CS 332: *Elements of Theory of Computation* — Professor Sofya Raskhodnikova
Boston University — October 3, 2019

Homework 4 – Due Friday, October 11, 2019 before noon

This homework contains 2 problems, worth 10 points each.

Reminder Collaboration is permitted, but you must write the solutions *by yourself without assistance*, and be ready to explain them orally to the instructor if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

Problems Please practice on exercises and solved problems in Chapter 3. The material they cover may appear on exams.

1. (**TM descriptions**) In this problem, you will look at three ways of describing TMs: (a) formal, (b) implementation level and (c) high level (or a description of the underlying algorithm).

   (a) Sipser, 3.2c. Please use the same representation for your configurations as on p. 168.

   (b) Give implementation level description of a TM, possibly nondeterministic and/or with multiple tapes, that recognizes the following language over the alphabet $\{0, 1\}$:
   $\{w \mid$ string w contains some substring x which starts at position i and is equal to the binary representation of i (with no leading 0s), for some positive integer $i\}$.

   (c) Sipser, 3.7

   (d) Formulate the language that M_{bad} in part (c) was intended to recognize and describe a correct TM that recognizes this language.

2. (**Closure properties**) Given a string w over alphabet $\Sigma = \{0, 1\}$, define

$$\text{BALANCE}(w) = \begin{cases} (01)^x & \text{if } x = y, \\ (01)^y 0^{x-y} & \text{if } x > y, \\ (01)^x 1^{y-x} & \text{if } x < y, \end{cases}$$

   where x is the number of 0s in w and y is the number of 1s in w.

   For example, BALANCE(000111)=010101 and BALANCE(0100000111)=0101010100. Note that w and BALANCE(w) have the same number of 0s and 1s, but in the beginning of the string BALANCE(w), the 0 and 1 characters alternate until one of them runs out.

   Define the corresponding operation on languages:

$$\text{BALANCE}(L) = \{\text{BALANCE}(w) \mid w \in L\}.$$

   Show that the class of decidable languages is closed under

   (a) concatenation

   (b) BALANCE

Think about complement, union (solution to 3.15(a) on p. 191 in Sipser), intersection and star on your own.

Show that the class of Turing-recognizable languages is closed under

(c) star

(d) BALANCE

Think about union (solution 3.16(a) on p. 191 in Sipser), intersection and concatenation on your own.