Homework 3 – Due Friday, September 28, 2018 before noon

Reminder  Collaboration is permitted, but you must write the solutions by yourself without assistance, and be ready to explain them orally to the course staff if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

Exercises  Please practice on exercises and solved problems in Chapter 2. The material they cover may appear on exams. Pay especial attention to Problem 2.18. It will be helpful for one of the problems on this homework.

Problems

1. (CFGs) Give CFGs that generate the following languages. Use as few variables as you can (and no more than requested). Unless specified otherwise, the alphabet is \( \Sigma = \{0, 1\} \).
   (a) \( L_1 = \{ w \mid w \text{ starts with } 0'1 \text{ and ends with } 10^i \text{ for some } i \geq 0 \} \).
       Use at most 3 variables.
   (b) Let \( \Sigma_2 = \{[0], [1], [\varepsilon], [1] \} \).
       \( L_2 = \{ w \in \Sigma_2^* \mid \text{the two rows of } w \text{ have the same sum} \} \).
       Use at most 2 variables.
   (c) \( L_3 = \{ w \mid w \text{ represents a binary number that starts with } 1 \text{ and is divisible by } 3 \} \).
       Use at most 4 variables.
   (d) \( L_4 = \{ x_1 \# x_2 \# \cdots \# x_k \mid k \geq 1, \text{each } x_i \in \{0, 1\}^*, \text{and } x_i = x_j^R \text{ for some } i \neq j \} \).
       Recall that \( w^R \) represents string \( w \) written backwards. Use at most 4 variables.

2. (PDAs) Draw state diagrams of PDAs (with as few states as you can) that recognize the following languages. Write an algorithmic description for you PDAs.
   (a) The language \( L_1 \) from Problem 1(a): (i) diagram; (ii) algorithmic description.
   (b) The language \( L_2 \) from Problem 1(b): (i) diagram; (ii) algorithmic description.

3. (Non-CFLs) Prove that the following languages are not context-free.
   (a) \( A = \{ 0^m1^n0^m1^n \mid m, n \geq 0 \} \).
   (b) Let \( \Sigma_2 = \{[0], [1], [\varepsilon], [1] \} \).
       \( B = \{ w \in \Sigma_2^* \mid \text{the bottom row of } w \text{ is the reverse of the top row of } w \text{ and the number of } 0s \text{ and } 1s \text{ in } w \text{ is the same} \} \).
   (c) The following language over the alphabet \( \{a, b, c\} \):
       \( C = \{ a^i x \mid i \geq 0, x \in \{b, c\}^* \text{, and if } i = 1 \text{ then } x = w w \text{ for some string } w \} \).
       (Careful: \( C \) satisfies the pumping lemma for CFLs! Make sure you understand why, but you don’t need to write it down.)

4* (Optional, no collaboration is allowed) Give a PDA that recognizes the following language over the alphabet \( \Sigma = \{0, 1\} : L = \{ xyz \mid x, z \in \Sigma^* \text{ and } y \in \Sigma^*1\Sigma^*, \text{where } |x| = |z| \geq |y| \} \).