LECTURE 4

Last time
• Axioms of probability
• How to compute probabilities
• Probability rules

Today
• Common fallacies
• Tree diagrams
• Monty Hall problem

Reminders
• HW2 due Thursday
• The collaboration policy you submitted should be signed and dated
A family has 2 children. Assume that each child is equally likely to be a boy or a girl. Which of the following is more likely?

A. They have 2 boys.
B. They have 2 girls.
C. They have 2 kids of different gender.
D. All the three possibilities above are equally likely.
Toss a coin twice. Which outcomes did you get?

A. HH
B. TT
C. Two different outcomes.
``Linda is 31, single, outspoken, and very bright. She majored in philosophy in college. As a student, she was deeply concerned with racial discrimination and other social issues, and participated in anti-nuclear demonstrations.”

Rank the likelihood of the following alternatives:

1) Linda is active in the feminist movement
2) Linda is a bank teller
3) Linda is a bank teller and active in the feminist movement
Conjunction Fallacy

Rank the likelihood of the following alternatives:

1) Linda is active in the feminist movement
2) Linda is a bank teller
3) Linda is a bank teller and active in the feminist movement

Between 80 and 95 percent of the subjects ranked: (1) > (3) > (2)
Conjunction Fallacy

Rank the likelihood of the following alternatives:

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Tree Diagram Method: Example

- An urn contains 3 red balls and 2 green balls
- Draw a ball uniformly at random
  - If the ball drawn is red, add a green ball to the urn
  - If the ball drawn is green, add a red ball to the urn
  - The drawn ball is not returned to the urn
- Draw a second ball uniformly at random
- Determine \( \Pr(\text{“second ball is red”}) \)
Tree Diagram Method: Urn

Pr("second ball is red") =
Monty Hall problem

- 1970 game show hosted by Monty Hall.
- You (the contestant) are shown 3 doors: behind one is a prize and behind the other two are two goats.
- You pick a door, but do not open it.
- Then one of the other two doors is opened to reveal a goat.
- You are given the option of sticking with your choice or picking the other unopened door.
- You win the prize iff you pick the door with the prize.
- Should you switch?

Picture source: http://en.wikipedia.org/wiki/Monty_Hall_problem
Modeling Assumptions

- The prize is equally likely to be behind each of the 3 doors.
- If the host has a choice of which door to open, he is equally likely to pick either of them.
When do you have a higher probability of winning?

A. If you stick with your first choice.
B. If you switch.
C. Both probabilities are the same.
# simulates a single trial of the experiment

def monty_hall_single_trial():
    winning_door = randint(1,4)  # the door with the car
    contestant_door = randint(1,4)  # the door chosen by contestant
    remaining_doors = [i for i in range(1,4) if((i is not winning_door) and (i is not contestant_door))]
    opened_door = remaining_doors[randint(0,len(remaining_doors))]  # the door opened by the host
    return (winning_door, contestant_door, opened_door)

# simulates the experiment for num_trial trials
# returns the probability of winning by switching

def monty_hall_simulation(num_trials = 10000):
    num_wins = 0
    for i in range(num_trials):
        (winning_door, contestant_door, opened_door) = monty_hall_single_trial()
        if winning_door != contestant_door:
            num_wins += 1  # contestant wins by switching
    return (num_wins / num_trials)
Putting it to a Test

Monty Hall Simulation

10000 trials

Probability

switch

not switch

0.7

0.6

0.5

0.4

0.3

0.2

0.1

0.0

Tiago Januario, Sofya Raskhodnikova; based on slides by Alina Ene
Monty Hall problem: intuition

- If you pick the right door initially, you win iff you stick with it.
- If you pick the wrong door initially, you win iff you switch.
- What is the probability you get it right initially?

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Sticking with it'' player wins with probability 1/3.
Switching player wins with probability 2/3.
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Formal Analysis: Tree Diagram

- car location
- contestant’s initial pick
- door revealed
- outcome
- probability
- win by switching?

Pr("win by switching")
Some People Prefer a Goat to a Car

https://xkcd.com/1282/
Birthday Paradox

• How many people do you have to have in a room
  – to know for sure that some people share the same birthday?
  – To have $\geq 1/2$ probability of having two people like that?
• Surprisingly few!
• It is called a paradox because it goes against intuition.
Birthday Paradox: Modeling

Suppose for simplicity that

- We have $d = 365$ days
- Each person was born on a uniformly random day
- There are $n = 23$ people

What is the probability that everybody has a different birthday?
Birthday Paradox: Analysis

$d = 365$ days, $n = 23$ people

What is the probability that everybody has a different birthday?