Homework 8 – Due Thursday, October 27, 2016 on Canvas

Please refer to HW guidelines from HW1, course syllabus, and collaboration policy.

Exercises These should not be handed in, but the material they cover may appear on exams:

1. (**Network Flow**) Chapter 7, Problems 1-3, 11.

Problems to be handed in, 10 points each, 2-page limit per problem (Don't forget to prove correctness and analyze time/space requirements of your algorithm.)

1. (**Flow decomposition**) A flow f is **acyclic** if the subgraph of directed edges with positive flow contains no directed cycles.
   (a) Prove that for a any flow f, there is an acyclic flow with the same value as f. (In particular, this implies that some maximum flow is acyclic.)
   (b) A **path flow** assigns positive values only to the edges of one simple directed path from s to t. Prove that every acyclic flow can be written as the sum of a finite number of path flows.
   (c) Describe a flow in a directed graph that *cannot* be written as the sum of path flows.
2. (**Reducing capacity of one edge**) Chapter 7, Problem 10.
3. (**Reducing flow by deleting k edges**) Chapter 7, Problem 12.

4* (**Optional, no collaboration**) Give an efficient algorithm to check whether a given flow network has a *unique* maximum flow. Your algorithm should take as much time (asymptotically) as one maximum-flow computation.