, add a green ball to the urn
  - If the ball drawn is green, add a red ball to the urn
  - The drawn ball is not returned to the urn
- Draw a second ball uniformly at random
- Determine Pr("second ball is red")



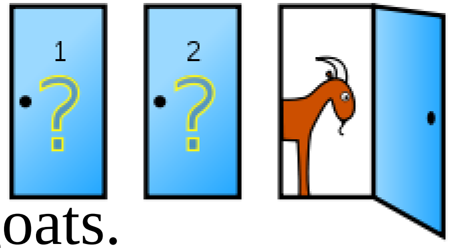
Tree Diagram Method: Urn

1st ball | 2nd ball | outcome | probability | Is 2nd ball red?



Pr("second ball is red") =

Monty Hall problem

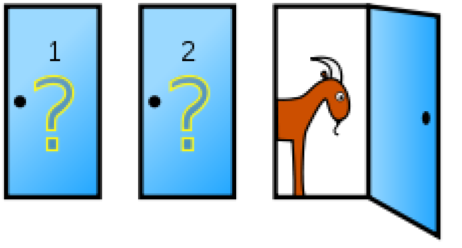


- 1970 game show hosted by Monty Hall.
- You (the contestant) are shown 3 doors: behind one is a prize and behind the other two are two goats.
- You pick a door, but do not open it.
- Then one of the other two doors is opened to reveal a goat.
- You are given the option of sticking with your choice or picking the other unopened door.
- You win the prize iff you pick the door with the prize.
- Should you switch?

Picture source: http://en.wikipedia.org/wiki/Monty_Hall_problem

Modeling Assumptions

- The prize is equally likely to be behind each of the 3 doors
- If the host has a choice of which door to open, he is equally likely to pick either of them.

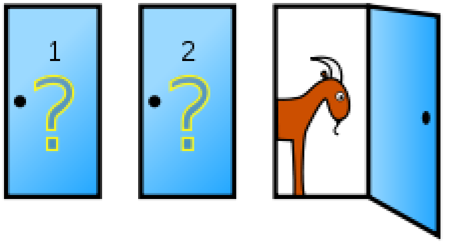


Picture source: http://en.wikipedia.org/wiki/Monty_Hall_problem

Top Hat Question (Join Code: 033357)

When do you have a higher probability of winning?

A. If you stick with your first choice.
B. If you switch.
C. Both probabilities are the same.



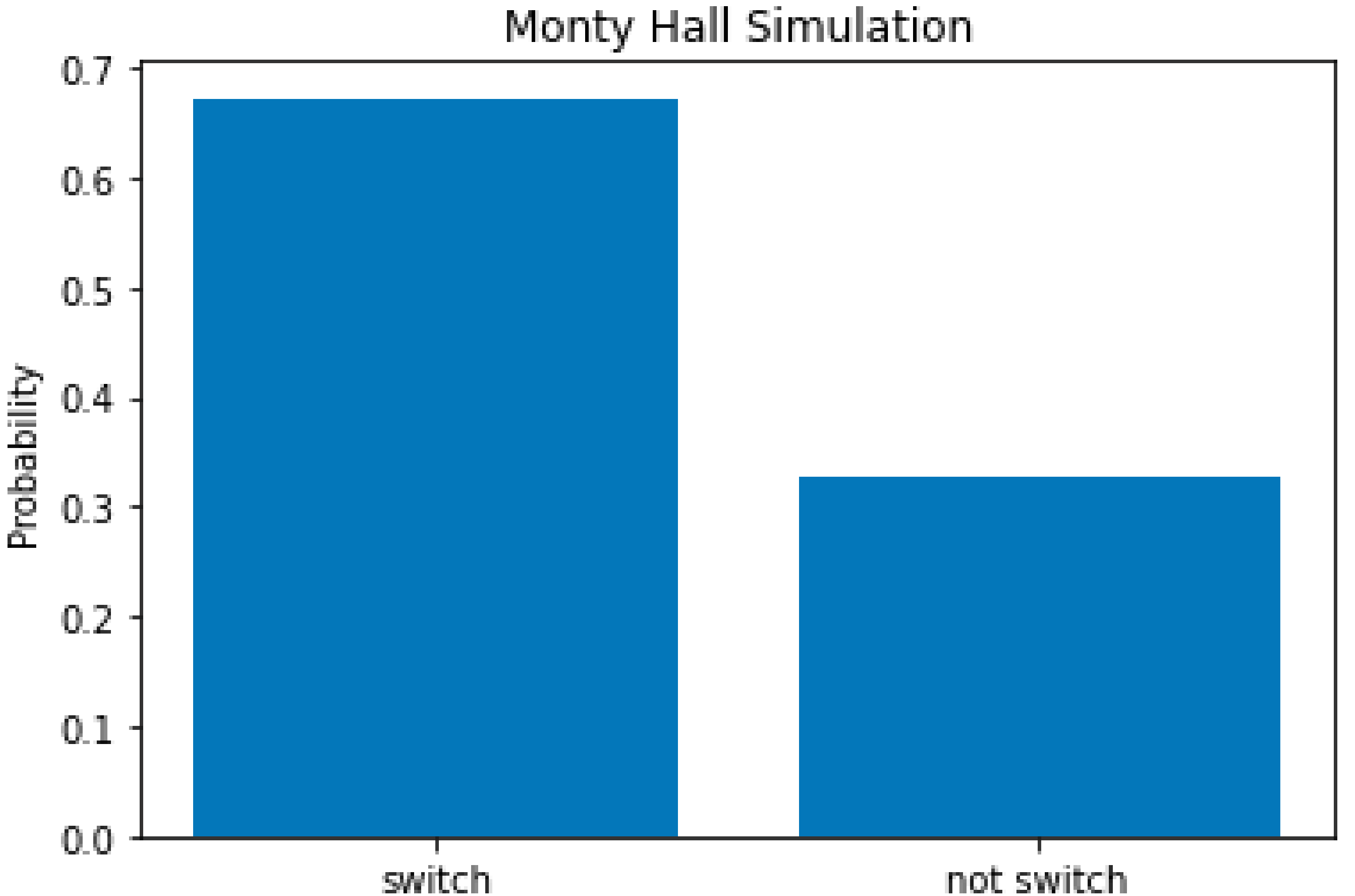
Picture source: http://en.wikipedia.org/wiki/Monty_Hall_problem

Putting it to a Test

```python
# simulates a single trial of the experiment
def monty_hall_single_trial():
    winning_door = randint(1,4) # the door with the car
    contestant_door = randint(1,4) # the door chosen by contestant
    remaining_doors = [i for i in range(1,4) if((i is not winning_door) and (i is not contestant_door))]
    opened_door = remaining_doors[randint(0,len(remaining_doors))] # the door opened by the host
    return (winning_door, contestant_door, opened_door)

# simulates the experiment for num_trial trials
# returns the probability of winning by switching
def monty_hall_simulation(num_trials = 10000):
    num_wins = 0
    for i in range(num_trials):
        (winning_door, contestant_door, opened_door) = monty_hall_single_trial()
        if winning_door != contestant_door:
            num_wins += 1 # contestant wins by switching
    return (num_wins / num_trials)
```

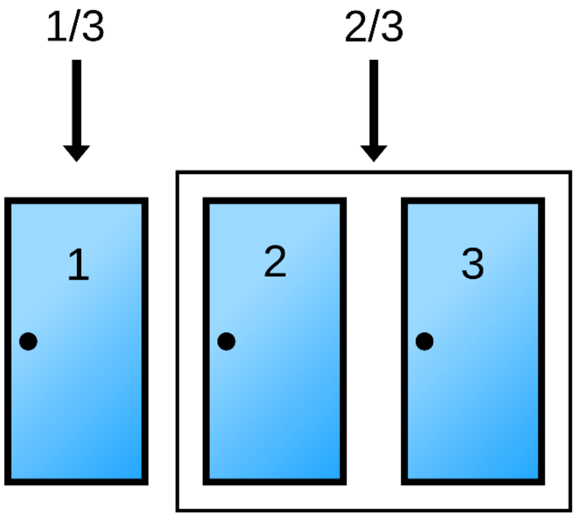
Putting it to a Test



10000 trials

Monty Hall problem: intuition

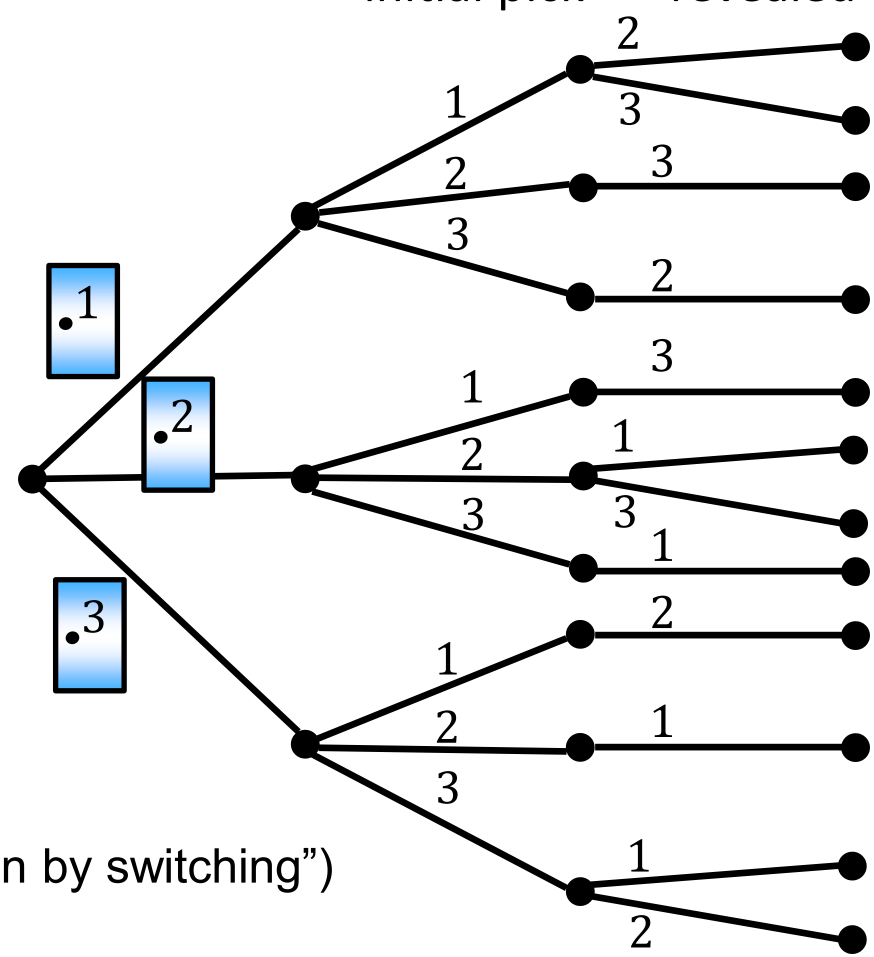
- If you pick the right door initially, you win iff you stick with it.
- If you pick the wrong door initially, you win iff you switch.
- What is the probability you get it right initially?




- ```Sticking with it'' player wins with probability 1/3.
- Switching player wins with probability 2/3.

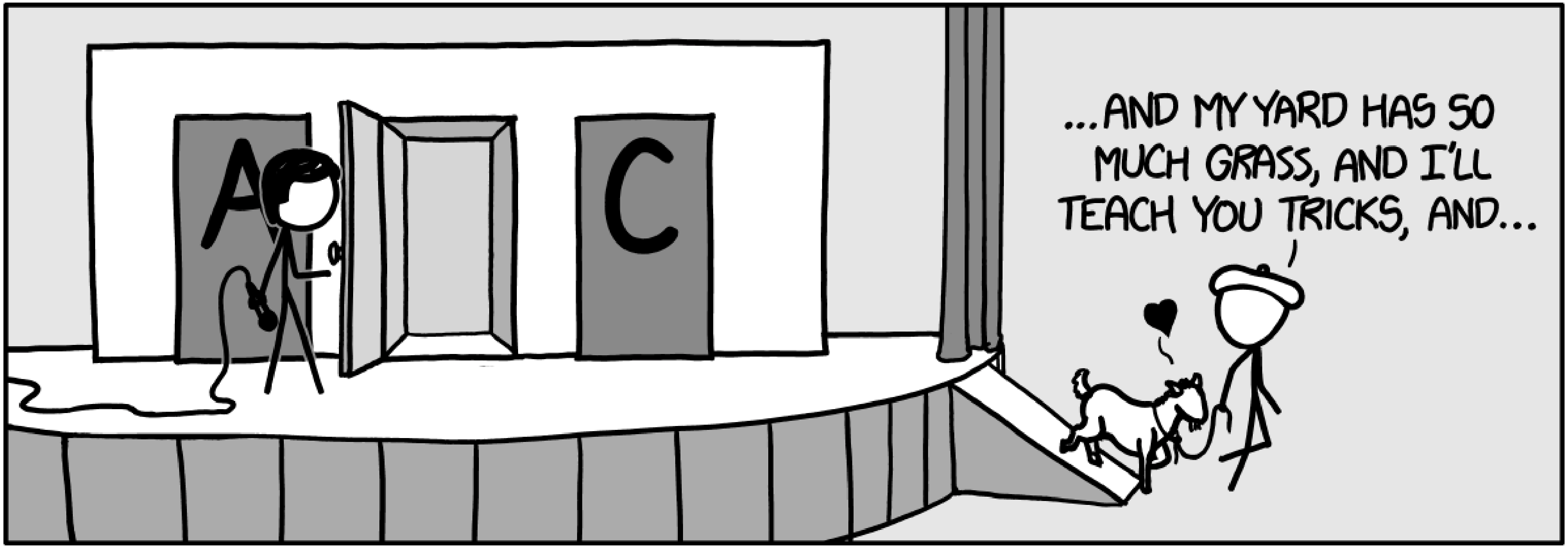
Formal Analysis: Tree Diagram

car location | contestant's initial pick | door revealed | outcome | probability | win by switching?



Pr("win by switching")

Some People Prefer a Goat to a Car



https://xkcd.com/1282/

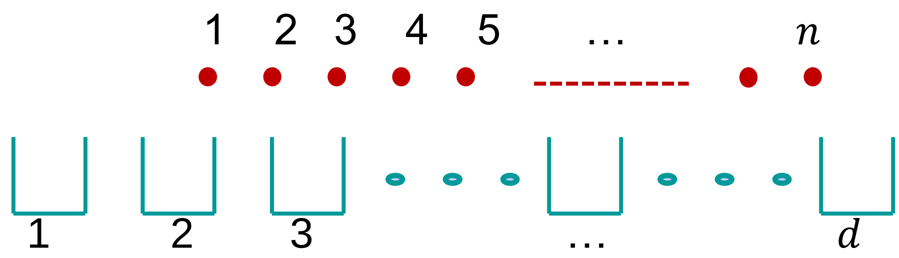
Birthday Paradox

- How many people do you have to have in a room
  – to know for sure that some people share the same birthday?
  – To have $\geq 1/2$ probability of having two people like that?
- Surprisingly few!
- It is called a paradox because it goes against intuition.

Birthday Paradox: Modeling

Suppose for simplicity that

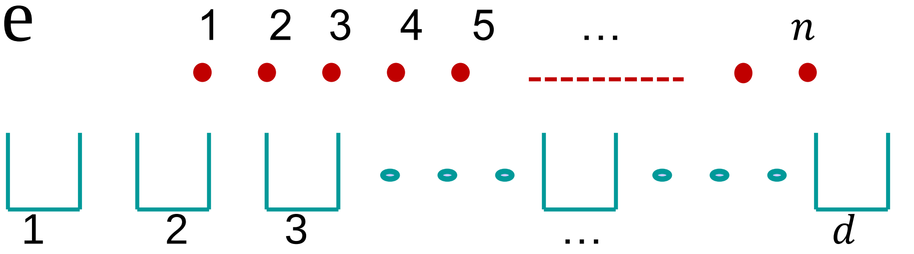
- We have $d = 365$ days
- Each person was born on a uniformly random day
- There are $n = 23$ people



What is the probability that everybody has a different birthday?

Birthday Paradox: Analysis

$d = 365$ days, $n = 23$ people



What is the probability that everybody has a different birthday?