Homework 7 – Due Wednesday, March 19, 2008 before the lecture

Please refer to the general information handout for the full homework policy and options.

Page limit You can submit **at most** 1 page per problem, even if the problem has multiple parts. If you submit a longer solution for some problem, only the first page will be graded. This homework contains 4 problems, worth 10 points each.

Reminder Collaboration is permitted, but you must write the solutions *by yourself without assistance*, and be ready to explain them orally to the instructor if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

Exercises Please practice on exercises and solved problems in chapter 7.

Problems

1. (**Asymptotic notation**) For each of the following, answer *true* or *false*. No justification is required.

   (a) $2^{10} = O(n)$

   (b) $16n = O(n)$

   (c) $n^4 = O(n^2 \log n)$

   (d) $n \log n + 10n = O(n^2)$

   (e) $3^n = O(2^n)$

   (f) $3^n = 2^{O(n)}$

   (g) $2^{2^n} = O(2^{2n})$

   (h) $n^n = O(n!)$

   (i) $n = o(n)$

   (j) $2n = o(n^2)$

   (k) $2^n = o(3^n)$

   (l) $1 = o(n)$

   (m) $2 \log n = o(\log n)$

   (n) $\frac{1}{3} = o(1)$

   (o) $\log_2 n = \Theta(\log_3 n)$

   (p) $2^n = \Theta(4^n)$

   (q) $n^5 = \Theta(32^{\log_2 n})$

   (r) $n^3 = \Omega(n^3)$

   (s) $\log n = \Omega(\log(\log n))$

   (t) $2^{5^n} = \Omega(5^{2^n})$

2. (**Modular exponentiation**) Book, 7.12.

3. (**P is closed under star**) Book, 7.14.

4. Let kPOST $= \{\langle S, k\rangle |$ S is a finite set of dominoes over Σ; k is an integer written in unary, and there is a sequence of at most k dominoes (allowing repeats) for which the top and bottom sequences are equal$\}$.

   (a) Prove that kPOST is in NP.

   (b) If k was not written in unary, would your solution to part (a) still work? Why or why not?