Intro to Theory of Computation

CS
332

Lecture 11

Last time:

- Midterm

Today:

- Turing Machines
- Turing Machine Variants

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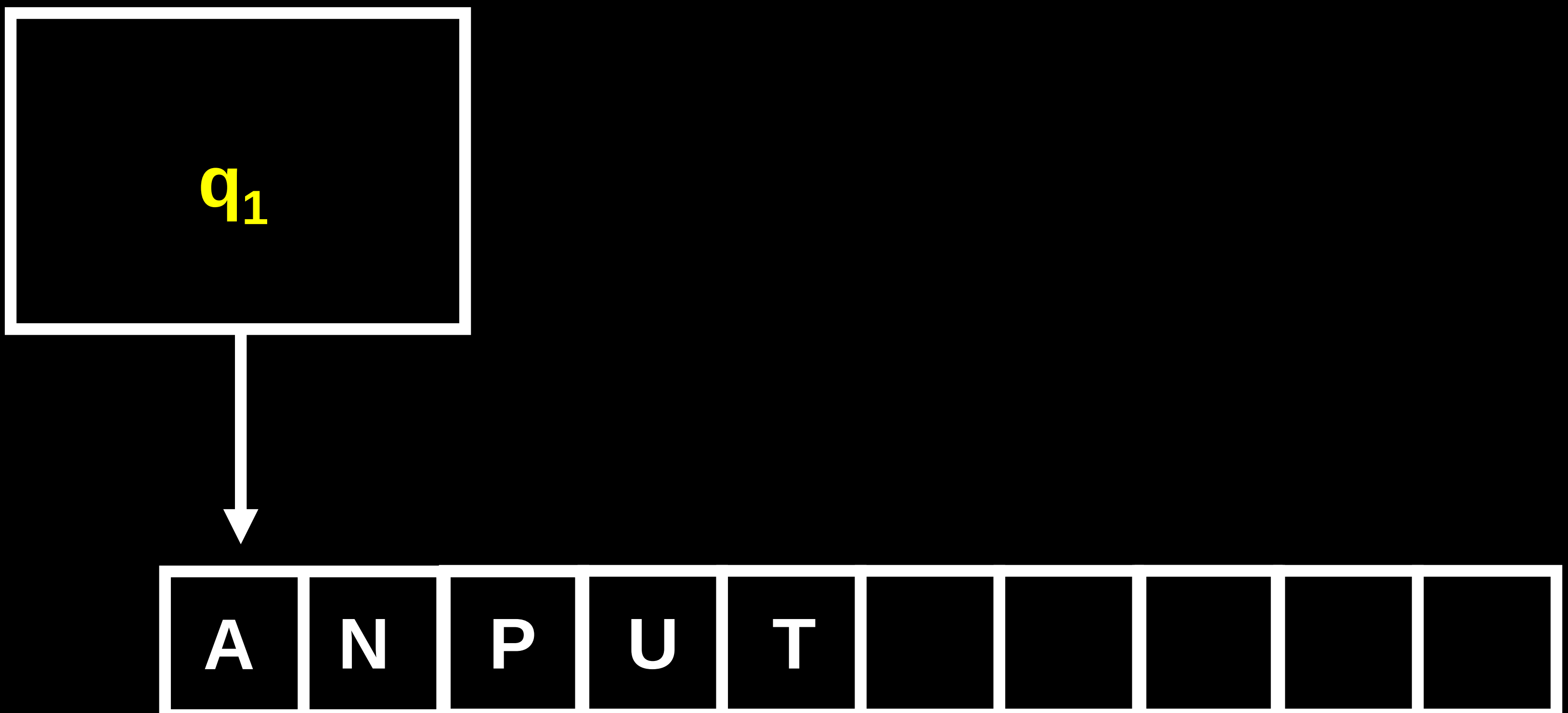
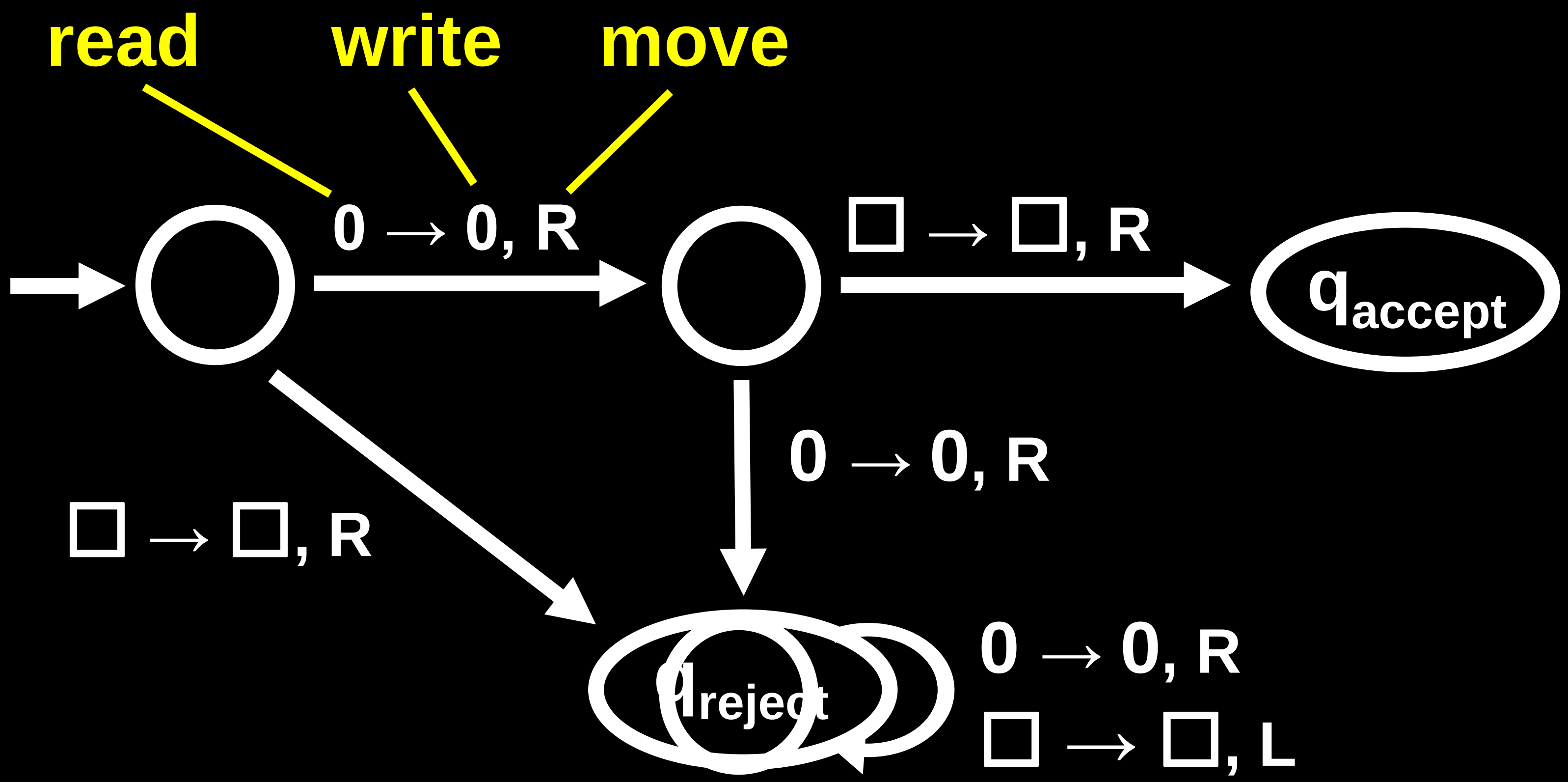
TURING MACHINE (TM)



UNBOUNDED (on the right) TAPE



A TM can loop forever

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TM versus PDA

TM can both write to and read from the tape

The head can move left and right

The input does not have to be read entirely

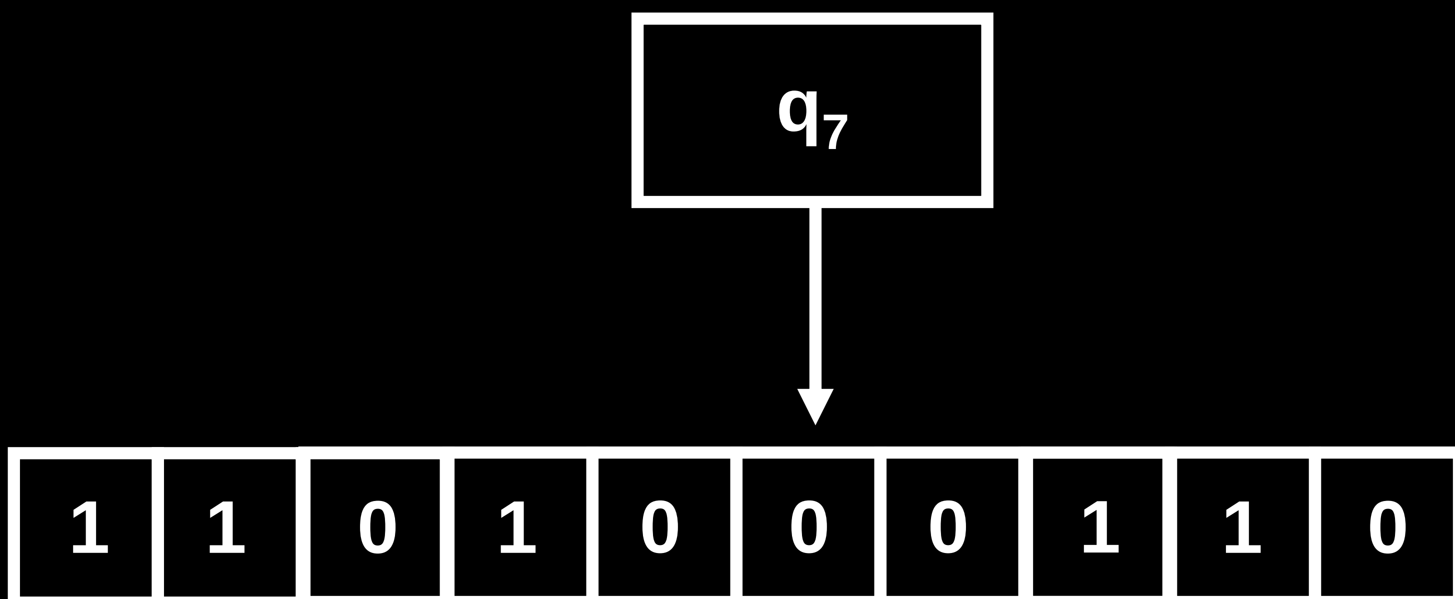
Accept and Reject take immediate effect

Infinite tape on the right, stick on the left

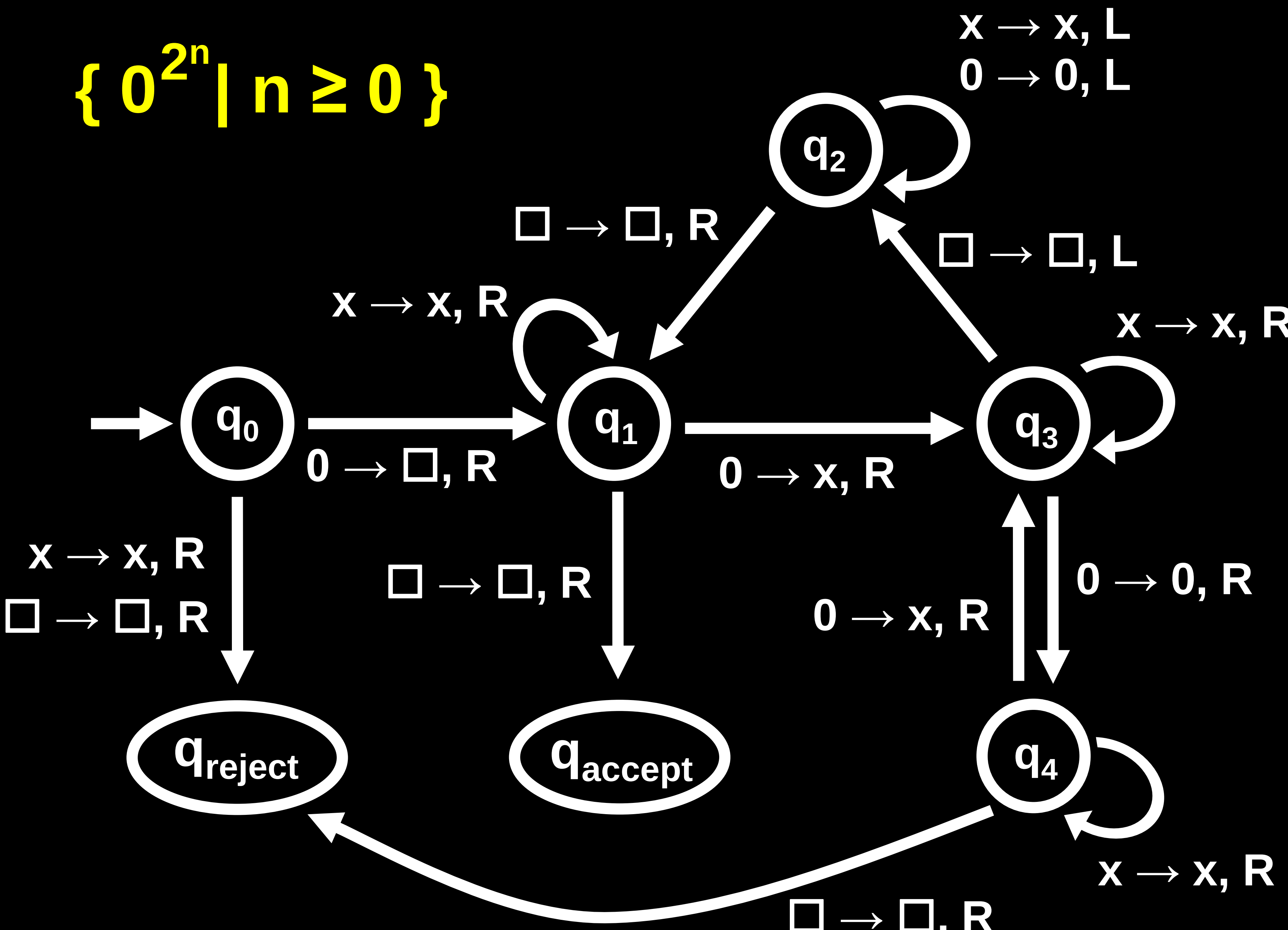
TM is deterministic (NTM is nondeterministic)

CONFIGURATIONS

$$11010q_700110$$

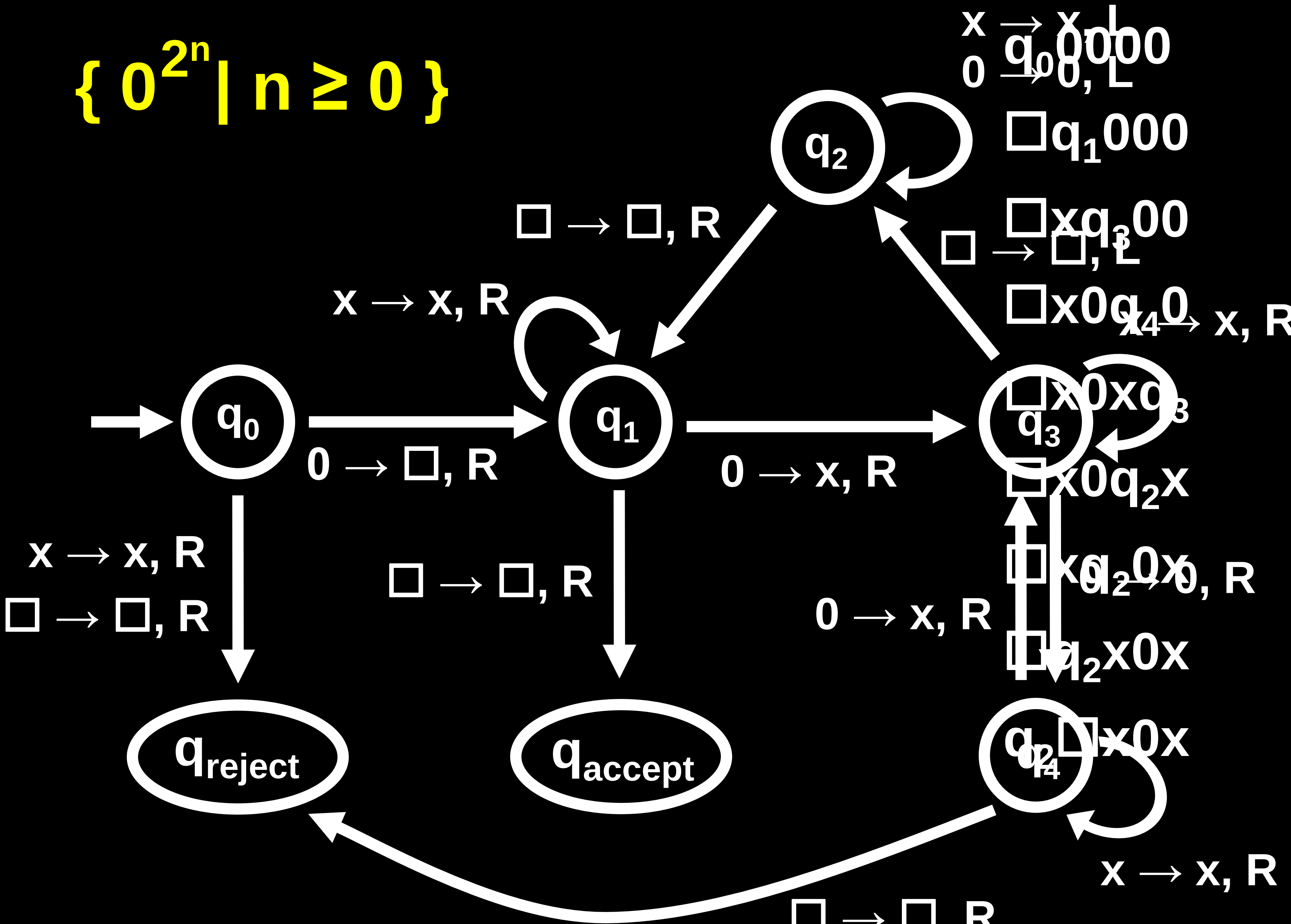


$\{ 0^{2^n} | n \geq 0 \}$



x → x, L
0 → 0, L
q2
□ → □, R
□ → □, L
x → x, R
x → x, R
q0
q1
q3
0 → □, R
0 → x, R
x → x, R
□ → □, R
□ → □, R
0 → 0, R
0 → x, R
qreject
qaccept
q4
x → x, R
□ → □, R

$\{ 0^{2^n} \mid n \geq 0 \}$



MUL = $\{1^i\#1^j\#1^k \mid ij = k \text{ and } i, j, k \geq 1\}$

11#111#111111

x1#111#111111

x1#yyy#zzz111

x1#111#zzz111

xx#yyy#zzzzzz

LP = $\{1^i\#x_1\#\ldots\#x_n \mid n \geq i \text{ and } x_i = x_1\}$

$$111\#101\#11\#101$$

$$x11\overline{\#}101\#11\#101$$

$$xx1\overline{\#}101\overline{\#}11\#101$$

$$xxx\overline{\#}\dot{1}\dot{0}\dot{1}\overline{\#}11\ddot{\#}\ddot{1}\ddot{0}\ddot{1}$$

CS 332 Formal Definition of a TM

A ***TM*** is a 7-tuple $\mathbf{P = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})}$

$\mathbf{Q}$ is a finite set of states

Σ is the input alphabet, where $\square \notin \Sigma$

Γ is the tape alphabet, where $\square \in \Gamma$ and $\Sigma \subseteq \Gamma$

$\delta : Q \times \Gamma \to Q \times \Gamma \times \{L,R\}$ is the transition function

$q_0, q_{accept}, q_{reject} \in Q$ are the start, accept and reject states

Implementation-level Description of a TM

- Describe (in English) the instructions for a TM
  - How to move the head
  - What to write on the tape
- Example
  1. Scan the tape from left to right and, for every 1 read until non-1 symbol is encountered
     - replace 1 with x,
     - find the next # on the right and replace it with $\overline{\#}$.
     - If no matching # found, **reject**.

Accepting and rejecting

A TM on input sting w may

either halt (enter q_{accept} or q_{reject})
or never halt (loop)

A TM is a decider if it halts on every input.

Language of a TM

A TM **recognizes** a language L if it accepts all strings in L and no other strings.

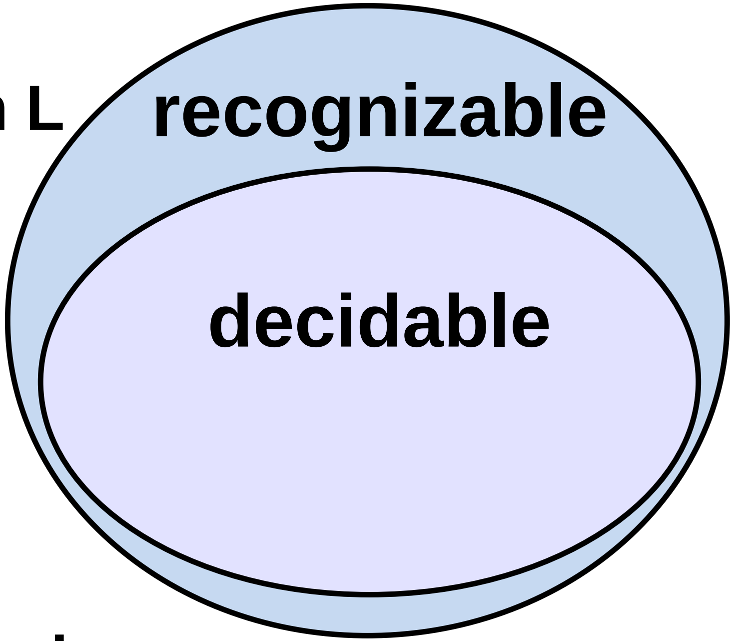
- A language is called **recognizable** (or enumerable) if some TM recognizes it.

A TM **decides** a language L if it accepts all strings in L and rejects all strings not in L.

- A language is called **decidable** (or recursive) if some TM decides it.

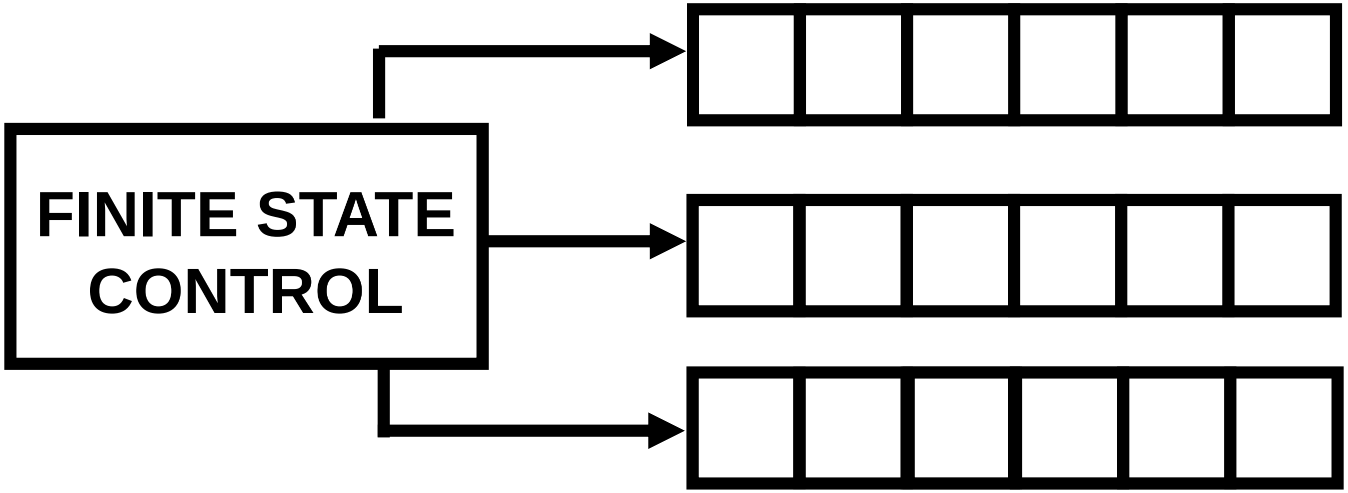
Recognizable vs. decidable languages

- **A language L is recognizable (*enumerable*) if some TM**
  1. **accepts strings in L and**
  2. **does not accept strings not in L by entering q_{reject} or looping.**



- **A language L is decidable (*recursive*) if some TM**
  1. **accepts strings in L and**
  2. **rejects strings not in L by entering q_{reject} .**

TM variant: multitape TM



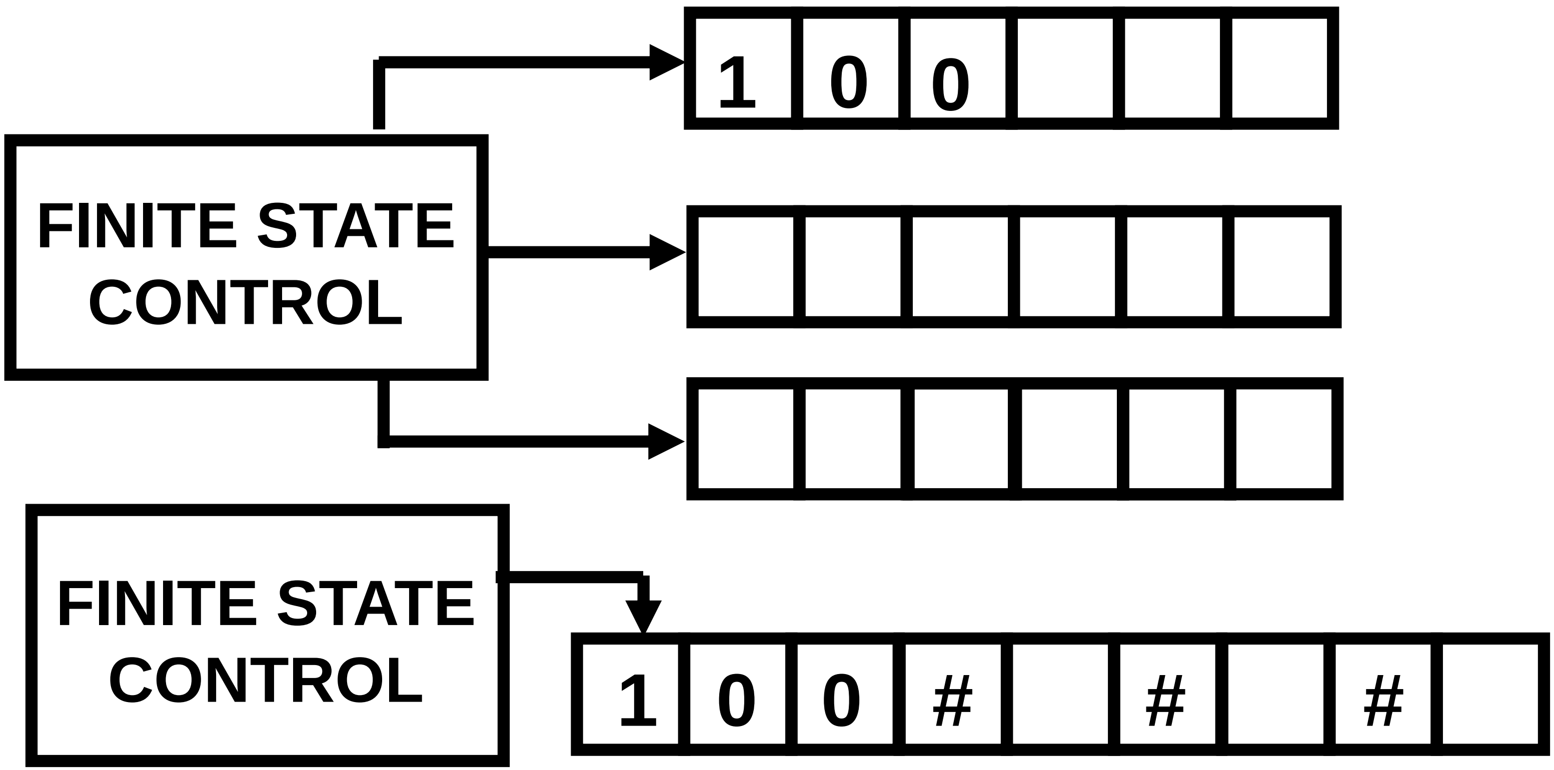
Fixed number of tapes, *k*

(can't change during computation)

$$\delta : Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L,R,S\}^k$$

Multitape TMs are equivalent to single-tape TMs

Theorem. Every multitape TM can be transformed into an equivalent single-tape TM.



SIMULATING MULTIPLE TAPES

●● ● ●

L#100# 0 # 1 #R

q_{jRSS} q_{i1} $q_{i1\square}$ $q_{i1\square\square}$ $q_{j101RSS}$

1. "Format" tape.

2. For each move of the k-tape TM:

   Scan left-to-right, finding current symbols

   Scan left-to-right, writing new symbols

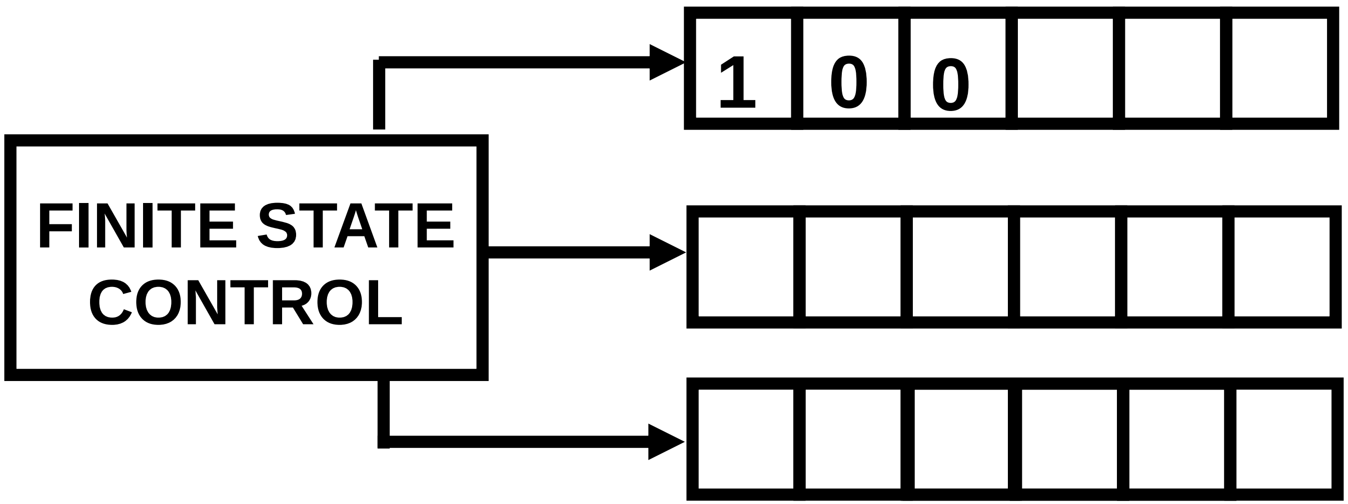
   Scan left-to-right, moving each tape head.

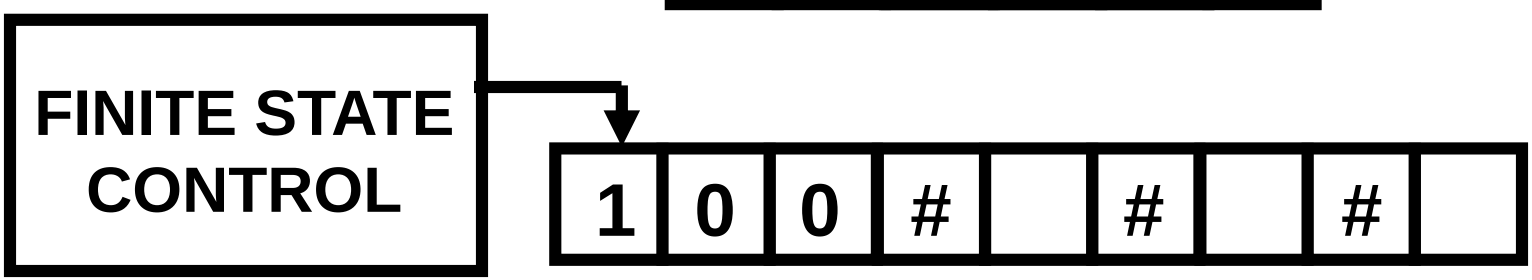
3. If a tape head goes off right end, insert blank.

   If tape head goes off left end, move back right.

Multitape TMs are equivalent to single-tape TMs

Theorem. Every multitape TM can be transformed into an equivalent single-tape TM





To show one type of machine can simulate another…

1. Explain how to initialize the new machine.
2. Explain how the new machine simulates each step of the old machine.

Exercise

Which of these statements are valid descriptions of nondeterministic steps (in a PDA)?

A. Nondeterministically read the input and push it onto the stack.

B. Nondeterministically either read a and push it onto the stack or read b and pop b from the stack.

C. Nondeterministically read the input character a and either push it onto the stack or pop b from the stack.

D. Nondeterministically push one of positive integers onto the stack.

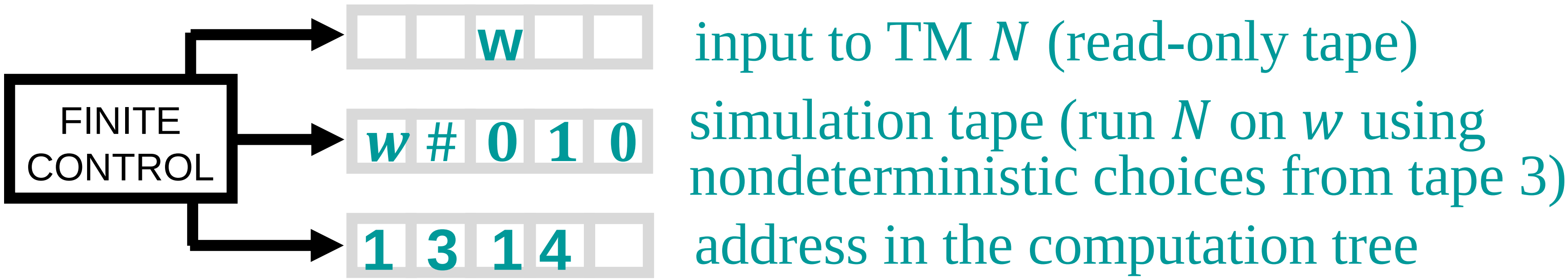
E. None of the above.

F. More than one choice above works.

NTMs are equivalent to TMs

Theorem. Every nondeterministic TM can be transformed into an equivalent deterministic TM.

Proof idea: Consider an NTM N. Use a 3-tape TM.



- Let b be the largest # of nondeterministic choices N has in a step. Use alphabet $\{1, \ldots, b\}$ for addresses.
- Do a BFS of the computation tree.

TMs are equivalent to…

TMs are equivalent to multitape TMs

TMs are equivalent to nondeterministic TMs

TMs are equivalent to doubly unbounded TMs

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Doubly unbounded TMs

A TM with doubly unbounded tape is like an ordinary TM but

- Its tape is infinite on the left and on the right.

Initially, only the input is written on the tape and the head is on the first nonblack symbol.