Homework 10 – Due Friday, December 6, 2019 <u>before</u> noon

This homework contains 4 mandatory and 1 optional problem, worth 10 points each.

Reminder Collaboration is permitted, but you must write the solutions *by yourself without assistance*, and be ready to explain them orally to the instructor if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

Exercises Please practice on exercises and solved problems in Chapter 7 and on problem 7.20. The material they cover may appear on exams.

Problems

1. (**Hamiltonian Path**) Read the reduction from *3SAT* to *HAMPATH* on page 314 of Sipser.
   (a) Is this construction also a valid polynomial-time reduction from *2SAT* to *HAMPATH*?
   (b) Draw the graph G that the reduction outputs on input formula $\phi = (\bar{x} \vee y) \wedge (x \vee \bar{y})$. For both satisfying assignments of ϕ, give a corresponding Hamiltonian path in G.
   (c) Draw the graph G that the reduction outputs on input formula $\phi = (x \vee y) \wedge (\bar{x} \vee \bar{y}) \wedge (\bar{x} \vee y) \wedge (x \vee \bar{y})$. Argue that G does not have a Hamiltonian path (not relying on the fact that we already proved that the reduction is correct).
   (d) Why would a polynomial-time reduction from *HAMPATH* to *2SAT* have surprising implications, but a reduction in the other direction does not?

2. (**Edge Orientation**) In a directed graph, the ***indegree*** of a node is the number of incoming edges and the ***outdegree*** of a node is the number of outgoing edges. Show that the following problem is NP-complete. Given an undirected graph G and a subset S of the nodes in G, determine whether it possible to convert G to a directed graph by assigning directions to each of its edges so that every node in S has indegree 0 or outdegree 0, and all remaining nodes in G have indegree at least 1.

3. (**Minesweeper**) Sipser, 7.32.

4. (**MIN-FORMULA**) Two Boolean formulas are ***equivalent*** if they have the same set of variables and are true on the same set of assignments to those variables (in other words, they describe the same Boolean function). For example, $\overline{x} \wedge \overline{y}$ and $\overline{x \vee y}$ are equivalent. A Boolean formula is ***minimal*** if no shorter Boolean formula is equivalent to it. Let MIN-FORMULA=$\{\langle\phi\rangle \mid \phi$ is a minimal Boolean formula$\}$.
   (a) Show that MIN-FORMULA is in PSPACE.
   (b) Your friend wants to convince you that $\overline{\text{MIN-FORMULA}}$ in NP.
   She says that if $\phi \in \overline{\text{MIN-FORMULA}}$ then there exists a formula ψ shorter than ϕ that is equivalent to ϕ. An NTM can verify that $\phi \in \overline{\text{MIN-FORMULA}}$ by guessing ψ.
   Explain why this argument fails to show that $\overline{\text{MIN-FORMULA}}$ in NP.

5*. (**Optional, no collaboration**)

A **2cnf-formula** is an AND of clauses, where each clause is an OR of at most two literals. Let 2SAT $= \{\langle\phi\rangle \mid \phi$ is a satisfiable 2cnf-formula$\}$. Show that 2SAT$\in$P.

Hint: The clause $(x \vee y)$ *is logically equivalent to each of the expressions* $(\overline{x} \rightarrow y)$ *and* $(\overline{y} \rightarrow x)$*.* Use a graph to represent the formula.