Homework 9 – Due Thursday, April 5, 2018 at 4:45pm

Submit solutions to all problems on separate sheets. They will be graded by different people.

Page limit  You can submit at most 1 sheet of paper per problem, even if the problem has multiple parts. If you submit a longer solution for some problem, only the first sheet of paper will be graded.

Reminder  Collaboration is permitted, but you must write the solutions by yourself without assistance, and be ready to explain them orally to the instructor if asked. You must also identify your collaborators and whether you gave help, received help, or worked something out together. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

Exercises  Please practice on exercises in Chapter 5 of Mitzenmacher-Upfal.

Problems

1. (Tight Poisson approximation) In this problem, you will prove Theorem 5.10 from MU for functions whose expectation is monotone increasing in \( m \).
   
   (a) Do part (a) of Exercise 5.14.
   
   (b) Do part (b) of Exercise 5.14.
   
   (c) Do part (a) of Exercise 5.15 only for the case when \( E[f(X_1^{(m)}, \ldots, X_n^{(m)})] \) is monotonically increasing in \( m \).
   
   (d) Do part (b) of Exercise 5.15.

2. (Poisson sampling) You are testing cookies by picking each sample uniformly and independently at random. Suppose that a batch of cookies contains \( p_1 \) fraction of defective cookies, \( p_2 \) fraction of ok cookies, and \( p_3 \) fraction of amazingly delicious cookies. You take a Poisson number of samples with mean \( \mu \). Let \( X, Y \) and \( Z \) be the number of defective, ok, and amazingly delicious cookies in your sample, respectively. Prove that \( X, Y \) and \( Z \) are independent Poisson random variables with means \( p_1 \mu, p_2 \mu, \) and \( p_3 \mu, \) respectively.

3. (Hashing with open addressing) Exercise 5.22.