Homework 0 – Due Friday, September 7, 2018 at 11:59am

Submit your signed copy of the Collaboration and Honesty Policy as well as solutions to problems 1 and 2 on Gradescope.

**Page limit** You can submit at most 1 page per problem, even if the problem has multiple parts. If you submit a longer solution for some problem, only the first page will be graded. For each problem, specify your name and collaborators (or state “Collaborators: none”) at the top of the sheet.

**Exercises** Please practice on exercises and solved problems in Sipser, Chapter 0 and odd-numbered exercises in Hammack, Chapters 1, 2, 4–6 (solutions are provided on the book’s web page). The material they cover may appear on exams.

**Problems**

0. (0 points) The following steps are required to get you started in the course.

   (a) Sign up on piazza at www.piazza.com/bu/fall2018/cs332.

   (b) Sign up as a student on Gradescope at www.gradescope.com, with course entry code 9Y75ZD. Use your BU email ID to sign up on Gradescope.

   (c) Read and sign the Collaboration and Honesty Policy and submit it on Gradescope. We will be able to grade your homework only after you hand this in.

   (d) Check out the following links and resources:

      i. course webpage: https://cs-people.bu.edu/sofya/cs332/

      ii. supplementary textbook to review proof techniques:


      iii. practice with automata: http://automatatutor.com/ and http://jflap.org/

1. (Logic and sets review, 10 points) Negate the following statements:

   (a) Problems 1 and 2 have to be handed in to the teaching fellow whereas the Collaboration and Honesty Policy has to be submitted on Gradescope.

   (b) Every student in the class will miss at least one lecture.

   Answer **yes** or **no** to the following question and give a 1-sentence explanation:

   (c) Your professor said that if fewer than 80% of the students hand in their homework on Wednesday, there will be a quiz on Thursday. About 95% of the students handed in their homework on Wednesday, but the professor still gave a quiz on Thursday. Did she lie?

   Answer the following questions about basic set operations:

   (d) If $A = \{x, y, z\}$ and $B = \{(1, 2), (3, 4)\}$, what is $A \times B$? What is its size?

   (e) If $A = \{\emptyset, \{1, 2\}\}$, what is the power set of $A$? What is its size?
2. **(Proof techniques review, 10 points)** Suppose we are trying to divide a class of \( n \) students into groups of either 4 or 5 students.

(a) Find an error in the following proof that a class with \( n \geq 8 \) students can be divided into groups of 4 or 5. That is, **identify the first incorrect sentence** and **explain** what went wrong.

**Proof.** The proof is by strong induction. Let \( P(n) \) be the proposition that a class with \( n \) students can be divided into teams of 4 or 5.

**Base case:** We prove that \( P(8), P(9), \) and \( P(10) \) are all true by showing how to break classes of these sizes into groups of 4 or 5 students:

\[
\begin{align*}
8 & = 4 + 4; \\
9 & = 4 + 5; \\
10 & = 5 + 5.
\end{align*}
\]

**Induction hypothesis:** Next, we must show that \( P(8), ..., P(n) \) imply \( P(n+1) \) for all \( n \geq 10 \). That is, we assume that \( P(8), ..., P(n) \) are all true and show how to divide up a class of \( n+1 \) students into groups of 4 or 5. We first form one group of 4 students. Then we can divide the remaining \( n - 3 \) students into groups of 4 or 5 by the assumption \( P(n - 3) \). This proves \( P(n+1) \), and so the claim holds by induction. \( \square \)

(b) Provide a correct strong induction proof that a class with \( n \geq 12 \) students can be divided into groups of 4 or 5.