Homework 10 – Due Friday, December 8, 2017 before noon

This homework contains 4 problems, worth 10 points each.

Reminder  Collaboration is permitted, but you must write the solutions by yourself without assistance, and be ready to explain them orally to the instructor if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

Exercises  Please practice on exercises and solved problems in Chapter 7 and on the following exercise. The material they cover may appear on exams.

1. (PATH) Sipser, 7.20.

Problems

1. (Systems of linear inequalities) A linear inequality over variables $x_1, \ldots, x_k$ is an inequality of the form $c_1 x_1 + \ldots + c_k x_k \leq b$, where $c_1, \ldots, c_k$ and $b$ are integers. E.g., $5x_1 - 3x_2 + x_3 \leq -1$ is a linear inequality. A system of linear inequalities is a set of inequalities over the same variables. Such a system has an integer solution if one can assign integer values to all variables in such a way that all inequalities are satisfied.

Formulate as a language LE the problem of deciding whether a given system of linear inequalities has an integer solution.

(a) Prove that LE is in NP.

(b) Give a polynomial time reduction from 3SAT to LE.

(Careful: make sure you are doing it in the direction specified.)

2. (CONTENTED-SAT) Let CONTENTED-SAT= $\{\langle \phi \rangle \mid \phi$ has at least 32 satisfying assignments $\}$. Show that CONTENTED-SAT is NP-complete.

3. (Minesweeper) Sipser, 7.32.

4. (Hamiltonian Path) Read the reduction from 3SAT to HAMPATH on page 314 of Sipser.

(a) Is this construction also a valid polynomial-time reduction from 2SAT to HAMPATH?

(b) Draw the graph $G$ that the reduction outputs on input formula $\phi = (\bar{x} \lor y) \land (x \lor \bar{y})$. For both satisfying assignments of $\phi$, give a corresponding Hamiltonian path in $G$.

(c) Draw the graph $G$ that the reduction outputs on input formula $\phi = (x \lor y) \land (\bar{x} \lor \bar{y}) \land (\bar{x} \lor y) \land (x \lor \bar{y})$. Argue that $G$ does not have a Hamiltonian path (not relying on the fact that we already proved that the reduction is correct).

(d) Why would a polynomial-time reduction from HAMPATH to 2SAT have surprising implications, but a reduction in the other direction does not?