Homework 5 – Due Friday, October 19, 2018 before noon

This homework contains 3 mandatory and 1 optional problem, worth 10 points each.

Reminder  Collaboration is permitted, but you must write the solutions by yourself without assistance, and be ready to explain them orally to the instructor if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

Problems  Please practice on exercises and solved problems in Chapter 3. The material they cover may appear on exams.

1. (Multi-stack PDA) Consider a PDA with multiple stacks.
   (a) Give an example of a language that a 2-stack PDA can recognize, but no usual PDA can recognize. Give an implementation-level description of the 2-stack PDA that recognizes your language.
   (b) Show that a 2-stack PDA can simulate a Turing machine.
   (c) Is there a language that a 3-stack PDA can recognize, but no 2-stack PDA can recognize? Explain.

2. (Recognizable languages)
   (a) Sipser, 3.7
   (b) Formulate the language that $M_{bad}$ in part (a) was intended to recognize and describe a correct TM that recognizes this language.
   (c) In the mortal matrix problem, the input is two $n \times n$ matrices with integer entries, and the goal is to determine whether they can be multiplied in some order, possibly with repetition, to yield the zero matrix. Formulate this problem as a language and describe a TM that recognizes this language.

3. (AGREE$_{DFA}$) Consider the problem of determining, given two DFAs, whether there is a string that both of them accept.
   (a) Formulate this problem as a language AGREE$_{DFA}$.
   (b) Show that AGREE$_{DFA}$ is decidable by using a decider for one of the languages from Chapter 4 in the book.
   (c) Show that AGREE$_{DFA}$ is decidable by testing the two DFAs on all strings up to a certain length. Calculate the size that works.