Homework 10 – Due Friday, April 10, 2020 by noon on Gradescope.

Page limit  You can submit at most 1 sheet of paper per problem, even if the problem has multiple parts. If you submit a longer solution for some problem, only the first sheet of paper will be graded.

Reminder  Collaboration is permitted, but you must write the solutions by yourself without assistance, and be ready to explain them orally to the instructor if asked. You must also identify your collaborators and whether you gave help, received help, or worked something out together. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

Exercises  Please practice on exercises in Chapters 5 and 6 of Mitzenmacher-Upfal.

Problems

1. (Finding Hamiltonian cycles, 2 page limit) Consider the algorithm for finding Hamiltonian cycles covered in class (Algorithm 5.2 in MU).

   (a) Hamiltonian cycles in directed graphs can be defined analogously to Hamiltonian cycles in undirected graphs. However, the algorithm we discussed does not work for finding Hamiltonian cycles in directed graphs. Explain why.

   In the next two parts, find suitable randomized strategies for placing edges in the adjacency lists so that conditions of Theorem 5.17 are satisfied.

   (b) Explain how to apply Algorithm 5.2 when $G$ is chosen according to $G_{n,M}$ instead of $G_{n,p}$ and the number of edges $M \geq cn \ln n$ for a sufficiently large constant $c$.

   *Hint:* A graph from $G_{n,p}$ can be generated by first choosing the number of edges $X$ (according to which distribution?) and then generating a graph from $G_{n,X}$.

   (c) MU explains how to apply Algorithm 5.2 when $p$ is known (and sufficiently large). Explain how to apply it when $p$ is not known (but sufficiently large). *Hint:* Use part (b) if the graph has sufficiently many edges; otherwise, fail.

2. (MaxSat) Exercise 6.1.

3. (Tournaments) Exercise 6.9.

No midterm-substitution problem this week.