Homework 1 – Due Friday, September 13, 2019 before noon

Reminder  Collaboration is permitted, but you must write the solutions by yourself without assistance, and be ready to explain them orally to the course staff if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

Exercises  Please practice on exercises and solved problems in Chapters 1. The material they cover may appear on exams.

Problems  This homework contains 1 automatically graded problem. Your solution to all other problems should be submitted on Gradescope. There are 3 mandatory problems and one optional problem, worth 10 points each.

1. (DFA/NFA constructions) Your solution to this problem will be automatically graded by automatatutor.com. Please create an account there using your name and @bu.edu official email address. This is important for recording your grades correctly. Use practice problems on the website to figure out the interface. We will post instructions on how to access the assignment on piazza. (We will rescale the points assigned by Automata Tutor, so that each part is worth 2 points.) If you run into any trouble, post questions on Piazza. If you experience difficulties with the website (e.g., the website is unresponsive), please take a screenshot and send it to the email given on http://automatatutor.com/index.

Give state diagrams of DFAs with as few states as you can recognizing the following languages.

(a) \( L_1 = \{ w \mid w \text{ begins with a } 1 \text{ and ends with a } 0 \} \).
(b) \( L_2 = \{ w \mid w \text{ represents a binary number that is congruent to } 1 \mod 3 \} \). In other words, this number minus 1 is divisible by 3. The number is presented starting from the most significant bit and can have leading 0s.
(c) \( L_3 = \{ w \mid w \text{ is a string of the form } x_1y_1x_2y_2\ldots x_ny_n \text{ for some natural number } n \text{ such that if } x \text{ is the integer with binary representation } x_1x_2\ldots x_n \text{ and } y \text{ is the integer with binary representation } y_1y_2\ldots y_n \text{ then } x > y \} \). Both \( x \) and \( y \) are presented starting from the most significant bit and can have leading 0s.

Give state diagrams of NFAs with as few states as you can recognizing the following languages.

(d) \( L_4 = \{ w \mid w \text{ contains an even number of occurrences of } 10 \} \).
(e) \( L_5 = \{ w \mid w \text{ contains substrings } 1000 \text{ and } 10 \text{ which do not overlap} \} \).

2. (a) (Addition with three rows) Book, 1.32;
   (b) (Unary multiple of \( n \)) Book, 1.36. Hint: Try proving it for small values of \( n \) first.

3. (Closure properties) Prove that the class of regular languages is closed under the following operations (for all parts, give formal descriptions of FAs you construct).
(a) Given a string $w$ of 0s and 1s, the flip of $w$ is obtained by changing all 0s in $w$ to 1s and all 1s in $w$ to 0s. Given a language $A$, the flip of $A$ is the language $\{w \mid \text{the flip of } w \text{ is in } A\}$. Prove that the class of regular languages is closed under the flip operation.

(b) Given languages $A$ and $B$ over alphabet $\Sigma$, the perfect interleaving of $A$ and $B$ is defined as

$$\{w \mid w = a_1b_1 \cdots a_kb_k, \text{ where } a_1 \cdots a_k \in A \text{ and } b_1 \cdots b_k \in B, \text{ each } a_i, b_i \in \Sigma\}.$$ 

Prove that the class of regular languages is closed under perfect interleaving.

4* (Optional, no collaboration is allowed, ONE-OUT operation) Let $A$ be any language. Define ONE-OUT($A$) to be the language containing all strings that can be obtained by removing one symbol from a string in $A$. That is, ONE-OUT($A$) = $\{xz \mid xyz \in A \text{ where } x, z \in \Sigma^*, y \in \Sigma\}$. Show that the class of regular languages is closed under the ONE-OUT operation. Give both a proof by picture and a more formal proof by construction, like on page 61 of the book.