Probability in Computing

CS 237

LECTURE 11

Last time

- Independent Events
- Sequences of Independent Events
- Bayes' Rule

Today

- Another example on Bayes' Rule
- Review

Reminders

- HW 6 is due Thursday

Reading

- LLM 18.18-18.19, 19.2
  P 1.4.1, 3.1.4

Reminders

Bayes' Rule

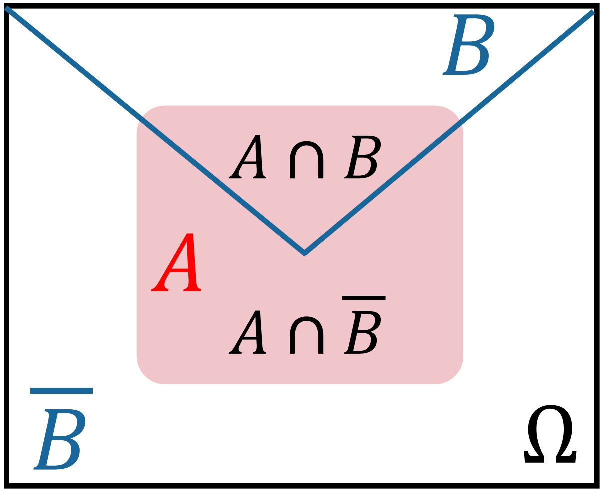
For two events A and B with $\Pr(A) \neq 0$:

$$\Pr(B \mid A) = \frac{\Pr(A \mid B) \cdot \Pr(B)}{\Pr(A)}$$

Law of Total Probability

For two events A and B:

$$\Pr(A) = \Pr(A|B) \cdot \Pr(B) + \Pr(A|\overline{B}) \cdot \Pr(\overline{B})$$



Accurate Medical Tests for Rare Conditions

Can an accurate medical test be wrong most of time?

Problem: A pregnant woman gets several medical tests.

- The doctor says the tests are "99.9% reliable."
- She finds out that a rare genetic disorder affecting about one out of every 4000 births was detected in her fetus.

Given this result and the test's reliability rate, what is the probability that the fetus has the disorder?

A. Less than 25%

B. Between 25% and 75%

C. More than 75%, but less than 99%

D. More than 99%

https://www.nytimes.com/2022/02/10/learning/lesson-plans/the-misleading-math-of-prenatal-tests.html

Terminology for Tests

Fill in the blank spaces with the phrases "false positive," "false negative," and "accurate test."

- *Hint*: "accurate test" should be used twice:

	Test is Positive	Test is Negative
Fetus Has Disorder		
Fetus Does Not Have Disorder		

https://www.nytimes.com/2022/02/10/learning/lesson-plans/the-misleading-math-of-prenatal-tests.html

Accurate Medical Tests for Rare Conditions

Problem: A pregnant woman gets several medical tests.

- The doctor says the tests are "99.9% reliable."
- A disorder affecting about one out of every 4000 births was detected.

Given this result and the test's reliability rate, what is the probability that the fetus has the disorder? **Apply Bayes' Rule!**

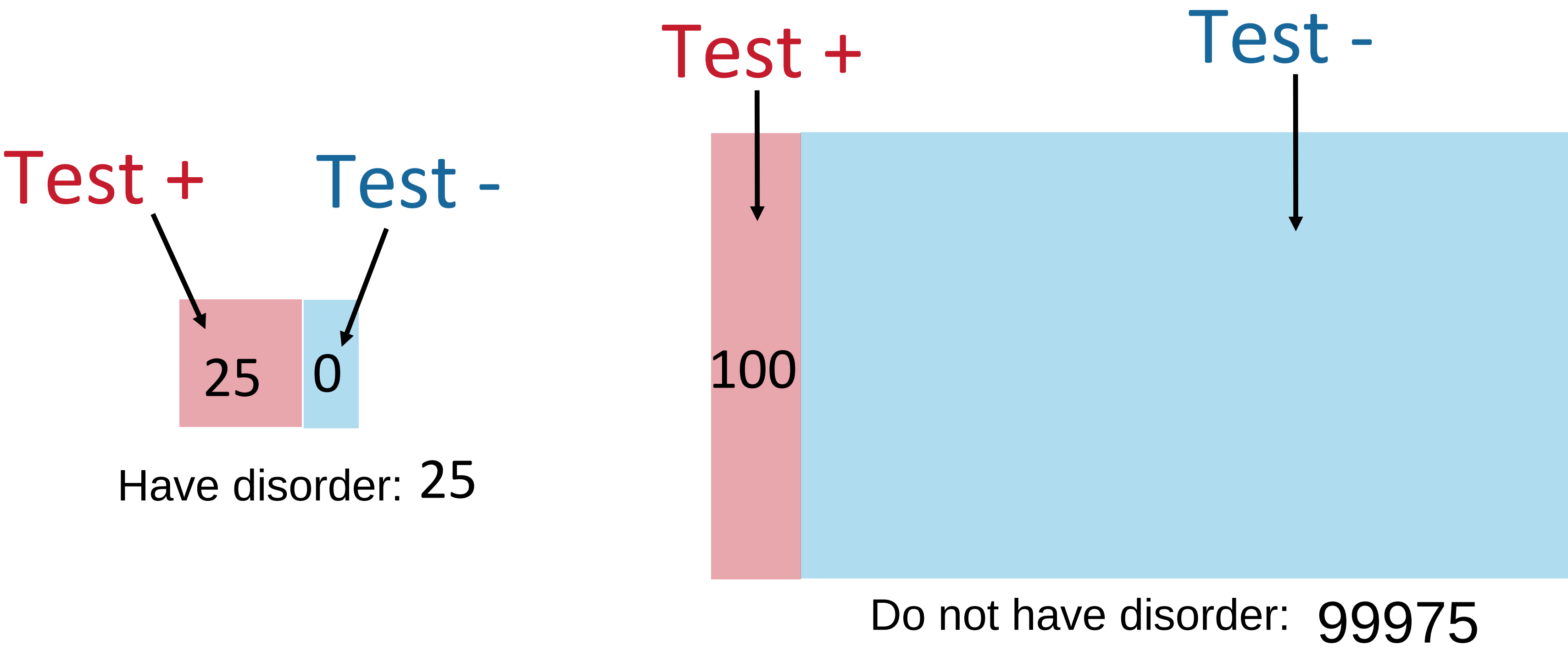
- Let P be the event that the test is positive
- Let D be the event that the fetus has the disorder

https://www.nytimes.com/2022/02/10/learning/lesson-plans/the-misleading-math-of-prenatal-tests.html

Medical Tests: Intuition in Pictures

- The doctor says the tests are “99.9% reliable.”
- A disorder affecting about one out of every 4000 births was detected.

Imagine taking a random sample of 100,000 births



False positives: $0.001 \times 99{,}975 = 99.975 \approx 100$

Tiago Januario, Sofya Raskhodnikova; Probability in Computing
https://www.nytimes.com/2022/02/10/learning/lesson-plans/the-misleading-math-of-prenatal-tests.html

Reminder: Independent Events

Definition: Independent Events

Two events A and B are independent if

$$\Pr(A \cap B) = \Pr(A) \cdot \Pr(B)$$

- **Independent ≠ disjoint!** When events A and B are disjoint: if A happens then B is guaranteed not to happen.

Top Hat question (Join Code: 033357)

There are 365 students signed up for CS237. Assume each student was born on a uniformly random day (excluding February 29).

Give a tight upper bound on the probability that

none of these students have a birthday today.

Hint: Use $1 - x \leq e^{-x}$, where $e \approx 2.71828$

A. 1/365
B. 1/3
C. $1/e$
D. $1/2$
E. 1

The Same Problem with Small Changes

There are $365 \cdot \frac{3}{5} = 219$ students signed up for CS237. Assume each student was born on a uniformly random day (excluding February 29).

Give a tight upper bound on the probability that

none of these students have a birthday today or tomorrow.

Hint: Use $1 - x \leq e^{-x}$, where $e \approx 2.71828$

Top Hat question (Join Code: 033357)

You have two fair coins: gold and silver.

Experiment: toss each coin 3 times

- G = "gold coin came up HEADS all 3 times"
- S = "silver coin came up HEADS all 3 times"

1. Are *G* and *S* independent?
2. Are *G* and *S* disjoint?

A. YES to both

B. NO to both

C. 1. YES 2. NO

D. 1. NO 2. YES

Top Hat question (Join Code: 033357)

You have a fair 4-sided die and two fair coins: gold and silver.

Experiment: roll the die, let D be the number rolled,
and toss each coin D times

- G = "gold coin came up HEADS all *D* times"
- S = "silver coin came up HEADS all *D* times"

1. Are *G* and *S* independent?
2. Are *G* and *S* disjoint?

A. YES to both

B. NO to both

C. 1. YES 2. NO

D. 1. NO 2. YES