LECTURE 11

Last time
• Chernoff Bounds

Today
• Chernoff-Hoeffding Bounds
• Estimating a parameter
• Set Balancing
Sums of independent RVs

Chernoff Bound (Both Tails). Let $X_1, \ldots, X_n$ be independent Bernoulli RVs. Let $X = X_1 + \cdots + X_n$ and $\mu = E[X]$. Then

- for any $\delta \in (0,1)$,

$$\Pr[|X - \mu| \geq \delta \mu] \leq 2e^{-\mu \delta^2/3}.$$
Sums of independent RVs

Hoeffding Bound. Let $X_1, \ldots, X_n$ be independent RVs with $E[X_i] = \mu_0$ and $\Pr[a \leq X_i \leq b] = 1$. Let $X = X_1 + \cdots + X_n$. Then

- (upper tail) $\Pr[X \geq \mu_0 n + \epsilon n] \leq e^{-2n\epsilon^2/(b-a)^2}$.
- (lower tail) $\Pr[X \leq \mu_0 n - \epsilon n] \leq e^{-2n\epsilon^2/(b-a)^2}$.
Exercise

• We throw $n$ balls uniformly and independently into $n$ bins.
Let $Y_1$ be the number of balls that fell into bin 1. Determine $m$ such that $\Pr[Y_1 > m] \leq \frac{1}{n^2}$. 