Homework 6 – Due Friday, October 26, 2018 before noon

Reminder Collaboration is permitted, but you must write the solutions by yourself without assistance, and be ready to explain them orally to the instructor if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

Problems Please practice on exercises and solved problems in Chapter 4. The material they cover may appear on exams.

1. (Decidable languages, 15 points)
   (a) (REV_{DFA}) Consider the problem of determining if the languages of two given DFAs are reverses of each other.
      i. Formulate this problem as a language REV_{DFA} and show that it is decidable.
      ii. Why does a similar approach fail to show that REV_{PDA} is decidable? (This language is defined analogously to REV_{DFA}, with PDAs instead of DFAs as inputs.)
   (b) (Prefix of a generated string) A string w is called a prefix of string s if s starts with w.
      i. Give a regular expression for all strings over alphabet Σ for which w is a prefix.
      ii. Let L = \{⟨G, w⟩ | G is a CFG, w is a string, and w is a prefix of some string s generated by G\}. Show that L is decidable.
      Hint: The result of Problem 2.18a in the book might be useful.
   (c) (More prefix fun) Given a DFA, we would like to determine if its language contains two different strings, w and s, such that w is a prefix of s. Formulate this problem and a language and prove it is decidable.
      Think, but do not hand it: Why wouldn’t the same approach work if the input is a PDA instead of a DFA?

2. (Countable and uncountable sets and diagonalization, 15 points)
   (a) A polynomial in variable x is an expression of the form \(c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots + c_d x^d\), where d is a non-negative integer and \(c_0, \cdots, c_d\) are constants, called coefficients. Let P be the set of polynomials with integer coefficients. Show that P is countable.
   (b) Let F be the set of all finite languages over alphabet \{0,1\}. Show that F is countable.
   (c) Let L be the set of all languages over alphabet \{0\}. Show that L is uncountable, using a proof by diagonalization.
   (d) Your friend told you that he found a new C++ library that contains many useful functions. One example is a function halt which takes two arguments: a program main.cpp and a valid input string x for that program. It returns 1 if main.cpp produces an output on input string x, and returns 0 if main.cpp runs forever on x. You want to convince your friend that halt cannot be always correct. However, your friend does not want to hear about TMs because they cannot possibly be relevant to C++ programs.
Give a diagonalization argument (similar to that on p. 207 of Sipser) to convince your friend that function \texttt{halt}, as specified above, does not exist. Your analogue of TM \( D \) on p. 207 should be a C++ program \texttt{src.cpp}. \textit{Caution:} You cannot model it directly on TM \( D \) because your friend’s claim is analogous to saying that \( \text{HALT}_{TM} \) is decidable, not that \( A_{TM} \) is decidable. Your program may call function \texttt{fread} that reads a file and returns the content of a file as a string. If you do not know C++, you can use any programming language with C-like syntax (or consult Ramesh on which ones he can grade).