# Review: Conditional Probability

## Conditional Probability

The conditional probability of event $E$ given event $F$ is

$$\Pr(E \mid F) = \frac{\Pr(E \cap F)}{\Pr(F)}.$$

***Well defined only if*** $\Pr(\boldsymbol{F}) \neq \mathbf{0}$

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# Review question

## Card dealing

We deal two cards. What is the probability that the second card is an ace, given that the first is an ace?

A. 3/52

B. 3/51

C. 4/52

D. 5/52

E. None of the answers above are correct.

# Product rule

For any two events $E_1$ and $E_2$,

$$\Pr(E_1 \cap E_2) = \Pr(E_1) \cdot \Pr(E_2 | E_1).$$

For all events $E_1, \ldots, E_n$,

$$\Pr(\cap_{i=1}^{n} E_i) = \Pr(E_1) \cdot \Pr(E_2 | E_1) \cdot \ldots \cdot (E_n | \cap_{i=1}^{n-1} E_i)$$

# Law of Total Probability

For any two events $A$ and $E$,

$$\Pr(A) = \Pr(A \cap E) + \Pr(A \cap \overline{E})$$
$$= \Pr(A|E) \cdot \Pr(E) + \Pr(A|\overline{E}) \cdot \Pr(\overline{E})$$

Let A be an event and let $E_1, \ldots, E_n$ be mutually disjoint events whose union is $\Omega$.

$$\Pr(A) = \sum_{i \in [n]} \Pr(A \cap E_i) = \sum_{i \in [n]} \Pr(A \mid E_i) \cdot \Pr(E_i).$$

# Bayes' Law

For any two events $A$ and $E$ with Pr(A) $\neq 0$,

$$\Pr(E|A) = \frac{\Pr(A|E) \cdot \Pr(E)}{\Pr(A)}$$

Let A be an event with Pr(A) $\neq 0$ and let $E_1, \dots, E_n$ be mutually disjoint events whose union is $\Omega$.

$$\Pr(E_j|A) = \frac{\Pr(E_j \cap A)}{\Pr(A)} = \frac{\Pr(A|E_j) \cdot \Pr(E_j)}{\sum_{i\in[n]} \Pr(A \mid E_i) \cdot \Pr(E_i)}$$