Probability in Computing

LECTURE 5

CS 237

Last time

- Common fallacies
- Tree diagrams
- Monty Hall problem

Today

- Spinners and Continuous Probability
- Geometric Probability
- Anomalies with Continuous Probability

Reminders

- HW2 due tonight
- Discussion 3 tomorrow

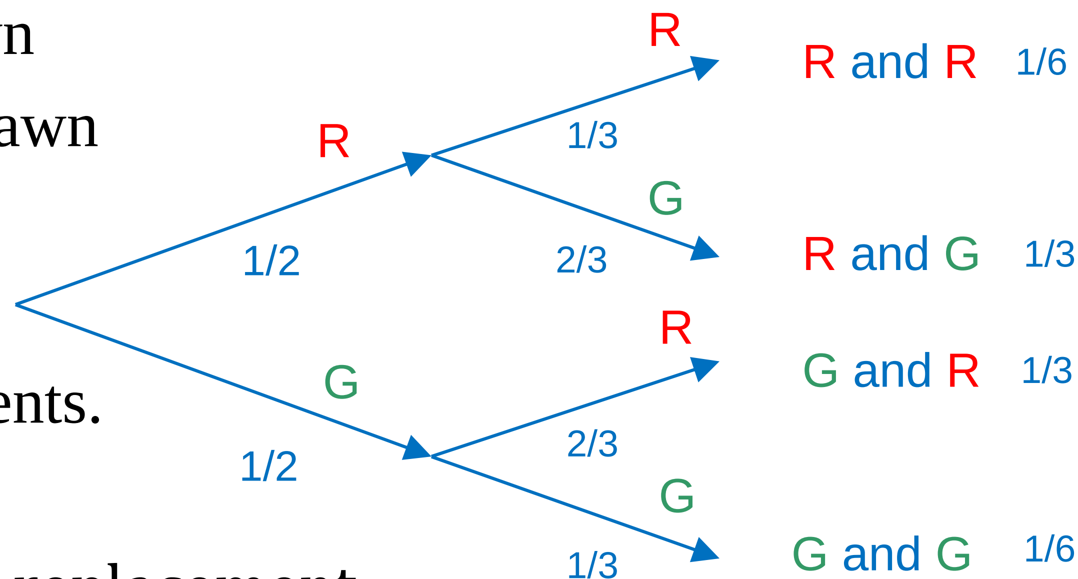
Top Hat question (Join Code: 413437)

Suppose you have a bag containing Red balls and Green balls. You draw one ball from the bag, then a second ball. Let

R = a red ball was drawn

G = a green ball was drawn

The tree diagram shows the experiment. Consider the following three statements.



1. Drawing was without replacement
2. The bag originally contained 2 Red and 2 Green balls
3. Pr(the 2 balls were of different colors) = 2/3

A. Only statement (1) is true.

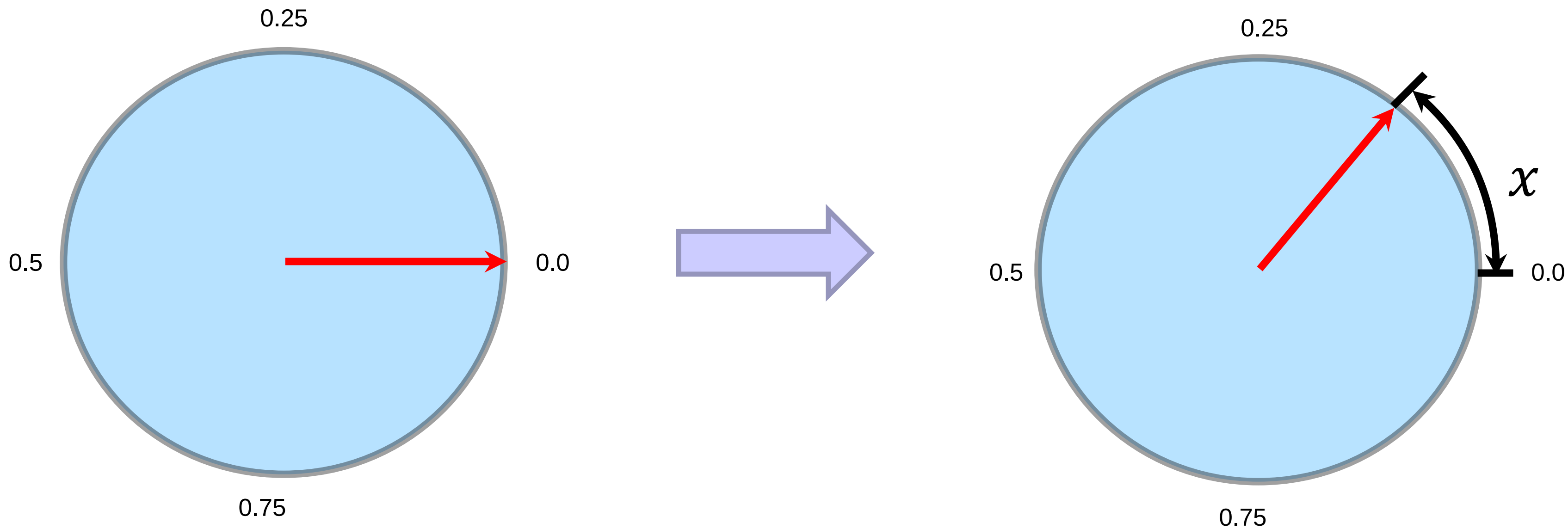
B. Only statement (2) is true.

C. Only (2) and (3) are true.

D. All three statements are true.

Continuous Probability

The Spinner Experiment

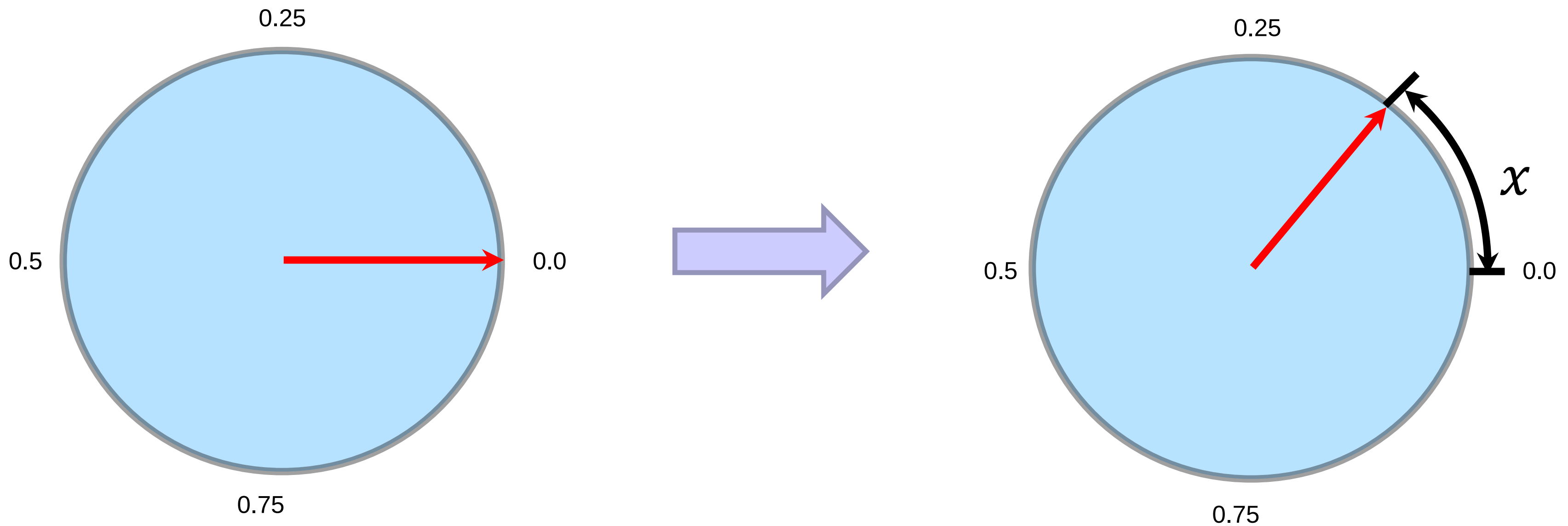


Assumptions:

- The circumference of the circle is 1.0
- When you spin the pointer, it is equally likely to end up anywhere around the circle

Continuous Probability

The Spinner Experiment



- Experiment: spin the pointer, see where it lands!
- Outcome: real number $x \in [0,1]$
- Sample space: $\Omega = [0,1]$

Continuous Probability

The Spinner Experiment

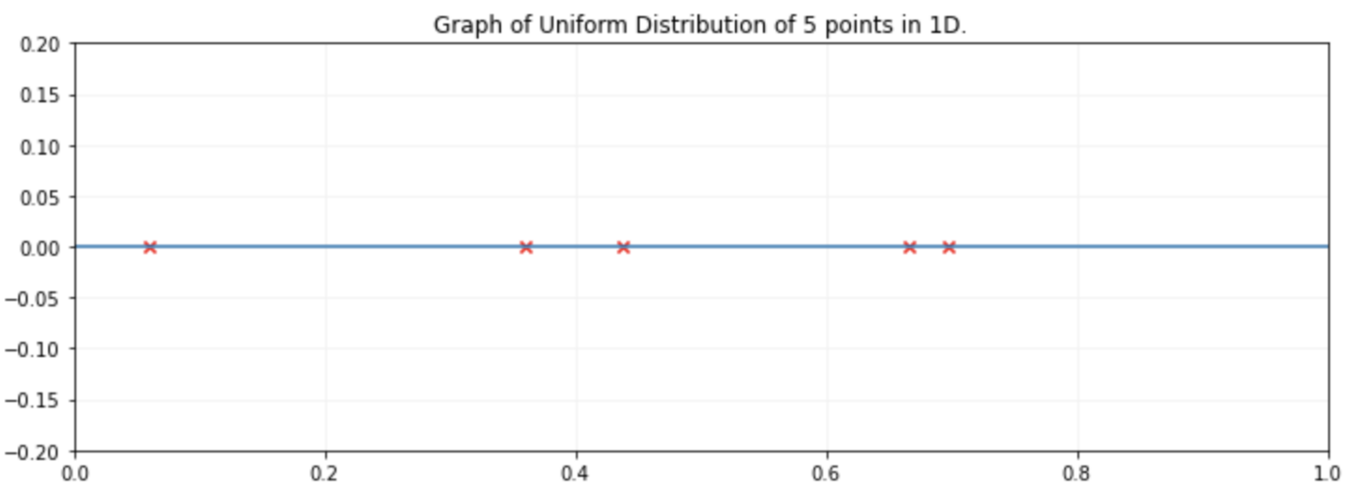
This is implemented by

the numpy function random()!

```
from numpy.random import random

for k in range(5):
    print(random())
```

```
0.2880928755517823
0.7162125723051093
0.6655015245054124
0.554162601188949
0.28532267939275413
```

```
[0.359507900573786, 0.43703195379934145, 0.6976311959272649, 0.06022547162926983, 0.6667667154456677]
```



Continuous Probability

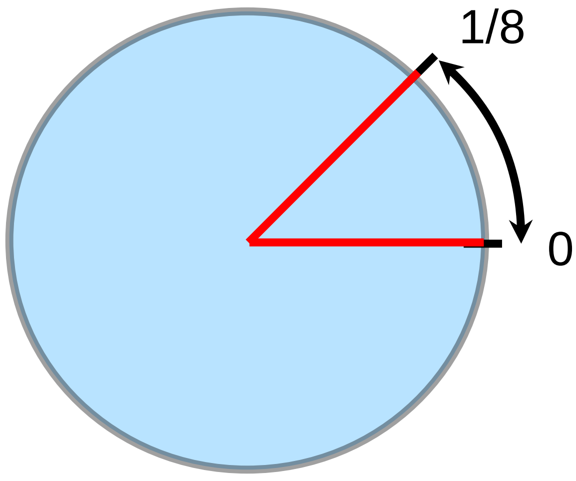
What is the probability that the dial lands in [0, 1/8]?

(A) 1/8

(B) 0

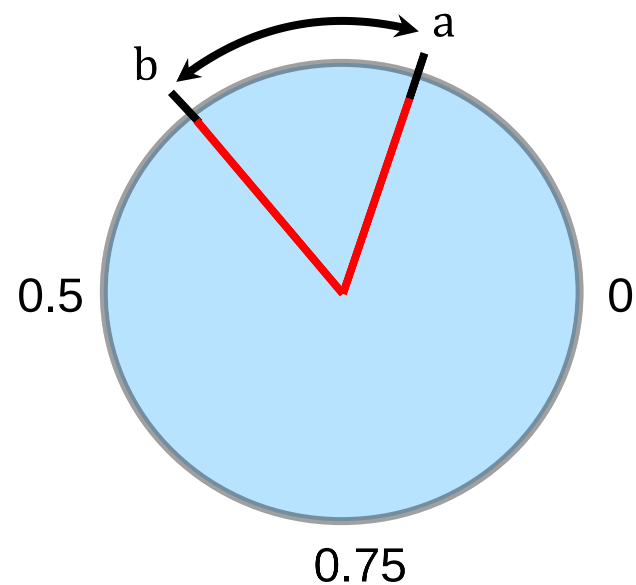
(C) 0.01

(D) none of the above

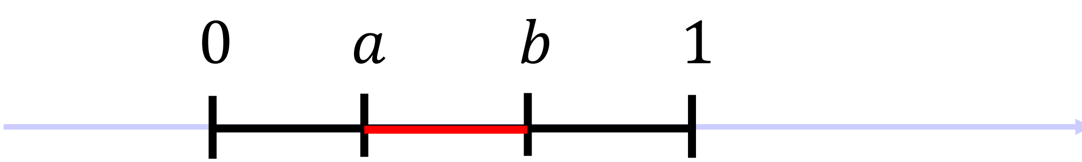


Continuous Probability

- Experiment: spin the pointer, see where it lands!
- Outcome: real number $x \in [0,1)$
- Sample space: $\Omega = [0,1)$
- Probability Function:



$$\Pr(x \in [a,b]) = \frac{\textit{length of } [a,b]}{\textit{length of } [0,1)} = b - a$$



$$[a,b] \subseteq [0,1)$$

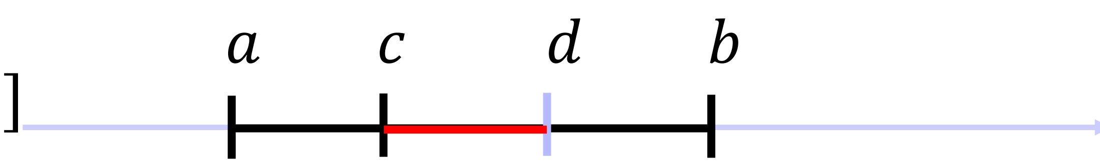
Continuous Probability

Uniform Sampling from an Interval [a..b]

- Experiment: Sample a point uniformly from [a..b]
- Outcome: real number $x \in [a, b]$
- Sample space: $\Omega = [a, b]$
- Probability Function:

$$Pr(x \in [c, d]) = \frac{\textit{length of } [c, d]}{\textit{length of } [a, b]} = \frac{d - c}{b - a}$$

$[c, d] \subseteq [a, b]$



Continuous Probability

Experiment: choose uniformly $x \in [0, 10]$


```python
from numpy.random import uniform

for k in range(5):
    print(uniform(0,10))
```

```
9.458670834431103
7.821264914591
4.367437002029626
1.1280161736707706
0.4086456784560122
```

$$Pr(x \in [0,6]) =$$

$$Pr(x \in [1,6] \cup [8,10]) =$$

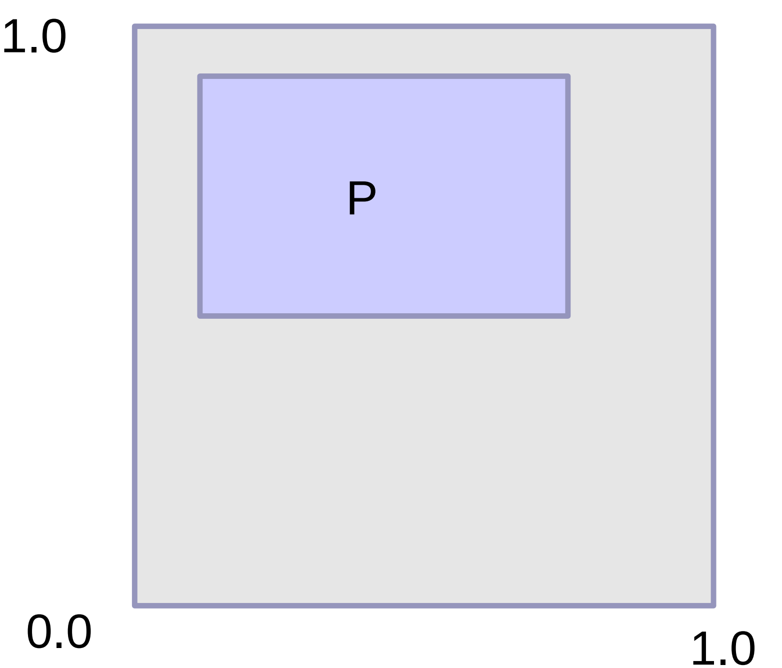
$$Pr(x \in [0,6] \cup [5,7]) =$$

Continuous Probability

Let's kick it up a notch! What about 2d?

- Experiment: Choose a random point in unit square, all points equally likely
- Event: hitting point inside rectangle P
- Sample space: the square
- Probability function:

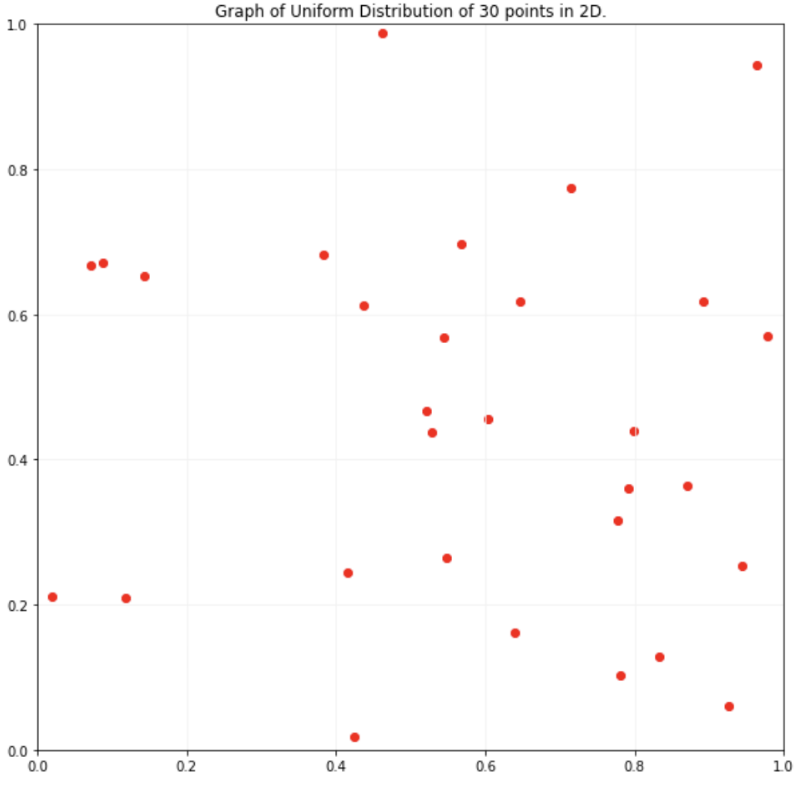
$$\Pr[p \in P] = \frac{\text{area of } P}{\text{area of square}}$$



Continuous Probability

In [41]:

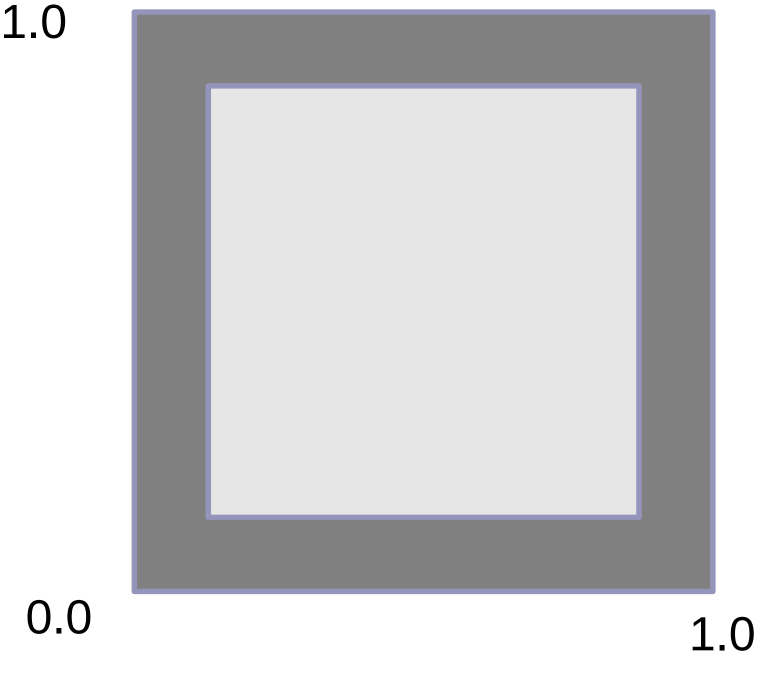
```
def random_plane_plot(num_trials):
    x_vals = [ random() for k in range(num_trials) ]
    y_vals = [ random() for k in range(num_trials) ]
    plt.figure(num=None, figsize=(10, 10))
    plt.title('Graph of Uniform Distribution of '+str(num_trials)+' points in 2D.',fontsize=12)
    plt.grid(color='0.95')
    plt.ylim(0, 1)
    plt.xlim(0,1)
    plt.scatter(x_vals, y_vals,marker="o",color="r")
    plt.show()


random_plane_plot(30)
```



Top Hat question (Join Code: 413437)

Suppose you throw a dart at a square target 1 meter on a side. What is the probability that it lands within 0.1 m of an edge?

A. $0.8^2 = 0.64$
B. $0.9^2 = 0.81$
C. $1 - 0.8^2 = 0.36$
D. $1 - 0.9^2 = 0.19$

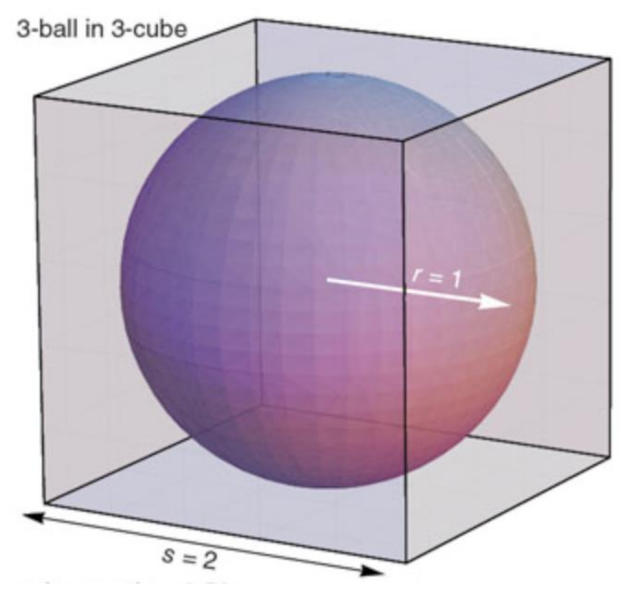


Continuous Probability

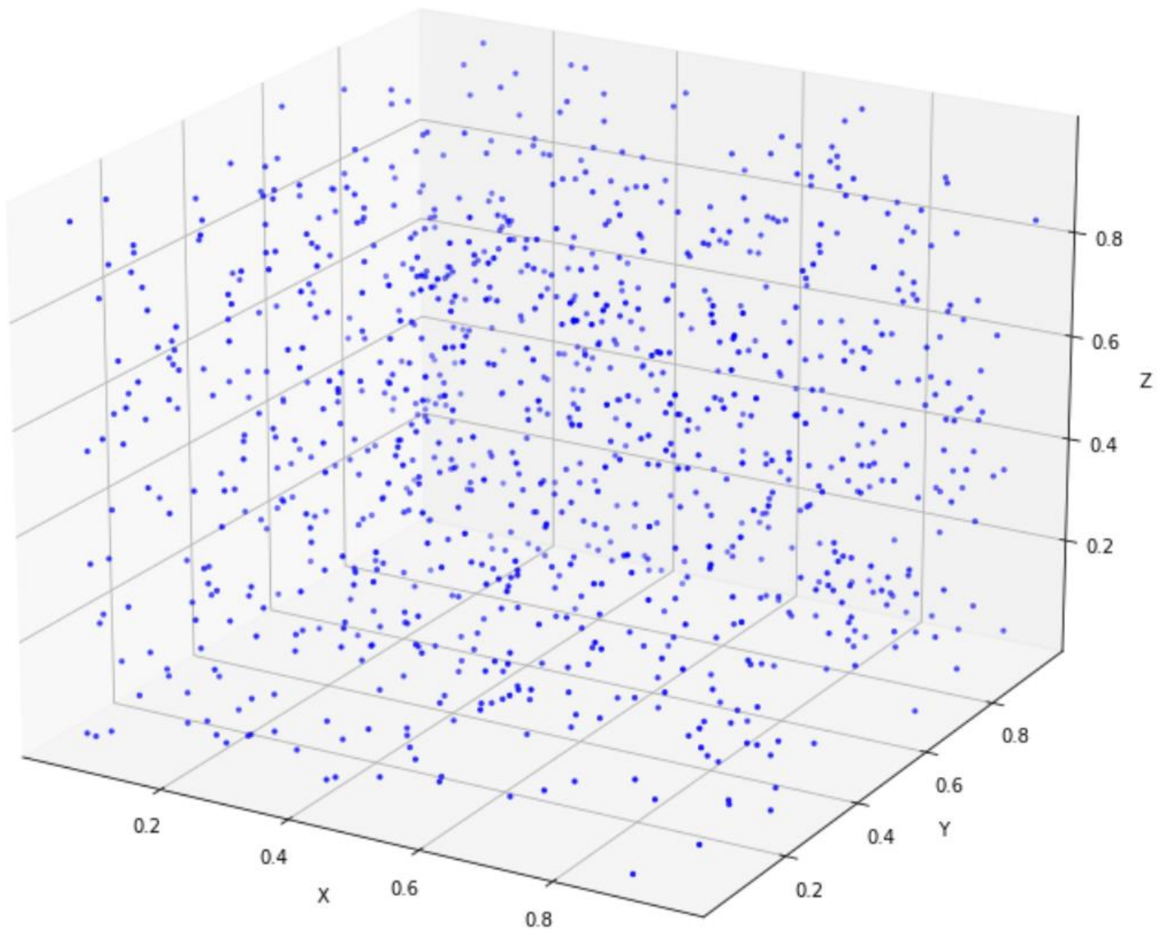
Why stop at 2d?

- Experiment: Choose a random point in 2x2x2 cube
- Event: hitting point inside unit sphere in cube
- Sample space: the cube
- Probability function:

$$\Pr[p \in\,] = \frac{\textit{volume of sphere}}{\textit{volume of cube}}$$



Continuous Probability



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Continuous Probability

Here's a (somewhat) realistic problem:

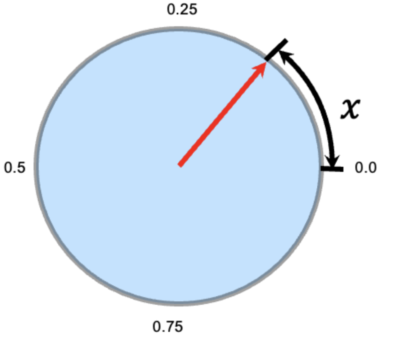
- Wayne and Sofya decide to meet at Starbucks between noon and 1pm to discuss HW 03.
- Each of them arrives at a time chosen uniformly at random between noon and 1pm.
- Once one of them arrives, they will wait for the other for 15 minutes and leave if they do not show up.
- What is the probability that Wayne and Sofya meet?

Continuous Probability

Anomalies with Continuous Probability

Let's go back to the spinner experiment:

What is the probability that the pointer lands exactly on 0.25?



(A) $p = 0$

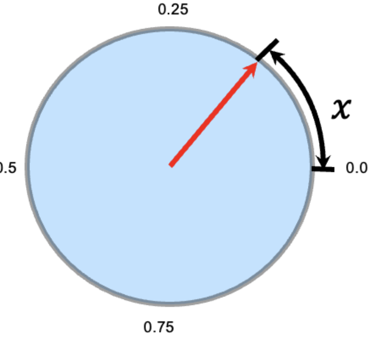
(B) $p > 0$

Continuous Probability

Anomalies with Continuous Probability

Let's go back to the spinner experiment:

What is the probability that the pointer lands exactly on 0.25?



(A) $p = 0$ (B) $p > 0$

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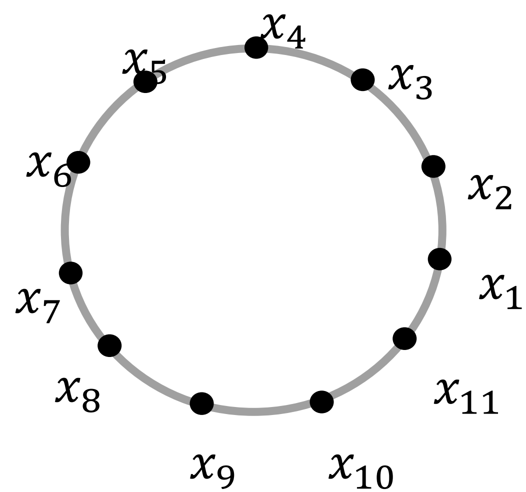
Continuous Probability

- What is the probability that the dial lands on 1/4?
- Suppose we suspect that it is, say, 0.1
- Pick 11 distinct points on the circle



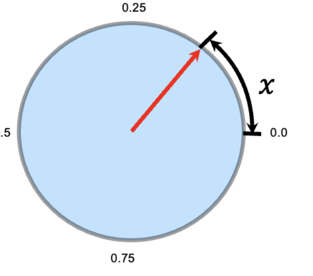
by symmetry: $Pr(lands\ on\ x_i) = 0.1\ for\ all\ i$

by additivity: $Pr(lands\ on\ one\ of\ x_1, \ldots, x_{11}) = 11 \cdot 0.1 = 1.1$

contradiction!

Top Hat question (Join Code: 413437)

Consider the spinner one more time.... We decided that



$$\Pr(\, x \in [0, 1/4] \,) \;=\; 1/4$$

But what is

$$\Pr(\, x \in [0, 1/4) \,) \;=\; ??$$

$$[a, b] \;=\; \{\, x \mid a \le x \le b \,\}$$

$$[a, b) \;=\; \{\, x \mid a \le x < b \,\}$$

A. $1/4 - \epsilon$

B. 1/4

C. The probability does not exist

Continuous Probability

- Discrete sample spaces: outcomes are countable
  - Toss a coin, roll two dice, toss a coin until we get heads, …
- Continuous sample spaces: outcomes are not countable
  - Spinner, darts, time, space, ...

Continuous Probability

- Discrete sample spaces: outcomes are countable
  - We assign a probability to each outcome
  - The probability of an event is the sum of the probabilities of the outcomes comprising the event
- Continuous sample spaces: outcomes are not countable
  - Individual outcomes have probability zero
  - We assign probabilities to the extent (length, area, volume, etc.) taken up by the event of interest