Homework 2 – Due Wednesday, January 30, 2007 <u>before</u> the lecture

Please refer to the general information handout for the full homework policy and options.

Reminder Collaboration is permitted, but you must write the solutions *by yourself without assistance*, and be ready to explain them orally to the instructor if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

Exercises Please practice on solved exercises and problems in Chapter 1. The material they cover may appear on exams.

Problems

1. (**Closure properties, $\leq$ 1.5 pages, 15 points**) Prove that the class of regular languages is closed under (a) complement, (b) intersection, (c) set difference. Recall that the set difference of A and B is defined as $A \setminus B = \{x \in A \mid x \notin B\}$.

   (d) Show by giving an example that, if M is an NFA that recognizes language C, swapping the accept and nonaccept states in M does not necessarily yield a new NFA that recognizes the complement of C.

   (e) Is the class of languages recognized by NFAs closed under complement? Explain you answer.

2. (**Number of states, $\leq$ 1 page, 10 points**) For each $k \geq 1$ let C_k be the language over $\Sigma = \{a, b\}$ consisting of all strings with two a's that are $k-1$ symbols apart. Using regular expressions, $C_k = \Sigma^* a \Sigma^{k-1} a \Sigma^*$.

   (a) Give a state diagram and a formal description of an NFA with $k+2$ states that recognizes C_k.

   (b) Prove that for each k, no DFA with fewer than 2^k states can recognize C_k.

3. (**Non-regular languages, $\leq$ 1.5 pages, 15 points**) Prove that the following languages are not regular. You may use the pumping lemma and the closure of the class of regular languages under union, intersection and complement.

   (a) $L_1 = \{0^n 1^m 0^n \mid m, n \geq 0\}$.

   (b) $L_2 = \{0^k \mid k \text{ is a prime number}\}$.

   (c) $L_3 = \{1^k y \mid y \in \{0, 1\}^* \text{ and } |y| = k\}$.

   (d) $L_4 = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$.

   (e) Prove that L_4 satisfies the conditions of the pumping lemma. (Specify pumping length p.)

   (f) Explain why parts (d) and (e) do not contradict the pumping lemma.