Intro to Theory of Computation

LECTURE 2

CS 332

Last time:

- Finite Automata

Today:

- Finite Automata
- Operations on languages
- Nondeterminism

Tomorrow: Homework 0 due
Homework 1 out

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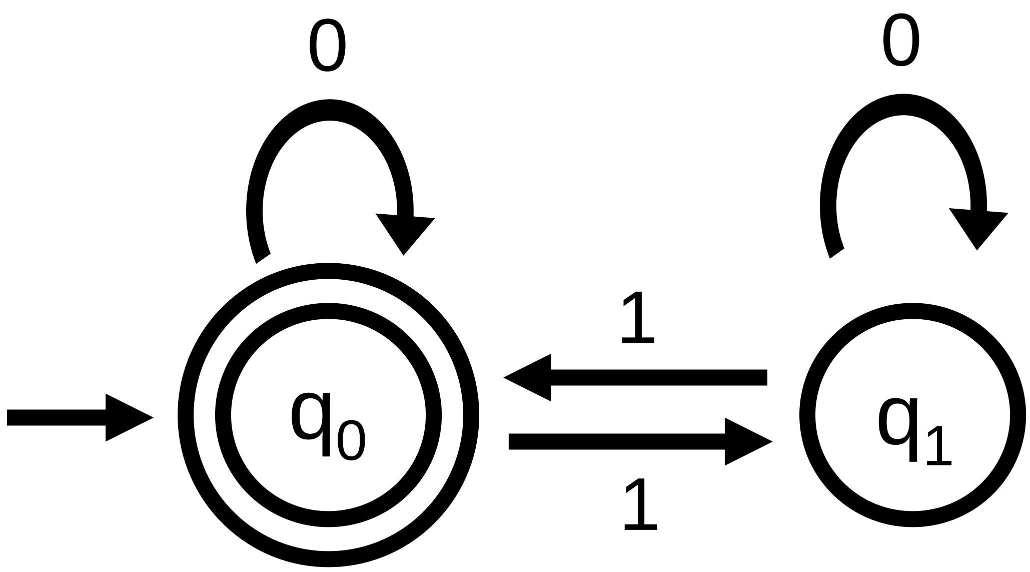
Review: FAs



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Review: Formal Definition

A ***finite automaton*** is a 5-tuple $\mathbf{M = (Q, \Sigma, \delta, q_0, F)}$

- $\mathbf{Q}$ is the set of states
- $\mathbf{\Sigma}$ is the alphabet
- $\mathbf{\delta : Q \times \Sigma \rightarrow Q}$ is the transition function
- $\mathbf{q_0 \in Q}$ is the start state
- $\mathbf{F \subseteq Q}$ is the set of accept states

$\mathbf{L(M)}$ = the ***language*** of machine M
= set of all strings machine M accepts

M ***recognizes*** the language L(M)

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Formal definition of FA

$M = (Q, \Sigma, \delta, q_0, F)$ where

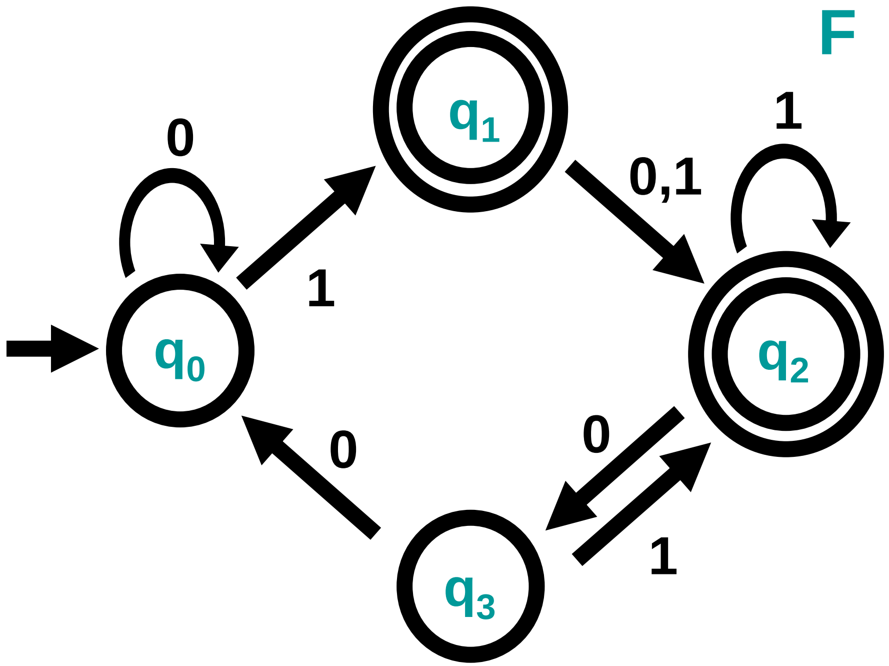
$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0,1\}$

$\delta : Q \times \Sigma \rightarrow Q$ transition function*

$q_0 \in Q$ is start state

$F = \{q_1, q_2\} \subseteq Q$ accept states



*

δ	0	1
q_0		
q_1		
q_2		
q_3		

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Regular languages

A language is regular if it is recognized by a finite automaton

L = { w | w contains 001} is regular

L = { w | w has an even number of 1s} is regular

Many interesting programs recognize regular languages

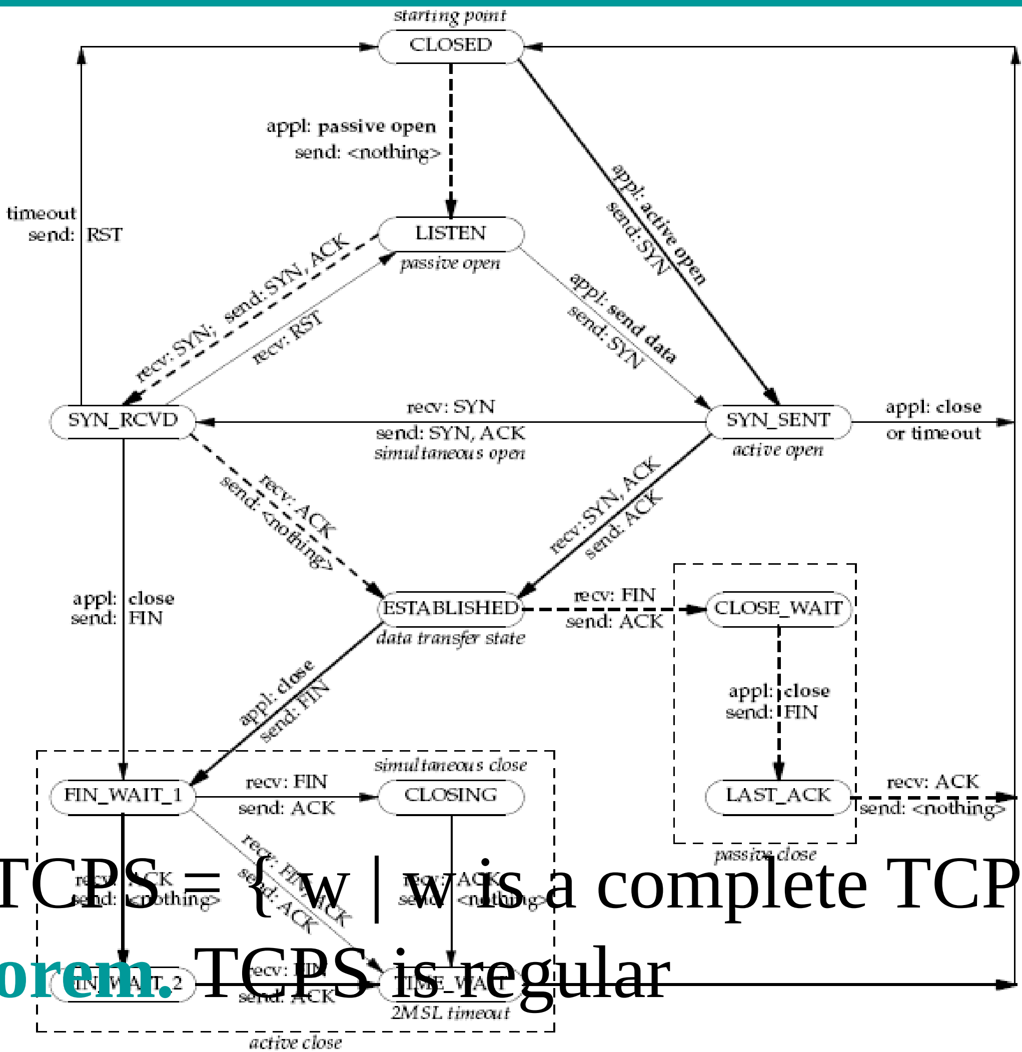
NETWORK PROTOCOLS

COMPILERS

GENETIC TESTING

ARITHMETIC

INTERNET TRANSMISSION CONTROL PROTOCOL



Let TCPS = { w | w is a complete TCP Session}

Theorem. TCPS is regular

Arithmetic

LET $\Sigma_3 = \left\{ \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix} \right\}$

- **A string over Σ_3 has three ROWS**
- **Each ROW $b_0 b_1 b_2 \dots b_N$ represents the integer $b_0 + 2b_1 + \dots + 2^N b_N$.**
- **Let ADD = $\{S \mid \text{ROW}_1 + \text{ROW}_2 = \text{ROW}_3\}$**

Theorem. ADD is regular.

CS 332 Compilers

COMMENTS :

Are delimited by /* */

Cannot have NESTED /* */

Must be CLOSED by */

***/ is ILLEGAL outside a comment**

COMMENTS = {strings over {0,1, /, *} with legal comments}

Theorem. COMMENTS is regular.

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Genetic testing

DNA SEQUENCES are strings over the alphabet {A,C,G,T}.

A GENE g is a special substring.

A GENETIC TEST searches a DNA SEQUENCE for a gene.

GeneticTest_g = {strings over {A,C, G, T} containing a copy of g}

Theorem. **GeneticTest_g is regular for every gene g.**

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Regular
Operations on languages

Complement: $\overline{A} = \{w \mid w \notin A\}$

Union:

Intersection:

Reverse:

Concatenation:

Star:

= {ε} ∪ A ∪ AA ∪ AAA ∪ AAAA ∪…

CS 332 Closure properties of the class of regular languages

THEOREM. The class of regular languages is **closed** under all 6 operations.

If A and B are regular, applying any of these operation yields a regular language.

Closure properties

Complement: $\overline{A} = \{ w \mid w \notin A \}$

Theorem. **The complement of a regular language is also a regular language.**

Proof:

Closure properties

Complement: $\overline{A} = \{ w \mid w \notin A \}$

Union: $A \cup B = \{ w \mid w \in A \text{ or } w \in B \}$

Closure under union

Theorem. The union of two regular languages is also a regular language

Proof: Consider two regular languages A_1 and A_2.

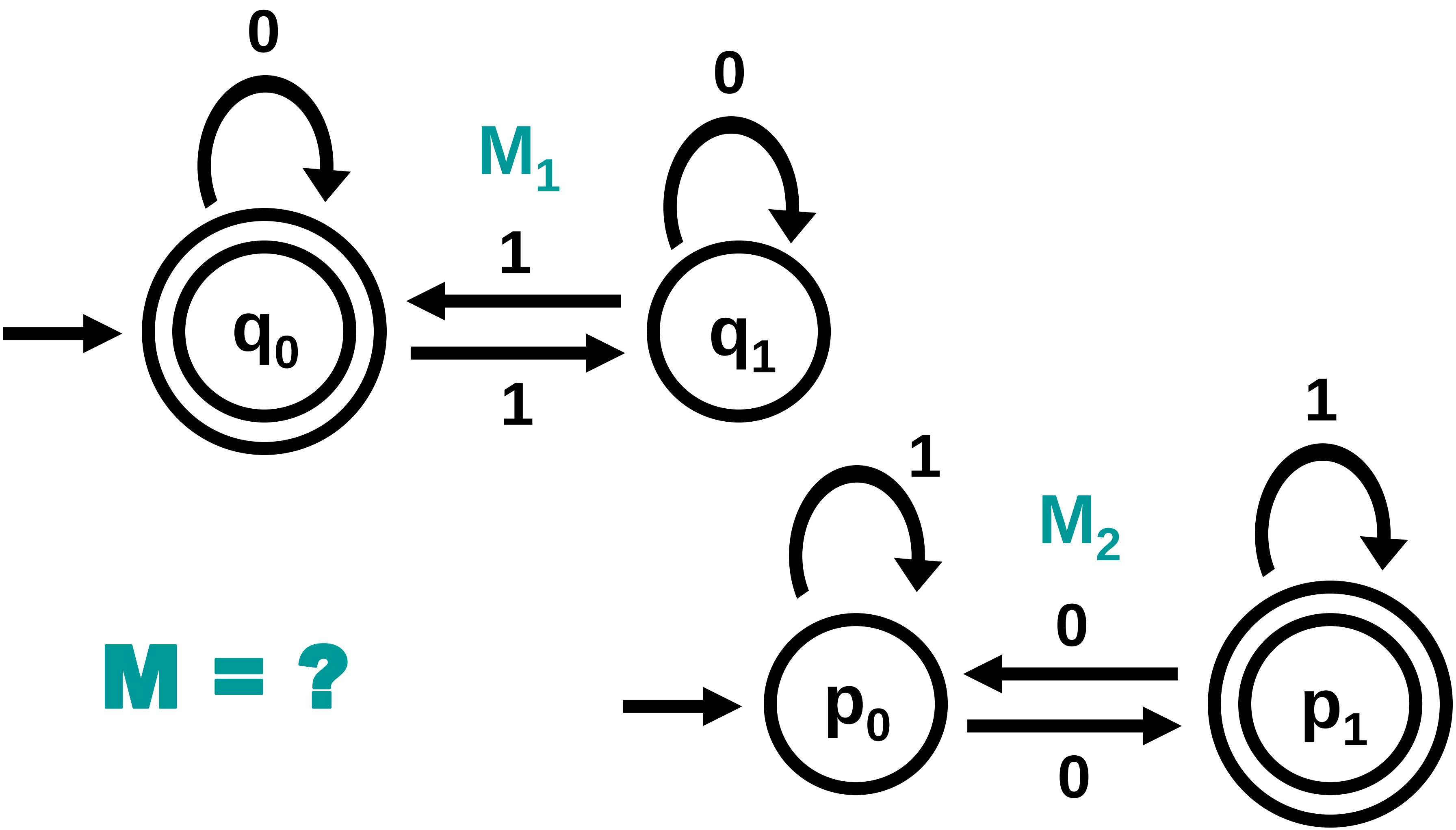
Prove that $A_1 \cup A_2$ is regular.

Let $M_1 = (Q_1, \Sigma, \delta_1, q_0^1, F_1)$ be finite automaton for A_1

and $M_2 = (Q_2, \Sigma, \delta_2, q_0^2, F_2)$ be finite automaton for A_2

Construct a finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ that recognizes $A = A_1 \cup A_2$

Example



M = ?

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Proof (continued)

Idea: Run both M_1 and M_2 at the same time!

Q = pairs of states, one from M_1 and one from M_2

$= \{ (q_1, q_2) \mid q_1 \in Q_1 \text{ and } q_2 \in Q_2 \}$

$= Q_1 \times Q_2$

$q_0 = (q_0^1, q_0^2)$

$F = \{ (q_1, q_2) \mid q_1 \in F_1 \text{ or } q_2 \in F_2 \}$

$\delta((q_1,q_2), \sigma) = (\delta_1(q_1, \sigma), \delta_2(q_2, \sigma))$

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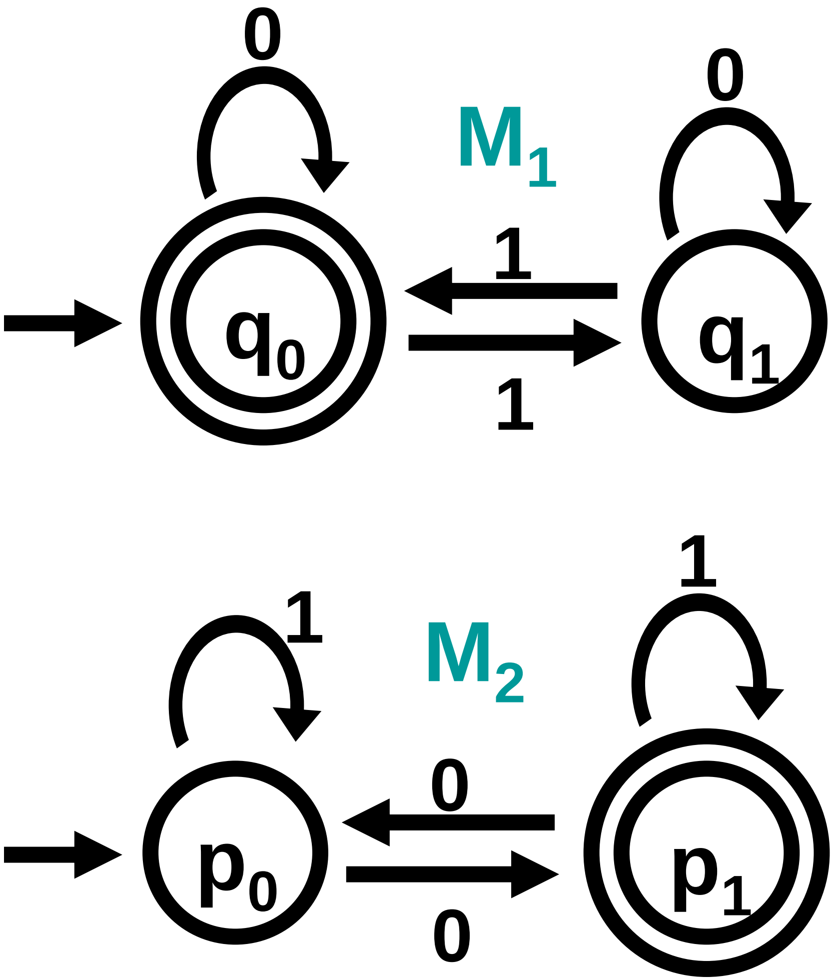
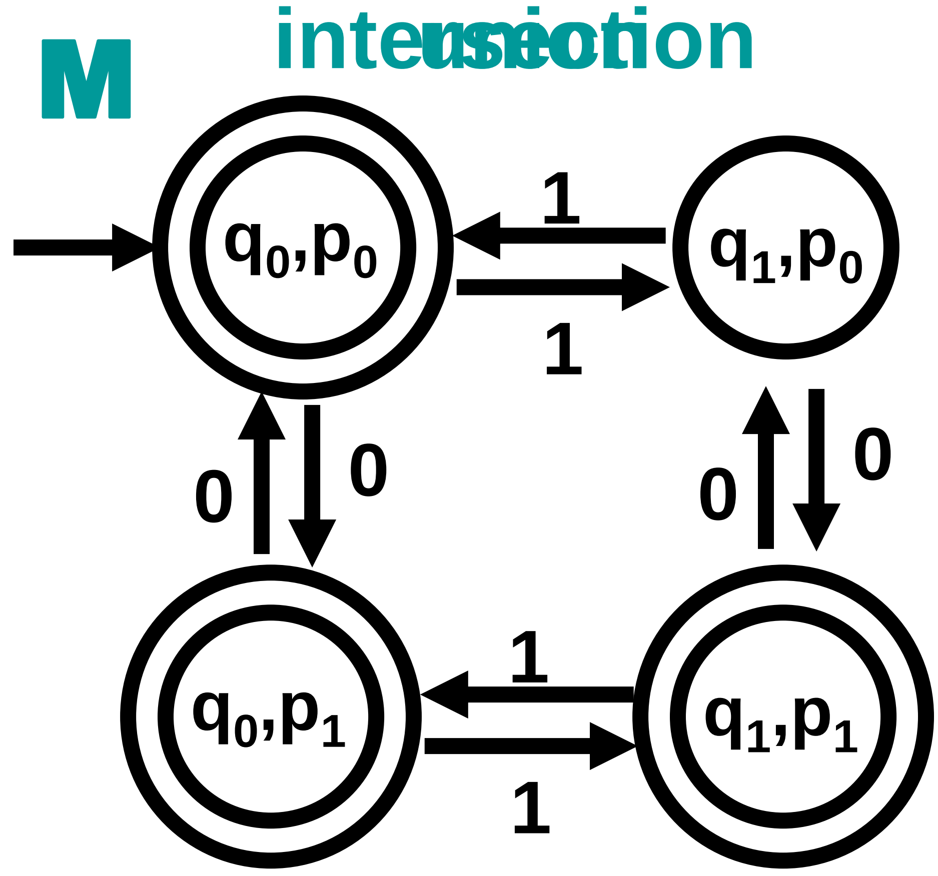
Example (continued)





Closure properties

Complement: $\neg A = \{ w \mid w \notin A \}$

Union: $A \cup B = \{ w \mid w \in A \text{ or } w \in B \}$

Intersection: $A \cap B = \{ w \mid w \in A \text{ and } w \in B \}$

Reverse: $A^R = \{ w_1 \ldots w_k \mid w_k \ldots w_1 \in A \}$

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Closure under reverse

Theorem. **The reverse of a regular language is also a regular language**

Proof: Let A be a regular language and M be a finite automaton that recognizes it.

Construct a finite automaton M' recognizing A^R.

Idea: Define M' as M with the arrows reversed. Swap start and accept states.

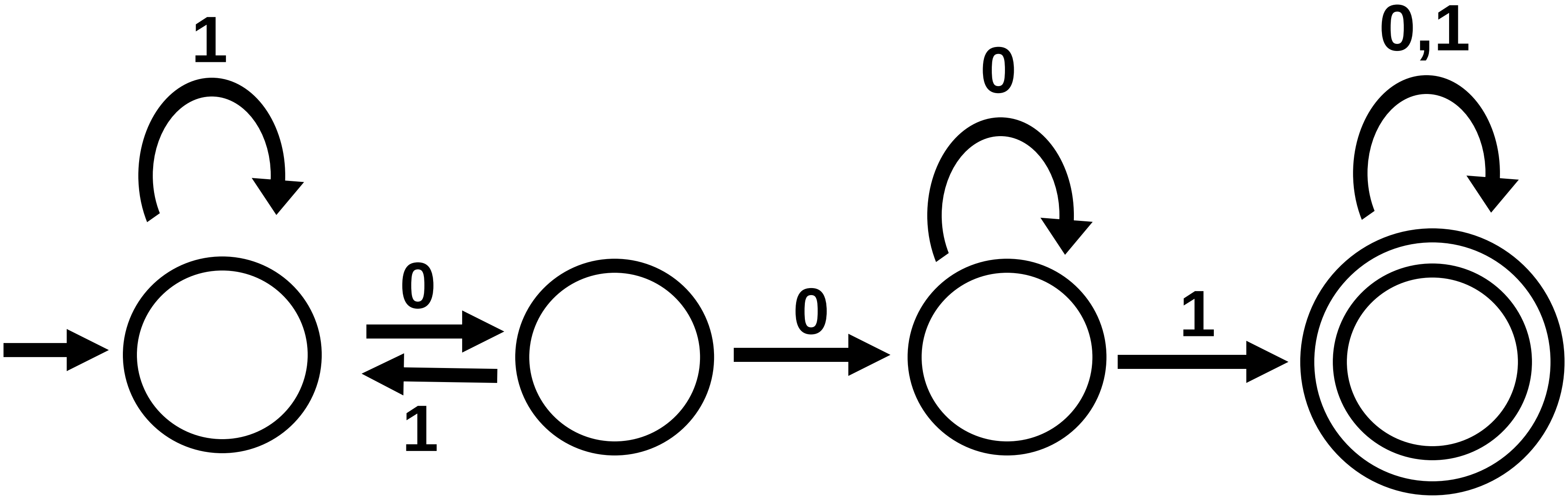
Closure under reverse

M' **IS NOT ALWAYS A DFA!**

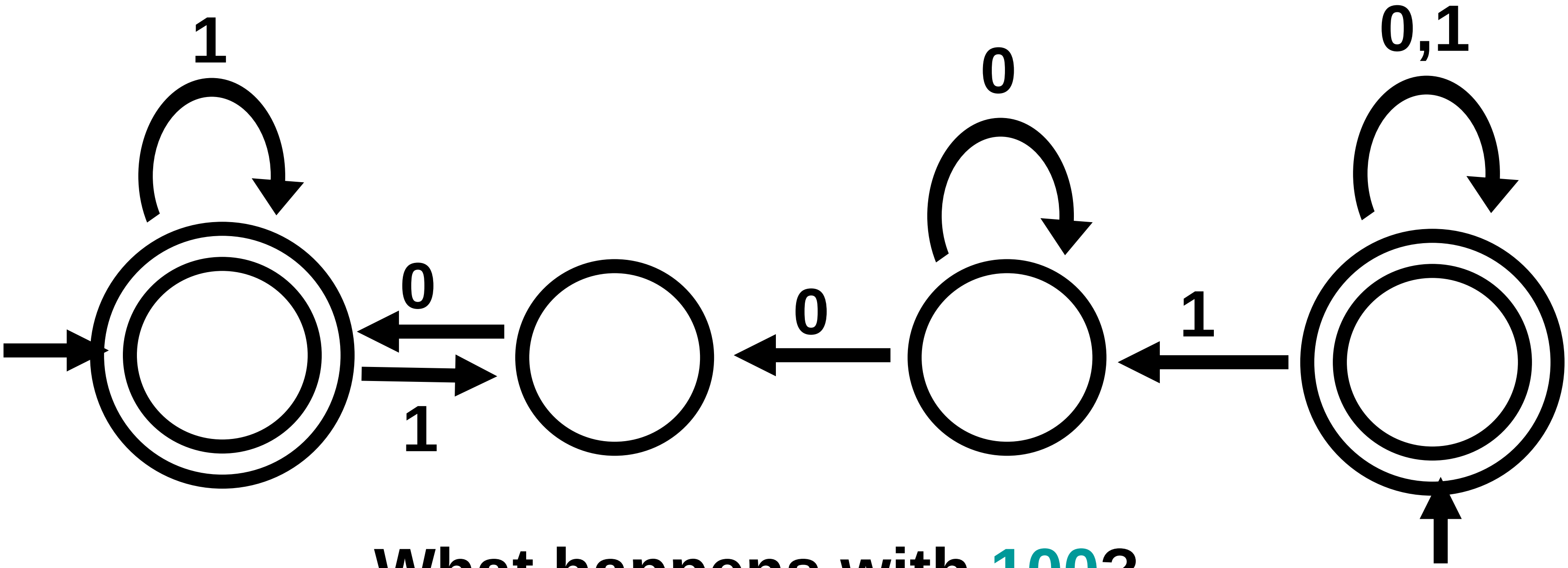
It may have many start states.

Some states may have too many outgoing edges, or none.

Example



NONDETERMINISM



What happens with 100?

Nondeterministic Finite Automaton (NFA) accepts if there is a way to make it reach an accept state.