Intro to Theory of Computation

CS 332

LECTURE 3

Last time:

- DFAs and NFAs
- Operations on languages

Today:

- Nondeterminism
- Equivalence of NFAs and DFAs
- Closure properties of regular languages

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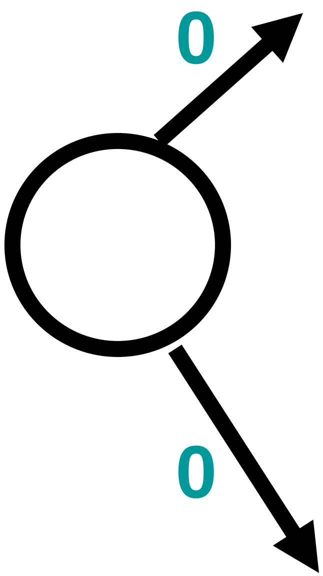
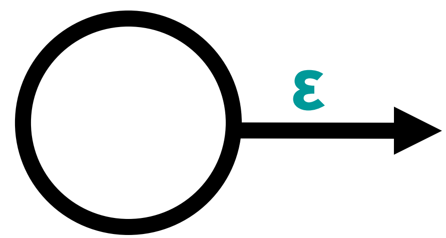
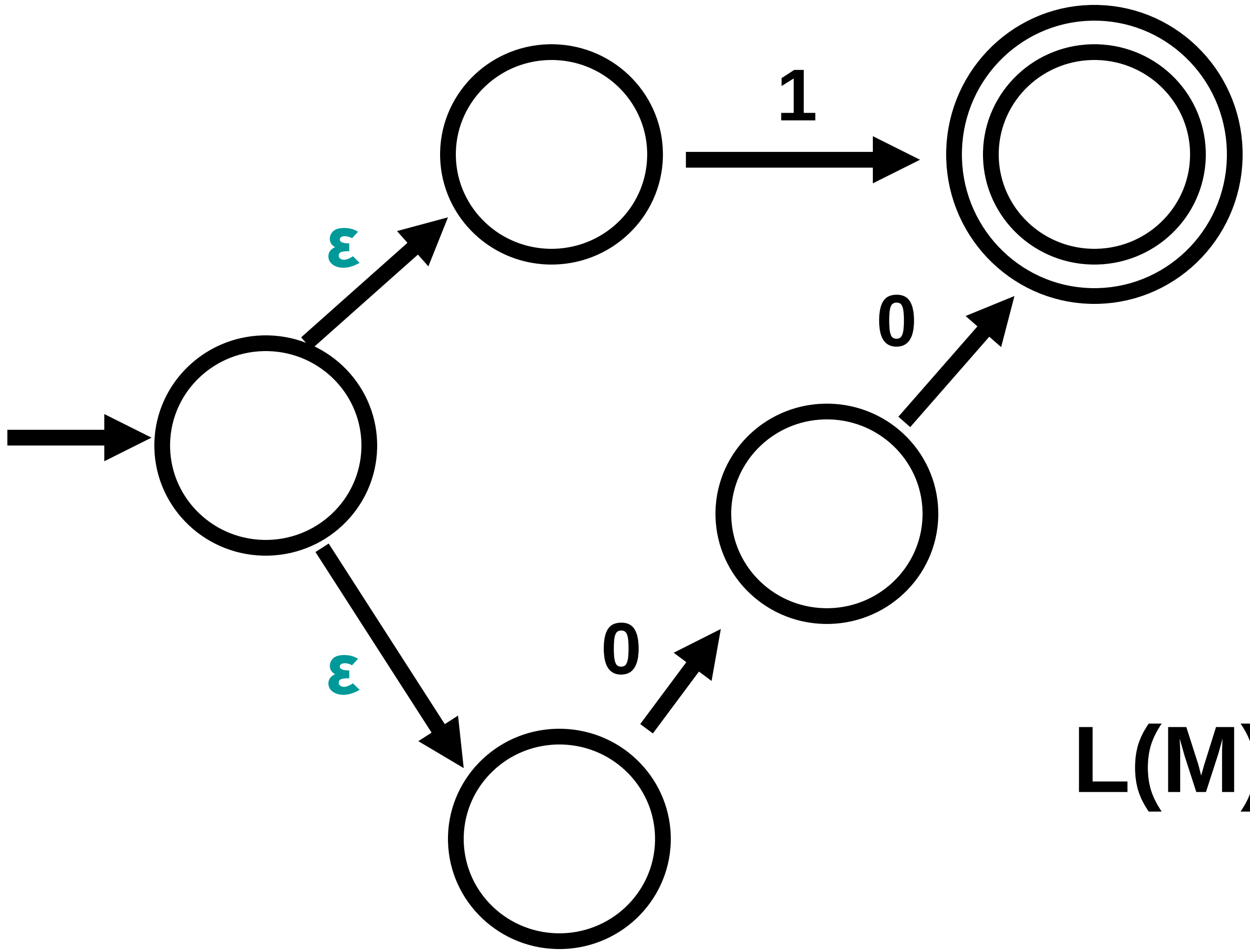
Nondeterminism

Nondeterministic Finite Automaton (NFA) accepts a string w if there is a way to make it reach an accept state on input w.



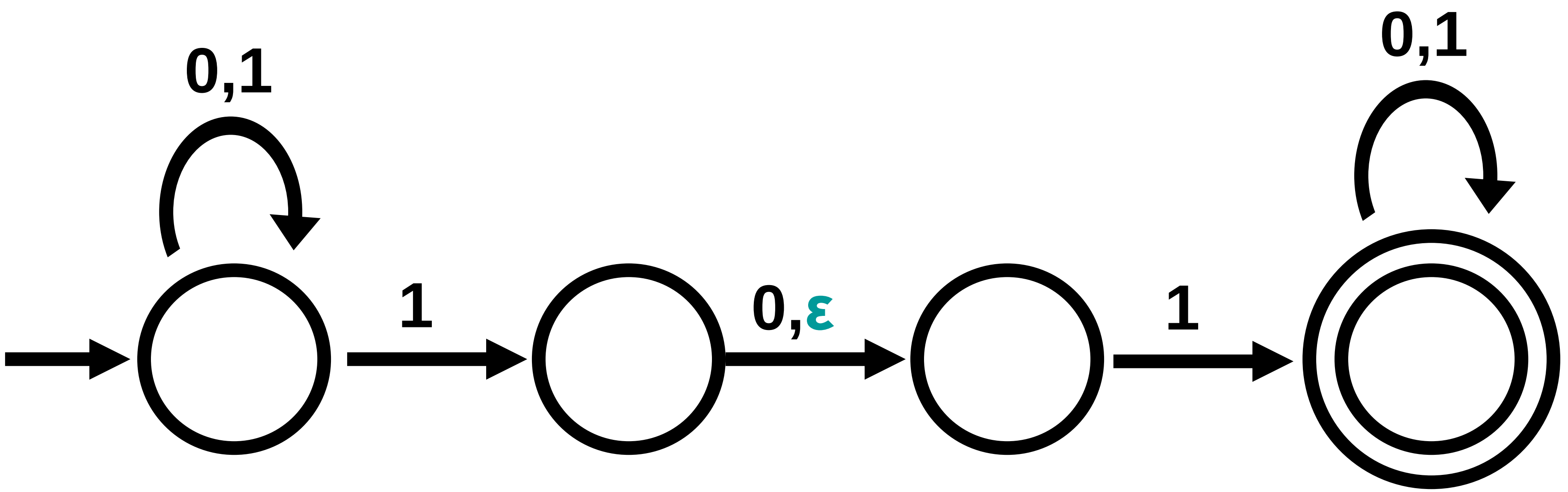


Example



L(M)={1,00}

Example

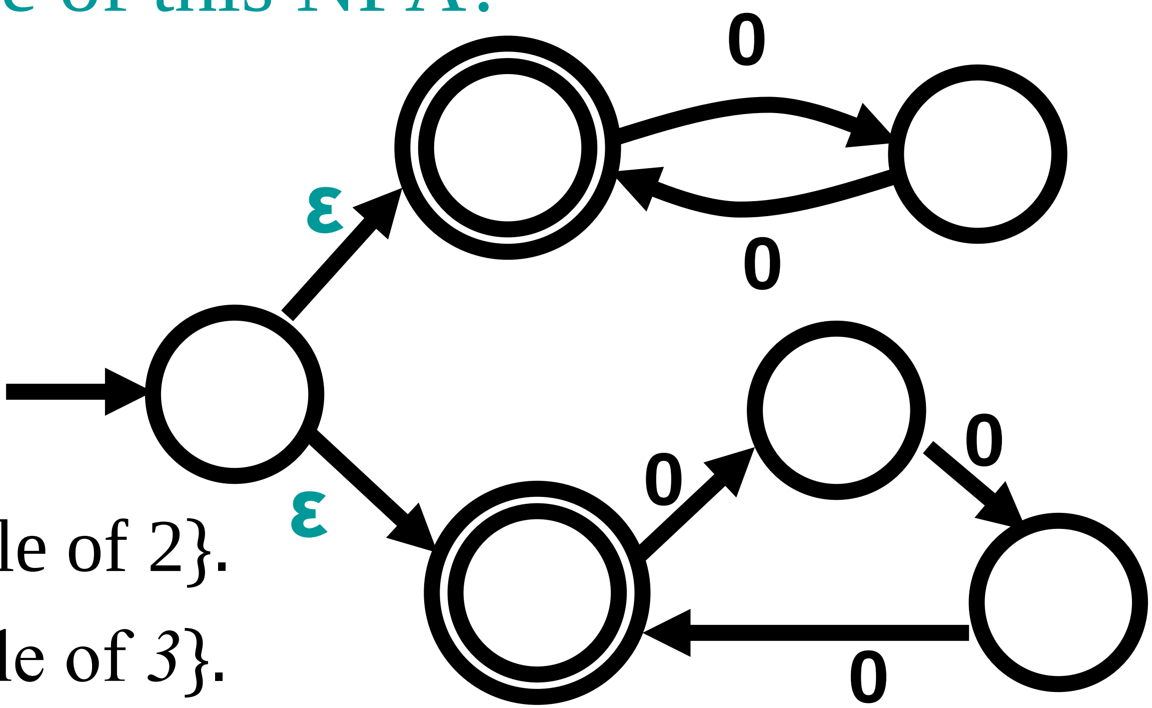


L(M)={w | w contains 101 or 11}

Exercise

What is the language of this NFA?

(0^k means $\underbrace{00\ldots0}_{k}$)



A. $\{0^k \mid k \text{ is a multiple of } 2\}$.

B. $\{0^k \mid k \text{ is a multiple of } 3\}$.

C. $\{0^k \mid k \text{ is a multiple of } 6\}$.

D. $\{0^k \mid k \text{ is a multiple of } 2 \text{ or } 3\}$.

E. None of the above.

Formal Definition

- An ***NFA*** is a 5-tuple $\mathbf{M = (Q, \Sigma, \delta, q_0, F)}$

  $\mathbf{Q}$ is the set of states

  $\mathbf{\Sigma}$ is the alphabet

  $\delta : Q \times \Sigma_\varepsilon \to P(Q)$ is the transition function

  $\mathbf{q_0 \in Q}$ is the start state

  $\mathbf{F \subseteq Q}$ is the set of accept states

- **$\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$ and $P(Q)$ is the set of subsets of Q**
- **M** ***accepts*** a string **w** if **there is** a path from $\mathbf{q_0}$ to an accept state that **w** follows.

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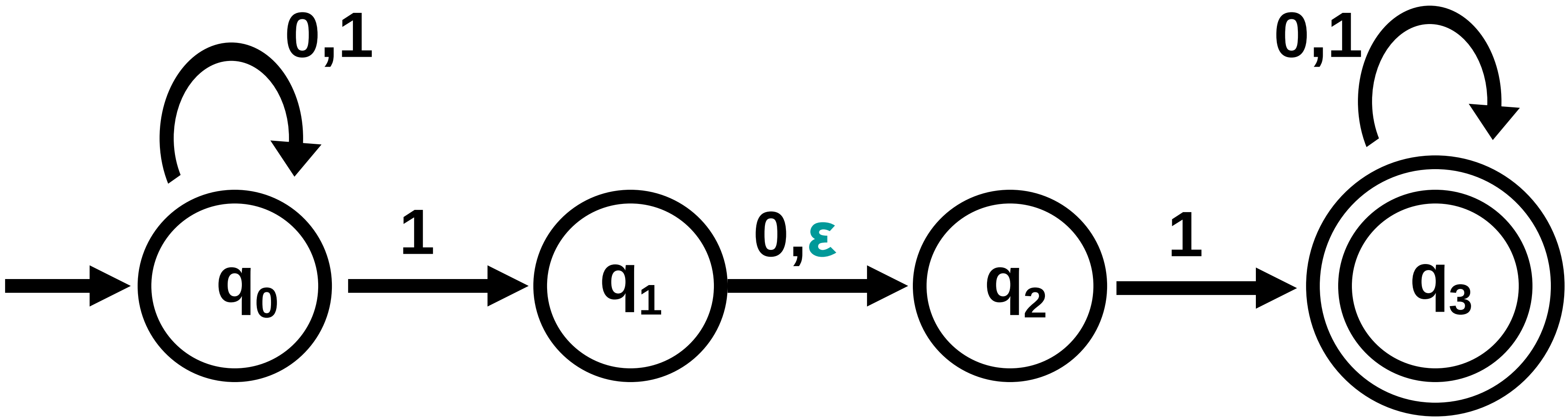
Example



$N = (Q, \Sigma, \delta, q_0, F)$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0,1\}$

$F = \{q_3\}$

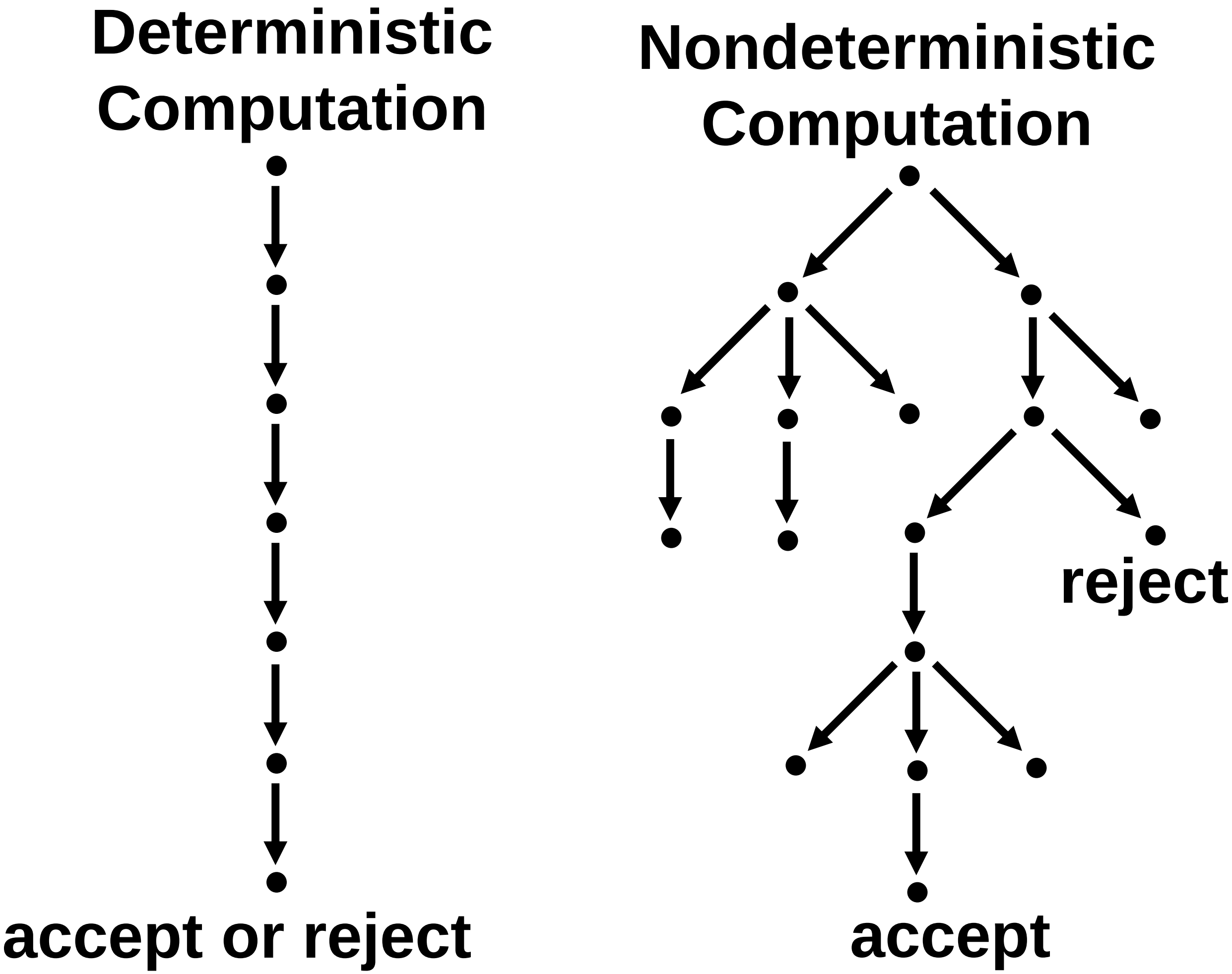
$\delta(q_0,0) = \{q_0\}$

$\delta(q_0,1) = \{q_0, q_1, q_2\}$

$\delta(q_1,\varepsilon) = \{q_1,q_2\}$

$\delta(q_2,0) = \varnothing$

Nondeterminism

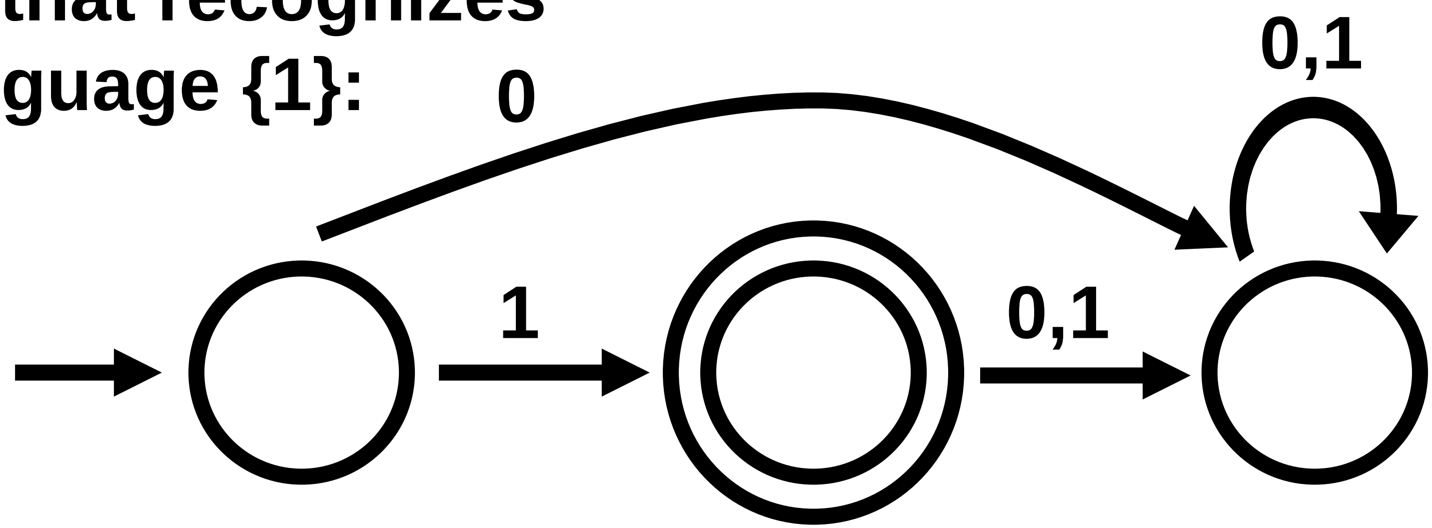


Ways to think about nondeterminism

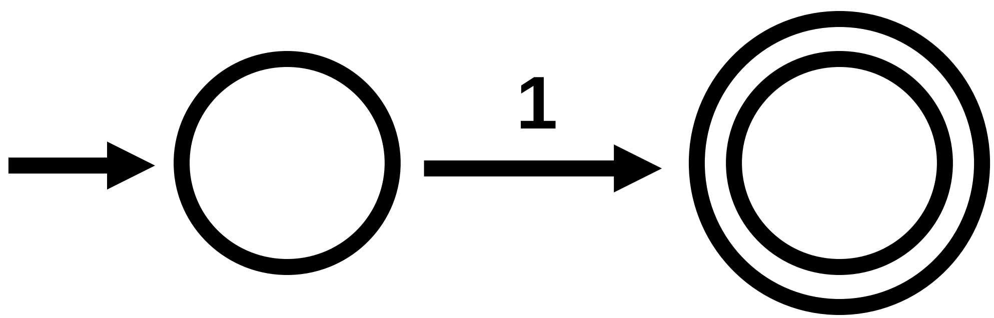
- parallel computation
- tree of possible computations
- guessing and verifying the "right" choice

NFAs ARE SIMPLER THAN DFAs

A DFA that recognizes the language {1}:



An NFA that recognizes the language {1}:



A DFA recognizing {1}

Theorem. Every DFA for language {1} must have at least 3 states.

Proof:

Equivalence of NFAs & DFAs

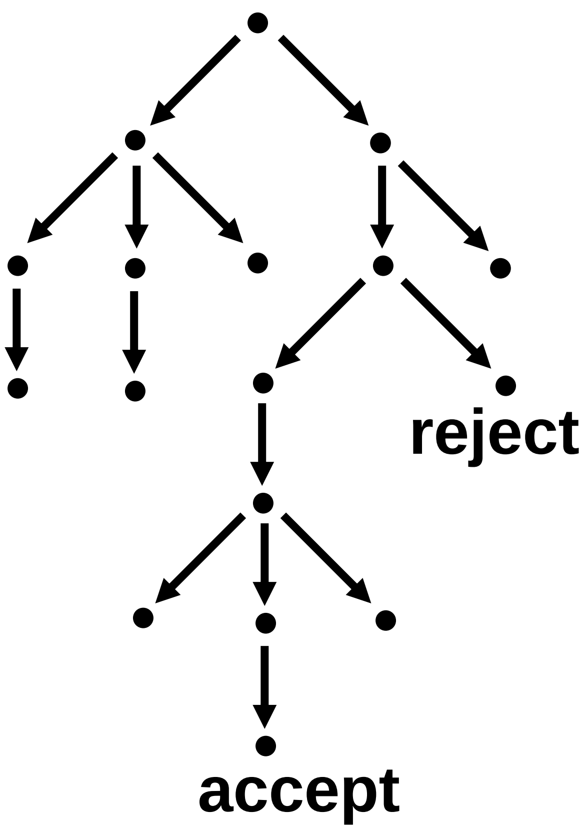
Theorem. **Every NFA has an equivalent DFA.**

Corollary: **A language is regular iff it is recognized by an NFA.**

NFA to DFA Conversion

Input: $N = (Q, \Sigma, \delta, q_0, F)$

Output: $M = (Q', \Sigma, \delta', q_0', F')$



Intuition: Do the computation in parallel, maintaining the set of states where all threads are.

Idea:

$$Q' = P(Q)$$

NFA to DFA Conversion

Input: $N = (Q, \Sigma, \delta, q_0, F)$

Output: $M = (Q', \Sigma, \delta', q_0', F')$

$Q' = P(Q)$

$\delta' : Q' \times \Sigma \rightarrow Q'$

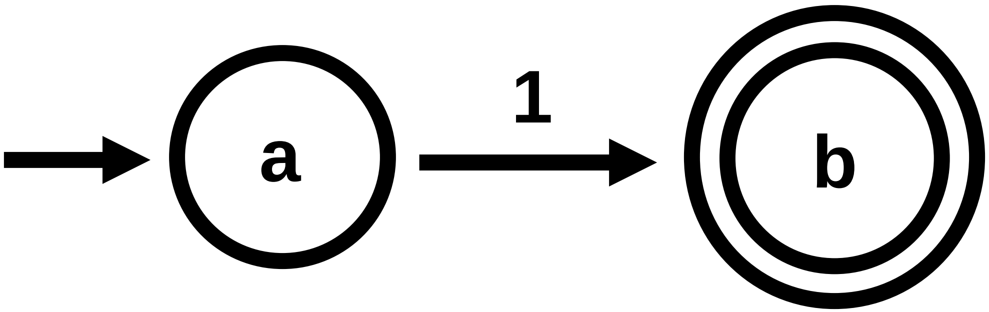
$\delta'(R,\sigma) =$ for all $R \subseteq Q$ and $\sigma \in \Sigma$.

$q_0' =$

$F' =$

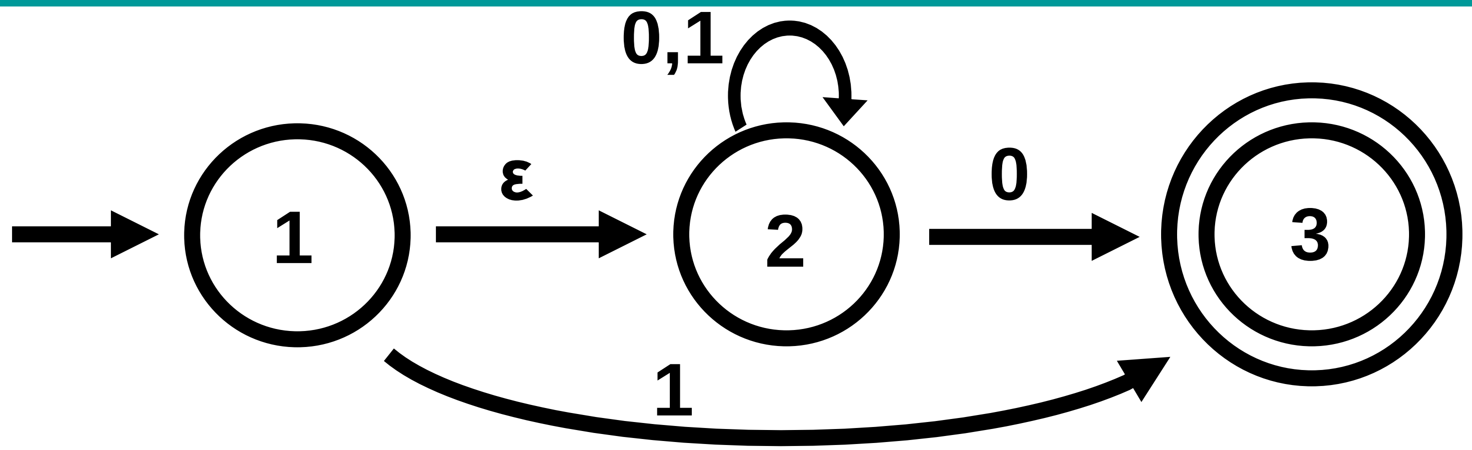
Example: NFA to DFA

1)



Examples NFA to DFA

2)



NFA to DFA Conversion

Input: $N = (Q, \Sigma, \delta, q_0, F)$

Output: $M = (Q', \Sigma, \delta', q_0', F')$

For $R \subseteq Q$, let $E(R)$ be the set of states reachable by ε-transitions from the states in R.

$Q' = P(Q)$

$\delta' : Q' \times \Sigma \rightarrow Q'$

$\delta'(R,\sigma) = \bigcup_{r \in R} (\delta(r,\sigma))$ for all $R \subseteq Q$ and $\sigma \in \Sigma$.

$q_0' = \quad (\{q_0\})$

$F' = \{ R \in Q' \mid R \text{ contains some accept state of } N\}$

Regular Operations on languages

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Complement: $\neg A = \{ w \mid w \notin A \}$

Union: $A \cup B = \{ w \mid w \in A \text{ or } w \in B \}$

Intersection: $A \cap B = \{ w \mid w \in A \text{ and } w \in B \}$

Reverse: $A^R = \{ w_1 \ldots w_k \mid w_k \ldots w_1 \in A \}$

Concatenation: $A \circ B = \{ vw \mid v \in A \text{ and } w \in B \}$

Star: $A^* = \{ w_1 \ldots w_k \mid k \geq 0 \text{ and each } w_i \in A \}$

Closure properties of the class of regular languages

THEOREM. The class of regular languages is **closed** under all 6 operations.

If A and B are regular, applying any of these operation yields a regular language.

Palindromes

A palindrome is a word or a phrase that reads the same forward and backward.

Examples

- mom
- madam
- Never odd or even.
- Stressed? No tips? Spit on desserts!

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Exercise

Let L be the set of words in English.

Then $L \cap L^R$ is

A. The set of English words in alphabetical order, followed by the same words in reverse alphabetical order.

B. {*w* | *w* is an English word or an English word written backwords}.

C. {*w* | *w* is an English word that is a palindrome}.

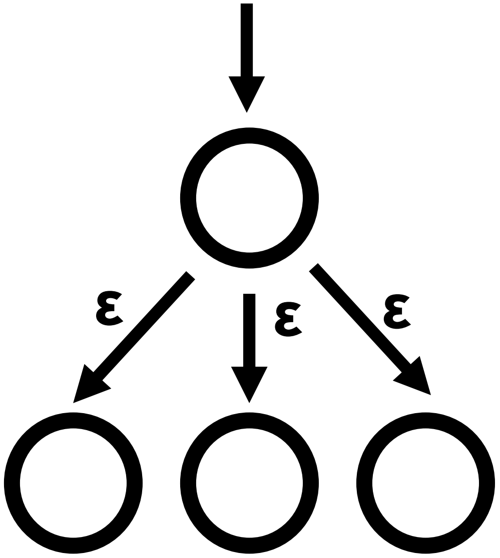
D. None of the above.

Closure under reverse

Theorem. **The reverse of a regular language is also regular**

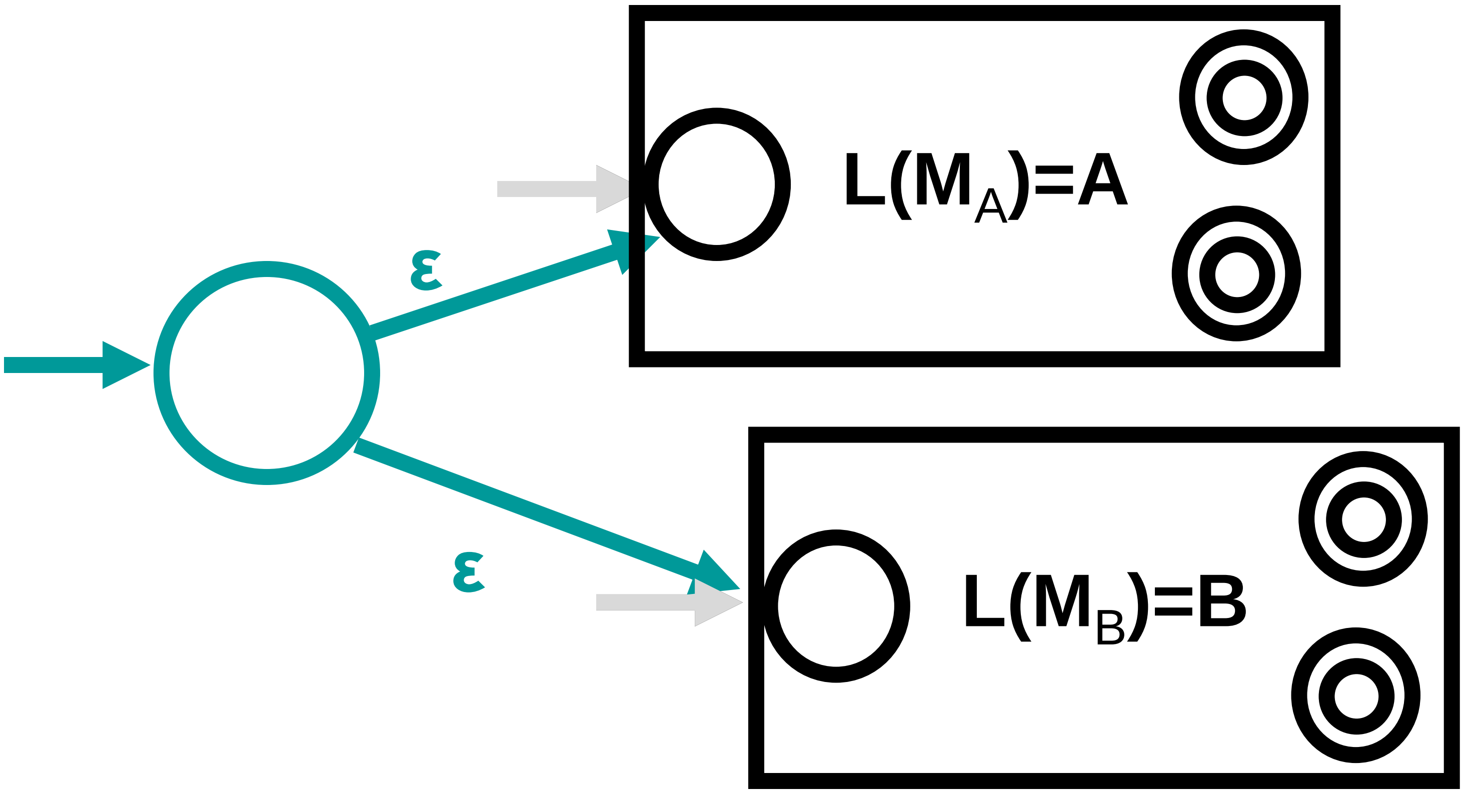
Proof**: Let L be a regular language and M be a DFA that recognizes it. Construct an NFA *M′* recognizing L^R:**

- **Define *M′* as M with the arrows reversed.**
- **Make the start state of M be the accept state in *M′*.**
- **Make a new start state that goes to all accept states of M by ε-transitions.**



New construction for A ∪ B

Construct an NFA M:

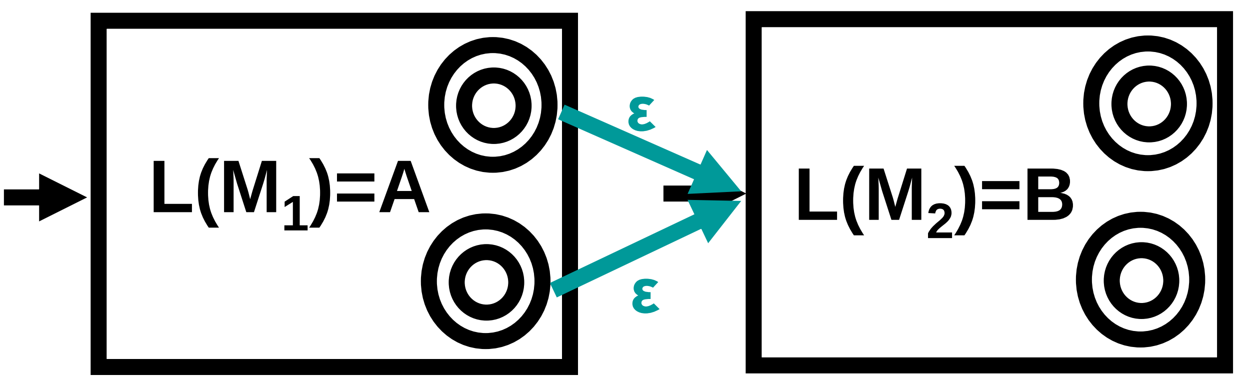


Concatenation operation

Concatenation: $A \circ B = \{ vw \mid v \in A \text{ and } w \in B \}$

Theorem. If A and B are regular, $A \circ B$ is also regular.

Proof: Given DFAs M_1 and M_2, construct NFA by connecting all accept states in M_1 to the start state in M_2.



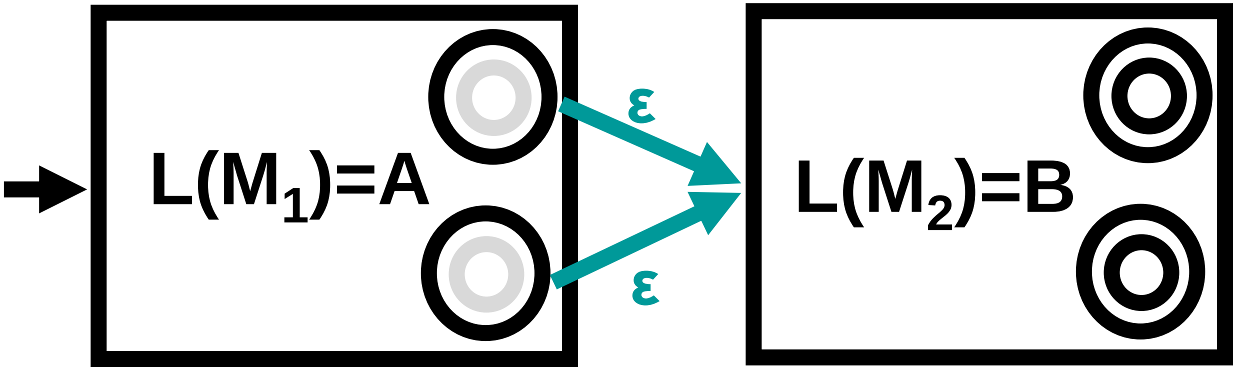
Concatenation operation

Concatenation: $A \circ B = \{ vw \mid v \in A \text{ and } w \in B \}$

Theorem. If A and B are regular, $A \circ B$ is also regular.

Proof: Given DFAs M_1 and M_2, construct NFA by connecting all accept states in M_1 to the start state in M_2.

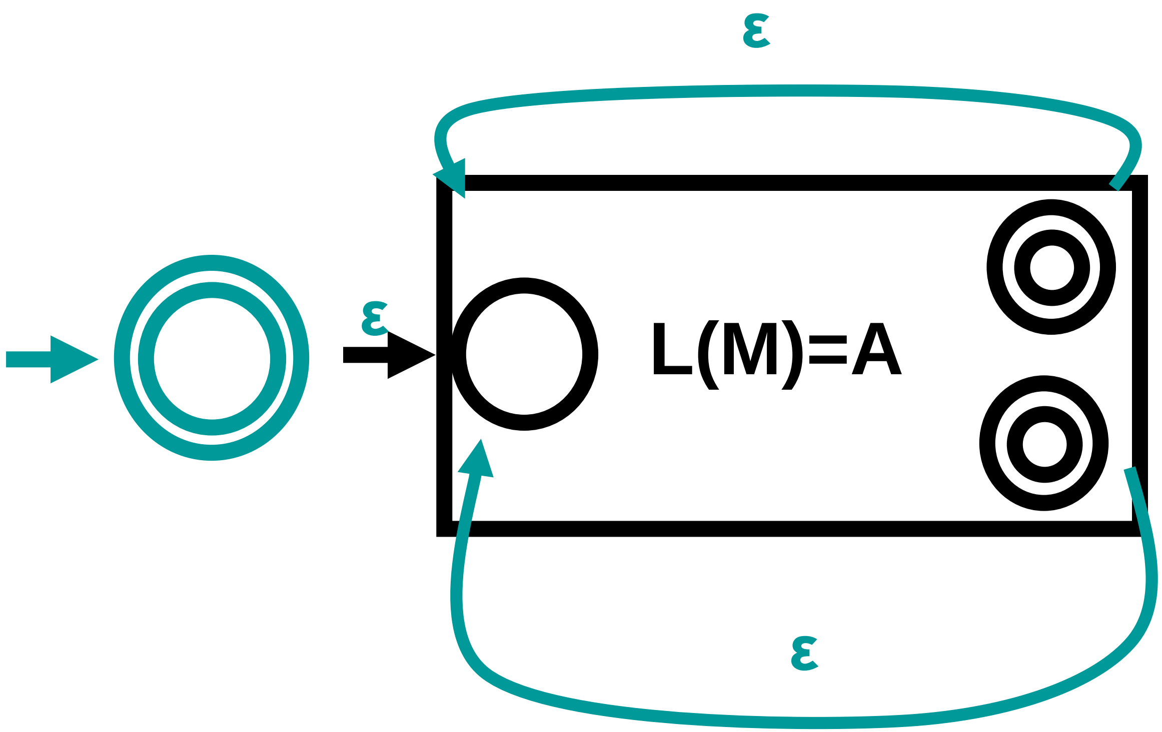
- Make all states in M_1 non-accepting.



Star operation

Star: $A^* = \{ w_1 \ldots w_k \mid k \geq 0$ and each $w_i \in A \}$

Theorem. **If A is regular, A* is also regular.**



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The class of regular languages is closed under

Regular operations

Union: $A \cup B = \{ w \mid w \in A \text{ or } w \in B \}$

Concatenation: $A \circ B = \{ vw \mid v \in A \text{ and } w \in B \}$

Star: $A^* = \{ w_1 \ldots w_k \mid k \geq 0 \text{ and each } w_i \in A \}$

Other operations

Complement: $\neg A = \{ w \mid w \notin A \}$

Intersection: $A \cap B = \{ w \mid w \in A \text{ and } w \in B \}$

Reverse: $A^R = \{ w_1 \ldots w_k \mid w_k \ldots w_1 \in A \}$