Homework 10 – Due Thursday, November 17, 2016 on Canvas

Please refer to HW guidelines from HW1, course syllabus, and collaboration policy.
These should not be handed in, but the material they cover may appear on exams:

1. (**Blood supply**) Chapter 7, problem 8.
2. (**Dining Problem**) Several families go out to dinner together. To increase their social interaction, they would like to sit at tables so that no two members of the same family are at the same table. Show how to find a seating arrangement that meets this objective (or prove that no such arrangement exists) by reducing this problem to Maximum Flow. Assume that the dinner contingent has p families, that the ith family has a_i members, that q tables are available and up to b_j people can be seated at table j.

Problems to be handed in

1. (**Node-capacitated networks**) Chapter 7, problem 13. To help you organize your answer (and to make the TA's life easier), beak your answer into parts as follows:
   (a) Give an algorithm for maximum flows in node-capacitated networks. Pay close attention to the argument of correctness of your algorithm.
   [*Hint:* One way to give an algorithm for this problem is to run Ford-Fulkerson on a modified version (say $G' = (V', E')$) of the original graph G, in which each vertex in V corresponds to two vertices (call them v_{in} and v_{out}) in V'. The graph G' should have an edge e' for every edge e in E along with some additional edges.]
   (b) Define an analogue of an s-t cut in a node-capacitated network, and define the "capacity" of your object.
   (c) Explain why the max-flow min-cut theorem holds for your analogue.
   If it is easier, you may want to prove correctness of your algorithm at the same time as you prove your max-flow/min-cut theorem.
2. (**Doctors Without Weekends**) Chapter 7, problem 19.
3. **Maximum-Cardinality Bipartite Matching**
   (a) Give a linear-programming (LP) formulation of the maximum-cardinality bipartite matching problem. The input is a bipartite graph $G = (U \cup V, E)$, where $E \subseteq U \times V$; the output is the largest matching in G. Your LP should have one variable for each edge.
   (b) Now dualize the LP from part (a). What do the dual variables represent? What does the objective function represent? What problem is this?