CS 332: *Theory of Computation*
Boston University

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September 5, 2019

Homework 1 – Due Friday, September 13, 2019 before noon

Reminder Collaboration is permitted, but you must write the solutions *by yourself without assistance*, and be ready to explain them orally to the course staff if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

Exercises Please practice on exercises and solved problems in Chapters 1. The material they cover may appear on exams.

Problems *This homework contains 1 automatically graded problem. Your solution to all other problems should be submitted on Gradescope.* There are 3 mandatory problems and one optional problem, worth 10 points each.

1. (**DFA/NFA constructions**) Your solution to this problem will be automatically graded by `automatatutor.com`. Please create an account there using your name and `@bu.edu` official email address. This is important for recording your grades correctly. Use practice problems on the website to figure out the interface. We will post instructions on how to access the assignment on piazza. (We will rescale the points assigned by Automata Tutor, so that each part is worth 2 points.) If you run into any trouble, post questions on `Piazza`. If you experience difficulties with the website (e.g., the website is unresponsive), please take a screenshot and send it to the email given on `http://automatatutor.com/index`.

   Give state diagrams of DFAs with as few states as you can recognizing the the following languages.

   (a) $L_1 = \{w \mid w \text{ begins with a 1 and ends with a 0}\}$.

   (b) $L_2 = \{w \mid w \text{ represents a binary number that is congruent to 1 modulo 3}\}$. In other words, this number minus 1 is divisible by 3. The number is presented starting from the most significant bit and can have leading 0s.

   (c) $L_3 = \{w \mid w$ is a string of the form $x_1y_1x_2y_2 \dots x_ny_n$ for some natural number n such that if x is the integer with binary representation $x_1x_2 \dots x_n$ and y is the integer with binary representation $y_1y_2 \dots y_n$ then $x > y\}$. Both x and y are presented starting from the most significant bit and can have leading 0s.

   Give state diagrams of NFAs with as few states as you can recognizing the the following languages.

   (d) $L_4 = \{w \mid w \text{ contains an even number of occurrences of 10}\}$.

   (e) $L_5 = \{w \mid w \text{ contains substrings 1000 and 10 which do not overlap}\}$.

2. (a) (**Addition with three rows**) Book, 1.32;

   (b) (**Unary multiple of** n) Book, 1.36. *Hint:* Try proving it for small values of n first.

3. (**Closure properties**) Prove that the class of regular languages is closed under the following operations (for all parts, give formal descriptions of FAs you construct).

(a) Given a string w of `0`s and `1`s, the *flip* of w is obtained by changing all `0`s in w to `1`s and all `1`s in w to `0`s. Given a language A, the flip of A is the language $\{w \mid \text{the flip of } w \text{ is in } A\}$. Prove that the class of regular languages is closed under the flip operation.

(b) Given languages A and B over alphabet Σ, the *perfect interleaving* of A and B is defined as

$$\{w | \ w = a_1 b_1 \cdots a_k b_k, \text{ where } a_1 \cdots a_k \in A \text{ and } b_1 \cdots b_k \in B, \text{ each } a_i, b_i \in \Sigma\}.$$

Prove that the class of regular languages is closed under perfect interleaving.

4* (**Optional, no collaboration is allowed, ONE-OUT operation**) Let A be any language. Define ONE-OUT(A) to be the language containing all strings that can be obtained by removing one symbol from a string in A. That is, ONE-OUT(A) $= \{xz \mid xyz \in A \text{ where } x, z \in \Sigma^*, y \in \Sigma\}$. Show that the class of regular languages is closed under the ONE-OUT operation. Give both a proof by picture and a more formal proof by construction, like on page 61 of the book.