

# *Algorithm Design and Analysis*

**CSE  
565**

## **LECTURE 1**

### **Analysis of Algorithms**

- Course information
- Why study algorithms?
- Stable matching problem

**Sofya Raskhodnikova**

# CSE 565 Course information

- 1. Staff**
- 2. Course websites**
- 3. Textbook**
- 4. Syllabus**
- 5. Prerequisites**
- 6. Name template**
- 7. Background survey**
- 8. Reading quizzes**
- 9. Exams**
- 10. Homework**
- 11. Collaboration policy**

# Algorithm Design and Analysis

*Theoretical study of how to solve computational problems*

- sorting a list of numbers
- finding a shortest route on a map
- scheduling when to work on homework
- answering web search queries

(Generally: precisely defined set of **inputs** and, for each input, **acceptable outputs**)

# Etymology of “Algorithm”

*Abu Abdullah Muhammad ibn Musa  
al-Khwarizmi (c. 780 -- 850 AD)*

- Persian astronomer and mathematician
- lived in Baghdad, father of algebra
- “On calculating with hindu numerals”  
a treatise in Arabic, 825
- “Algoritmi de numero Indorum”  
translation into Latin, 12th century
- Author’s name, mistaken for a plural noun, came to mean “calculation methods”

# Algorithms

- Definition: Finite set of unambiguous instructions for solving a problem.
  - An algorithm is **correct** if on all legitimate inputs, it outputs the right answer in a finite amount of time
- Can be expressed as
  - pseudocode
  - text in a natural language (e.g. English)
  - computer code
  - pictures

# Data Structures

- **Data structures** are ways to store information for which there are **algorithms** for performing particular operations (retrieving/manipulating information), e.g.
  - linked lists
  - hash tables
  - heaps
  - red-black trees
  - skip lists

# Why study algorithms?

- *Language* for thinking about program behavior
  - analyzing correctness and resource usage
  - most computing research is algorithmic
- Standard set of algorithms and design techniques
- Feasibility (what can and cannot be done)
  - halting problem, NP-completeness
- Successful companies (Google, Akamai, ...)
- Computation is fundamental to understanding the world
  - cells, brains, social networks, physical systems, ...
- Exercise for your brain
- Fun!

# Performance isn't everything

- Typical goal: Find most space- and time-efficient algorithm for given problem.
- What else is important?
  - modularity
  - maintainability
  - functionality
  - robustness
  - user-friendliness
  - programmer time
  - simplicity
  - extensibility
  - reliability



# Course Objectives

## Material

- Classical algorithms
- Analysis of algorithms
- Design techniques

## Skills

- Algorithmic Thinking
- Problem-solving & mathematical skills
- Technical writing

# Prerequisite: CSE 465 or equivalent

## Some of things you should have covered:

- **Reading/writing Proofs** (including induction, contradiction, ...)
- Asymptotic notation (e.g.  $\text{``big-O''}$ )
- Elementary data structures: lists, stacks, queues, sorted arrays
- Graphs and trees
  - Depth- and breadth-first search, shortest paths, ...
- Basic design paradigms:
  - divide and conquer
  - greedy algorithms
  - dynamic programming
- Basic mathematical tools: arithmetic and geometric series, counting permutations, vectors and matrices, etc

# Co-requisite: 464/468 or equivalent

- Finite automata, Turing machines, undecidability, complexity measures, “P” nondeterminism and “NP”
- What we will need:
  - abstractions of computation
  - P / NP / NP-completeness

# A first problem: Stable Matching

# Matching Residents to Hospitals

- **Goal:** Given a set of preferences among hospitals and medical school students, design a **self-reinforcing** admissions process.
- **Unstable pair:** applicant  $x$  and hospital  $y$  are **unstable** if
  - $x$  prefers  $y$  to its assigned hospital, and
  - $y$  prefers  $x$  to one of its admitted students
- **Stable assignment:** no unstable pairs.
  - Individual self-interest will prevent any applicant/hospital deal from being made.
- 2012 Nobel prize in economics for work on matching algorithms (“mechanisms”)

# Stable Matching Problem

- **Goal:** Given  $n$  men and  $n$  women, find a "suitable" matching.
  - Participants rate members of opposite sex.
  - Each man lists women in order of preference from best to worst.
  - Each woman lists men in order of preference from best to worst.

	favorite ↓ 1st	2nd	least favorite ↓ 3rd
Xavier	Amy	Bertha	Clare
Yancey	Bertha	Amy	Clare
Zeus	Amy	Bertha	Clare

*Men's Preference Profile*

	favorite ↓ 1st	2nd	least favorite ↓ 3rd
Amy	Yancey	Xavier	Zeus
Bertha	Xavier	Yancey	Zeus
Clare	Xavier	Yancey	Zeus

*Women's Preference Profile*

# Stable Matching Problem

- **Unstable pair:** man  $m$  and woman  $w$  are **unstable** if
  - $m$  prefers  $w$  to his assigned match, and
  - $w$  prefers  $m$  to her assigned match
- Unstable pairs have an incentive to elope
- **Stable matching:** no unstable pairs.

	favorite ↓ 1st	2nd	least favorite ↓ 3rd
Xavier	Amy	Bertha	Clare
Yancey	Bertha	Amy	Clare
Zeus	Amy	Bertha	Clare

*Men's Preference Profile*

	favorite ↓ 1st	2nd	least favorite ↓ 3rd
Amy	Yancey	Xavier	Zeus
Bertha	Xavier	Yancey	Zeus
Clare	Xavier	Yancey	Zeus

*Women's Preference Profile*

# Stable Matching Problem

- **Input:** preference lists of  $n$  men and  $n$  women
- **Goal:** find a stable matching if one exists

	favorite ↓ 1 <sup>st</sup>	2 <sup>nd</sup>	least favorite ↓ 3 <sup>rd</sup>
Xavier	Amy	Bertha	Clare
Yancey	Bertha	Amy	Clare
Zeus	Amy	Bertha	Clare

*Men's Preference Profile*

	favorite ↓ 1 <sup>st</sup>	2 <sup>nd</sup>	least favorite ↓ 3 <sup>rd</sup>
Amy	Yancey	Xavier	Zeus
Bertha	Xavier	Yancey	Zeus
Clare	Xavier	Yancey	Zeus

*Women's Preference Profile*



# Stable Matching Problem

- Q. Is assignment X-C, Y-B, Z-A stable?

	favorite ↓ 1 <sup>st</sup>	2 <sup>nd</sup>	least favorite ↓ 3 <sup>rd</sup>
Xavier	Amy	Bertha	Clare
Yancey	Bertha	Amy	Clare
Zeus	Amy	Bertha	Clare

*Men's Preference Profile*

	favorite ↓ 1 <sup>st</sup>	2 <sup>nd</sup>	least favorite ↓ 3 <sup>rd</sup>
Amy	Yancey	Xavier	Zeus
Bertha	Xavier	Yancey	Zeus
Clare	Xavier	Yancey	Zeus

*Women's Preference Profile*

# Stable Matching Problem

- Q. Is assignment X-C, Y-B, Z-A stable?
- A. No. Bertha and Xavier will hook up.

	favorite ↓		least favorite ↓
	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>
Xavier	Amy	Bertha	Clare
Yancey	Bertha	Amy	Clare
Zeus	Amy	Bertha	Clare

*Men's Preference Profile*

	favorite ↓		least favorite ↓
	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>
Amy	Yancey	Xavier	Zeus
Bertha	Xavier	Yancey	Zeus
Clare	Xavier	Yancey	Zeus

*Women's Preference Profile*

# Stable Matching Problem

- Q. Is assignment X-A, Y-B, Z-C stable?

	favorite ↓ 1st	2nd	least favorite ↓ 3rd
Xavier	Amy	Bertha	Clare
Yancey	Bertha	Amy	Clare
Zeus	Amy	Bertha	Clare

*Men's Preference Profile*

	favorite ↓ 1st	2nd	least favorite ↓ 3rd
Amy	Yancey	Xavier	Zeus
Bertha	Xavier	Yancey	Zeus
Clare	Xavier	Yancey	Zeus

*Women's Preference Profile*

# Stable Matching Problem

- Q. Is assignment X-A, Y-B, Z-C stable?
- A. Yes. X and Y got their first choice; Z is the last choice for every woman. No man can participate in an unstable pair.

	favorite ↓ 1st		least favorite ↓ 3rd
	1st	2nd	3rd
Xavier	Amy	Bertha	Clare
Yancey	Bertha	Amy	Clare
Zeus	Amy	Bertha	Clare

*Men's Preference Profile*

	favorite ↓ 1st		least favorite ↓ 3rd
	1st	2nd	3rd
Amy	Yancey	Xavier	Zeus
Bertha	Xavier	Yancey	Zeus
Clare	Xavier	Yancey	Zeus

*Women's Preference Profile*

# Existence of Stable Matching

- Q. Do stable matchings always exist?
- A. Not obvious a priori.

# Stable Roommate Problem

- **Stable roommate problem**

- $2n$  people; each person ranks others from **1** to  $2n-1$ .
- Assign roommate pairs so that no unstable pairs.

	<i>1<sup>st</sup></i>	<i>2<sup>nd</sup></i>	<i>3<sup>rd</sup></i>
<i>Adam</i>	B	C	D
<i>Bob</i>	C	A	D
<i>Chris</i>	A	B	D
<i>Doofus</i>	A	B	C

A-B, C-D  $\Rightarrow$  B-C unstable  
A-C, B-D  $\Rightarrow$  A-B unstable  
A-D, B-C  $\Rightarrow$  A-C unstable

- **Observation.** Stable matchings do not always exist for stable roommate problem.

# An Algorithm for Stable Matching

- Propose-and-reject algorithm. [Gale-Shapley 1962]

```
Initialize each person to be free.
while (some man is free and hasn't proposed to every woman) {
    Choose such a man m
    w = 1st woman on m's list to whom m has not yet proposed
    if (w is free)
        assign m and w to be engaged
    else if (w prefers m to her fiancé m')
        assign m and w to be engaged, and m' to be free
    else
        w rejects m
}
```

*Men's Preference Profile*

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

*Women's Preference Profile*

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor



*Men's Preference Profile*

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	<b>Bertha</b>	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

Victor proposes to Bertha.

*Women's Preference Profile*

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	<b>Victor</b>	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Victor proposes to Bertha.

- Bertha accepts since previously unmatched.

*Men's Preference Profile*

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

*Women's Preference Profile*

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Wyatt proposes to Diane.

### *Men's Preference Profile*

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### *Women's Preference Profile*

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Wyatt proposes to Diane.

- Diane accepts since previously unmatched.

*Men's Preference Profile*

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

*Women's Preference Profile*

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Xavier proposes to Bertha.

### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Xavier proposes to Bertha.

- Bertha dumps Victor and accepts Xavier.

*Men's Preference Profile*

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

*Women's Preference Profile*

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Victor proposes to Amy.

### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Victor proposes to Amy.

- Amy accepts since previously unmatched.



*Men's Preference Profile*

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

*Women's Preference Profile*

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Yancey proposes to Amy.

### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Yancey proposes to Amy.

- Amy rejects since she prefers Victor.

*Men's Preference Profile*

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

*Women's Preference Profile*

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Yancey proposes to Diane.

### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

Yancey proposes to Diane.

- Diane dumps Wyatt and accepts Yancey.

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

### Men's Preference Profile

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Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
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Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Wyatt proposes to Bertha.

### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Wyatt proposes to Bertha.

- Bertha rejects since she prefers Xavier.

### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Wyatt proposes to Amy.

### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Wyatt proposes to Amy.

- Amy rejects since she prefers Victor.



### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Wyatt proposes to Clare.

### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Wyatt proposes to Clare.

- Clare accepts since previously unmatched.

### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

Zeus proposes to Bertha.

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Zeus proposes to Bertha.

- Bertha rejects since she prefers Xavier.

### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Zeus proposes to Diane.

### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

Zeus proposes to Diane.

- Diane rejects Yancey and accepts Zeus.

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Yancey proposes to Clare.

### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Yancey proposes to Clare.

- Clare rejects since she prefers Wyatt.



### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Yancey proposes to Bertha.

### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Yancey proposes to Bertha.

- Bertha rejects since she prefers Xavier.

### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Yancey proposes to Erika.

### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Yancey proposes to Erika.

- Erika accepts since previously unmatched.

### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

STOP

- Everyone matched.
- Stable matching!

# Propose-and-Reject Algorithm

- Propose-and-reject algorithm. [Gale-Shapley 1962]

```
Initialize each person to be free.
```

```
while (some man is free and hasn't proposed to every woman) {  
    Choose such a man m  
    w = 1st woman on m's list to whom m has not yet proposed  
    if (w is free)  
        assign m and w to be engaged  
    else if (w prefers m to her fiancé m')  
        assign m and w to be engaged, and m' to be free  
    else  
        w rejects m  
}
```

# Proof of correctness

**Three claims:** The algorithm always

1. terminates,
2. matches everyone (matching is “perfect”), and
3. outputs a stable matching

### Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

### Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

STOP

- Everyone matched.
- Stable matching!



# Proof of Correctness: Termination

- **Claim.** Algorithm terminates after at most  $n^2$  iterations of while loop.

- **Proof.** Each time through the loop a man proposes to a new woman.

There are only  $n^2$  possible proposals. ■

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>
Victor	A	B	C	D	E
Wyatt	B	C	D	A	E
Xavier	C	D	A	B	E
Yancey	D	A	B	C	E
Zeus	A	B	C	D	E

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>
Amy	W	X	Y	Z	V
Bertha	X	Y	Z	V	W
Clare	Y	Z	V	W	X
Diane	Z	V	W	X	Y
Erika	V	W	X	Y	Z

An instance where  $n(n-1) + 1$  proposals required

# Propose-and-Reject Algorithm

- **Observation 1.** Men propose to women in decreasing order of preference.
- **Observation 2.** Once a woman is matched, she never becomes unmatched; she only "trades up."

Men's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

Women's Preference Profile

	0 <sup>th</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

# Proof of Correctness: Perfection

- **Claim.** All men and women get matched.
- **Proof:** (by contradiction)
  - Suppose, for sake of contradiction, some guy, say Zeus, is not matched upon termination of algorithm.
  - Then some woman, say Amy, is not matched upon termination.
  - By Observation 2, Amy was never proposed to.
  - But Zeus proposes to everyone, since he ends up unmatched. ■

# Proof of Correctness: Stability

- **Claim.** No unstable pairs.

- **Proof:** (by contradiction)

- Suppose A-Z is an unstable pair: they prefer each other to their partners in Gale-Shapley matching  $S^*$ .

- *Case 1:* Z never proposed to A.

- ⇒ Z prefers his GS partner to A. ← men propose in decreasing order of preference
  - ⇒ A-Z is stable.

- *Case 2:* Z proposed to A.

- ⇒ A rejected Z (right away or later)

- ⇒ A prefers her GS partner to Z. ← women only trade up

- ⇒ A-Z is stable.

- In either case A-Z is stable, a contradiction. ▀

# Stable Roommate Problem

- **Stable roommate problem**

- $2n$  people; each person ranks others from **1** to  $2n-1$ .
- Assign roommate pairs so that no unstable pairs.

	<i>1<sup>st</sup></i>	<i>2<sup>nd</sup></i>	<i>3<sup>rd</sup></i>
<i>Adam</i>	B	C	D
<i>Bob</i>	C	A	D
<i>Chris</i>	A	B	D
<i>Doofus</i>	A	B	C

A-B, C-D  $\Rightarrow$  B-C unstable  
A-C, B-D  $\Rightarrow$  A-B unstable  
A-D, B-C  $\Rightarrow$  A-C unstable

- **Exercise.** Where does the correctness proof break down for the roommates version?

# Two more exercises

- Does the order in which unmatched men are selected change the final matching?
- If women propose (instead of men), what changes? Is it better to propose or to receive proposals?

# Efficient Implementation

- We describe  $O(n^2)$  time implementation.

(in Python: [http://euler.slu.edu/~goldwasser/courses/slu/csci314/2006\\_Spring/lectures/marriageAlgorithm/](http://euler.slu.edu/~goldwasser/courses/slu/csci314/2006_Spring/lectures/marriageAlgorithm/))

- Assume men have IDs  $1, \dots, n$ , and so do women.
- Engagements data structures:
  - a list of free men, e.g., a queue.
  - two arrays `wife[m]`, and `husband[w]`.
    - set entry to 0 if unmatched
    - if  $m$  is matched to  $w$  then `wife[m] = w` and `husband[w] = m`
- Men proposing data structures:
  - an array `men-pref[m, i] = ith women on m's list`
  - an array `count[m] = how many proposals m made.`

# Efficient Implementation

- Women rejecting/accepting data structures
  - Does woman  $w$  prefer man  $m$  to man  $m'$ ?
  - For each woman, create **inverse** of preference list of men.
  - Constant time queries after  $O(n)$  preprocessing per woman.

Amy	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>	6 <sup>th</sup>	7 <sup>th</sup>	8 <sup>th</sup>
Pref	8	3	7	1	4	5	6	2

Amy	1	2	3	4	5	6	7	8
Inverse	4 <sup>th</sup>	8 <sup>th</sup>	2 <sup>nd</sup>	5 <sup>th</sup>	6 <sup>th</sup>	7 <sup>th</sup>	3 <sup>rd</sup>	1 <sup>st</sup>

```
for i = 1 to n
  inverse[pref[i]] = i
```

Amy prefers man 3 to 6  
since  $\text{inverse}[3] < \text{inverse}[6]$   
2                      7



# Summary

- **Stable matching problem.** Given  $n$  men and  $n$  women, and their preferences, find a stable matching if one exists.
- **Gale-Shapley algorithm.** Guarantees to find a stable matching for **every** problem instance.
  - (Also proves that a stable matching always exists)
- **Time and space complexity:**  
 $O(n^2)$ , linear in the input size.

# Assignments

- Reading for Wednesday:  
Chapters 2.1-2.4 of KT book  
(Asymptotic Analysis)
- To be completed on Canvas by Tuesday night:
  - Reading Quiz
  - Background survey