Analyzing Algorithms

How do we know if our code is good at it's job?

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BU Summer Challenge

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- But there are many algorithms to solve the same problem
- How do we assess what algorithms are "better" than others?
- There's not always one "best" algorithm for a problem. It often depends on our situation

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- Today we will focus on “running time”.

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- Algorithms can behave differently for two arrays of the same size
  - *Worst case analysis:* What's the maximum number of steps an algorithm could take?

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- We have to loop through the entire array, and each iteration we do 2 actions so roughly $2n$ steps

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    - If $s < m$, then s is to the left of m (or not in the array)
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- We repeat the above step with the half of the array that s must be in if it is in the array.

Binary Search Example

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- So an array of size n takes roughly $2 \cdot \log_2 n$ steps.

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- This sum actually equals $\frac{n(n+1)}{2} = \frac{1}{2}n^2 + \frac{1}{2}n$

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- Each iteration makes $n - 1$ comparisons, for a total of $n(n - 1)$ iterations.

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- In general, there are 2^n combinations
- If we wanted to check whether a formula was ever true for any combination, we have to check 2^n combinations.

Comparing These Running Times

	Search	Binary Search	Selection Sort	Bubble Sort	Booleans
Steps	$2n$	$2\log_2(n)$	$\frac{1}{2}(n^2+n)$	n^2-n	2^n

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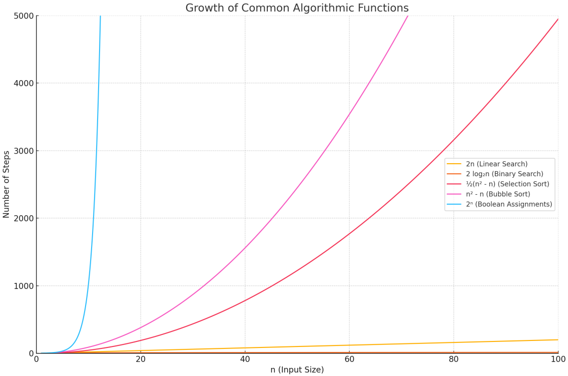

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- If an algorithm doesn't take longer as the input gets longer, we say it is $O(1)$.

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- A $\mathcal{O}(2^n)$ algorithm would be inefficient
- In real life, $O(n)$ and $O(n \log_2(n))$ algorithms are often called efficient
- Even $O(n^2)$ might be too much in practice