Homework 0 – Due Friday, September 6, 2019 before noon

Submit your signed copy of the Collaboration and Honesty Policy as well as solutions to problems 1 and 2 on Gradescope.

Page limit  You can submit at most 1 page per problem, even if the problem has multiple parts. If
you submit a longer solution for some problem, only the first page will be graded. For each problem,
specify your name and collaborators (or state “Collaborators: none”) at the top of the sheet.

Exercises  Please practice on exercises and solved problems in Sipser, Chapter 0 and odd-numbered
exercises in Hammack, Chapters 1, 2, 4–6 (solutions are provided on the book’s web page). The
material they cover may appear on exams.

Problems

0. (0 points) The following steps are required to get you started in the course.

(a) Sign up on piazza at www.piazza.com/bu/fall2019/cs332.
(b) Sign up as a student on Gradescope at www.gradescope.com
   Use your BU email ID to sign up on Gradescope.
(c) Read and sign the Collaboration and Honesty Policy and submit it on Gradescope. We will
   be able to grade your homework only after you hand this in.
(d) Check out the following links and resources:
   i. course webpage: https://cs-people.bu.edu/sofya/cs332/
   ii. supplementary textbook to review proof techniques:
   iii. practice with automata: http://automatatutor.com/ and http://jflap.org/
(e) Fill the sample nameplate on the course webpage with your name, print it, and bring to every
   class.

1. (Logic and sets review, 10 points) Negate the following statements:

(a) The first DFA has an even number of states, but the second DFA does not.
(b) What doesn’t kill you, makes you stronger.

Write the contrapositive of the following statement:

(c) Your homework assignment is not graded if you have not handed in a signed copy of the
   Collaboration policy.

Answer the following questions about basic set operations:

(d) If $A = \{x, y, z\}$ and $B = \{x, y\}$, what is $A \times B$? What is its size?
(e) If $A = \{x, y, z\}$, what is the power set of $A$? What is its size?
2. **(Proof techniques review, 10 points)** Type (preferred) or write the answer to this problem on a single sheet of paper. Specify your name, section and collaborators (or state “Collaborators: none”) at the top of the sheet.

(a) **(Induction.)** Find the error in the following proof that all BU students have the same GPA. Indicate the first sentence in the proof that is incorrect. (There must be an incorrect statement in the proof, since the claim is wrong.)

**Claim:** In every set of n BU students, all students have the same GPA.

**Proof:** By induction on n.

**Base case:** $n = 1$. In every set containing just one BU student, all students clearly have the same GPA.

**Induction step:** For $k \geq 1$ assume that the claim holds for $n = k$ and prove that it holds for $n = k + 1$. Take any set $S$ of $k + 1$ BU students. Remove one student from this set to obtain the set $S_1$ with $k$ students. By the induction hypothesis, all students in $S_1$ have the same GPA. Now replace the removed student and remove a different one to obtain the set $S_2$. By the same argument, all students in $S_2$ have the same GPA. Therefore, all the students in $S$ must have the same GPA.

(b) **(Direct proof.)** Give a direct proof that the following automaton accepts an infinite number of strings.

(c) **(Contradiction.)** Prove the same thing as in part (b) by contradiction.