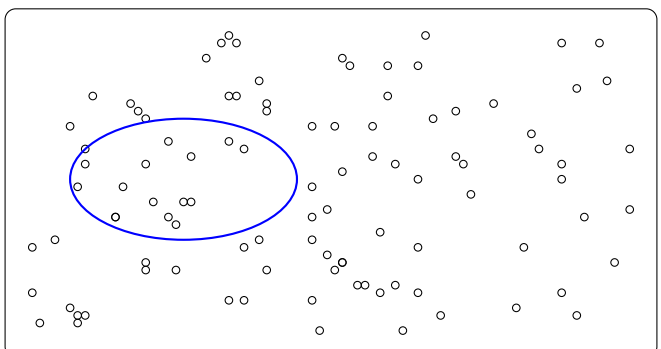
Weight reduction in distributed protocols: new algorithms and analysis

Tolik Zinovyev

Boston University

Committee selection

- Have n parties, want to select a small subset and delegate work to them (e.g., in consensus)
- The protocol should remain reliable / secure



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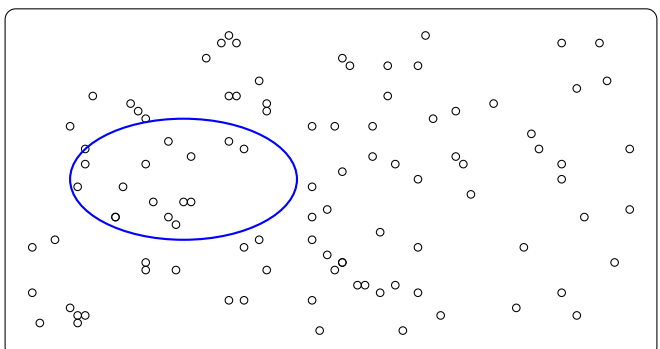

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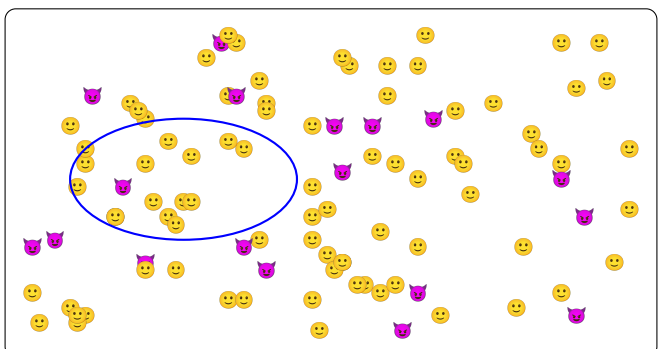

Committee selection (cont.)

Typical goal:

- 20% of parties are malicious, need to elect a committee with $< \frac{1}{3}$ parties malicious
- some security is lost: $\frac{1}{5} \to \frac{1}{3}$

$$t_i = \begin{cases} 1 \text{ if party } i \text{ is chosen} \\ 0 \text{ otherwise} \end{cases}$$

$A \subseteq [n]$ – adversary set, want

$$\text{for all } A \text{ with } |A| \leq \frac{n}{5} : \quad \Pr\left[\sum_{i \in A} t_i < \frac{1}{3} \sum_{i=1}^{n} t_i\right] \text{ is big}$$

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- select each party with probability p:
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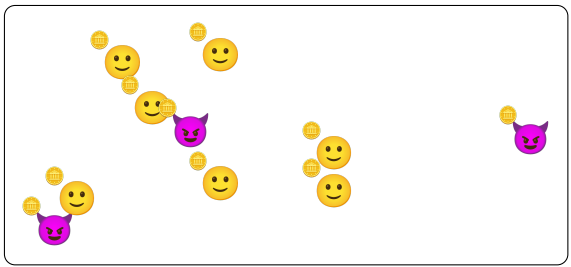

- about 20% of selected parties will be malicious; use tail bounds to analyze bad event
- security error $2^{-\lambda}$ requires committee size $\approx 50\lambda$ (for $\frac{1}{5} \to \frac{1}{3}$)

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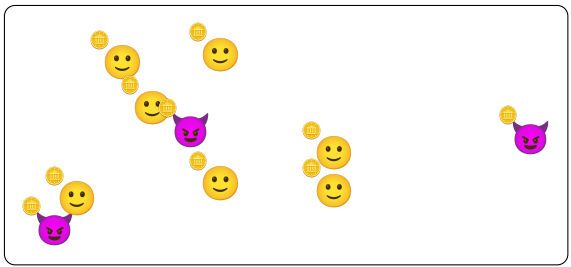

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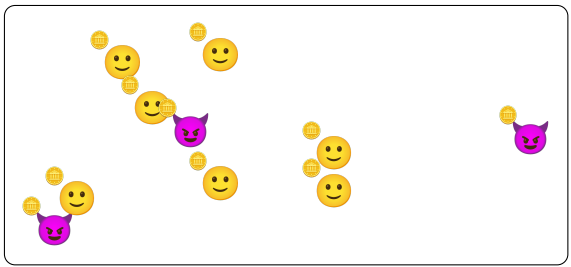

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Introducing weights

- the n parties have weights $w_1, w_2, ..., w_n \in \mathbb{Z}_{\geq 0}$
- need to assign new weights $t_1, t_2, ..., t_n \in \mathbb{Z}_{\geq 0}$
  - say "party i gets t_i tickets"
- new problem statement: adversary has weight at most 20%; he must have $< \frac{1}{3}$ tickets

need to sample
$t_1, t_2, ..., t_n \in \mathbb{Z}_{\geq 0}$ such that

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Introducing weights (cont.)

- what Algorand uses
- their solution: select each *unit* of weight with probability p
  - equivalently, $t_i \sim \texttt{Binomial}(w_i, p)$;
- often want security against adaptive corruptions
  - the adversary is allowed to corrupt parties throughout the protocol execution

- Algorand achieves it: does extra work to "hide" t_i

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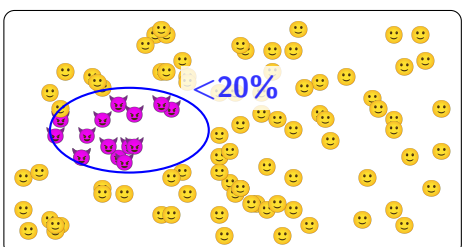

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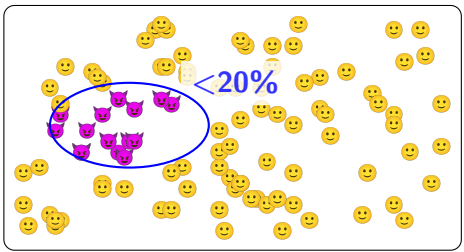

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Improvements in weighted committee selection

- Take weight distribution $(w_1, ..., w_n)$ into account
- Fait Accompli [GKR23]:
  - selects biggest parties deterministically and everyone else randomly as in Algorand
  - big players selected almost certainly anyway
  - randomly sample from less weight $\Longrightarrow$ less variance
  - improves committee size vs. error tradeoff
- Swiper [TF23] + others [BHS23, FMT24, DPTX25]:
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  - cons: committee size $\Omega(n)$ in worst case (e.g., $w_1 = w_2 = ... = w_n = 1$)
  - pros: security against adaptive corruptions guaranteed
  - pros: (?) deterministic committees are smaller when security parameter is large
- both show that realistic weight distributions allow small committees

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Objective function

Minimize the size of the committee $(t_1, t_2, ..., t_n)$: how to quantify?

- option (1) : number of distinct parties on the committee: the number of $i \in [n]$ with $t_i > 0$
  - useful in e.g. Algorand
- option (2) : the total new weight: $\sum_{i=1}^{n} t_i$
  - useful when the protocol scales poorly with the weights (e.g., secret sharing)

Fait Accompli solves (1) and (2); Swiper and **this work** solves (2).

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Problem statement

Assume $0 < \alpha < \beta < 1$. Given $w_1, ..., w_n \in \mathbb{Z}_{\geq 0}$, find $t_1, ..., t_n \in \mathbb{Z}_{\geq 0}$ such that

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- $\alpha \to \beta$, deterministic:

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Adopt terminology: party i gets t_i "tickets".

- pure optimization problem
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Coming next...

- ☑ Problem statement
- ☐ **Swiper overview**
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Swiper [TF23] overview: verifying a solution

Input: $(w_1, t_1), ..., (w_n, t_n) \in \mathbb{Z}^2_{\geq 0}$

Output: True/False – test if the solution $(t_1, ..., t_n)$ is valid; need to check

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Swiper: knapsack problem, dynamic programming solution in time $O(n \cdot T)$, where $T = \sum_{i=1}^{n} t_i$.

for $k \in [n]$ **do**

  for $t \in \{0, 1, ..., T\}$ **do**

    compute min weight the adversary has to corrupt from $[k]$ to get t tickets total:

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return $\neg \exists t$ s.t. $t \geq \beta \sum_{i=1}^{n} t_i \wedge \mathrm{dp}[t] \leq \alpha \sum_{i=1}^{n} w_i$;

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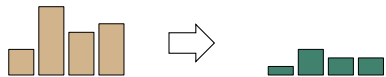
Swiper [TF23] overview

Linear scaling:

- assignment $(t_1, ..., t_n) = (w_1, ..., w_n)$ works, $(t_1, ..., t_n) = 0^n$ doesn't
- set in-between: $t_i = \lfloor s w_i + c \rfloor$ $(c = \mathrm{const})$
- run binary search to find locally minimal s that generates a valid ticket assignment
  - validity testing described in previous slide



Equivalently:

- define potential solutions $\vec{t_1}, \vec{t_2}, ...$ with $\mathrm{size}(\vec{t_j}) = j$ tickets
- find locally minimal j such that $\vec{t_j}$ is valid but $\vec{t_{j-1}}$ is not

Swiper paper proves that when $c = \alpha$, all $\vec{t_M}, \vec{t_{M+1}}, ...$ are valid, where

$$M = \left\lfloor \frac{\alpha(1-\alpha)}{\beta - \alpha} n + 1 \right\rfloor = O(n).$$

Running time analysis:

- each search iteration: validity testing in time $O\left(n \cdot \sum_{i=1}^{n} t_i\right) \leq O(n \cdot M) = O(n^2)$
- total: $O(n^2 \log n)$

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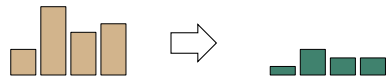

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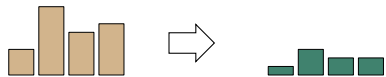

Equivalently:

- define potential solutions $\vec{t_1}, \vec{t_2}, ...$ with $\text{size}(\vec{t_j}) = j$ tickets
- find locally minimal j such that $\vec{t_j}$ is valid but $\vec{t_{j-1}}$ is not

Swiper paper proves that when $c = \alpha$, all $\vec{t_M}, \vec{t_{M+1}}, ...$ are valid, where

$$M = \left\lfloor \frac{\alpha(1-\alpha)}{\beta - \alpha} n + 1 \right\rfloor = O(n).$$

Running time analysis:

- each search iteration: validity testing in time $O\left(n \cdot \sum_{i=1}^{n} t_i\right) \leq O\left(n \cdot M\right) = O\left(n^2\right)$
- total: $O\left(n^2 \log n\right)$

Swiper [TF23] overview

Linear scaling:

- assignment $(t_1, ..., t_n) = (w_1, ..., w_n)$ works, $(t_1, ..., t_n) = 0^n$ doesn't
- set in-between: $t_i = \lfloor s w_i + c \rfloor$ $(c = \text{const})$
- run binary search to find locally minimal s that generates a valid ticket assignment
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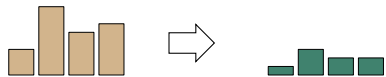

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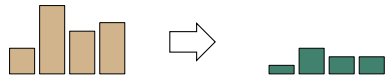
Coming next...

- ☑ Problem statement
- ☑ Swiper overview
- ☐ **Contribution 1: improving Swiper**
- ☐
- ☐
- ☐
- ☐

Improving Swiper: *super Swiper*



- Replaces binary search with linear search
  - $j \leftarrow 1$;
    while $\overrightarrow{t_j}$ is not valid **do**
    $\quad j \leftarrow j + 1$;
    return $\overrightarrow{t_j}$;
  - binary search can get stuck in a "local minimum"
  - linear search improves output
- reduces running time from $O(n^2 \log n)$ to $O(n + R^2 \log R)$, where $R \leq O(n)$ is the number of tickets $\sum_{i=1}^{n} t_i$ in the output
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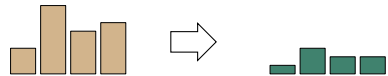

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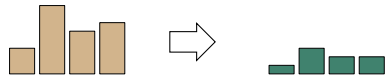

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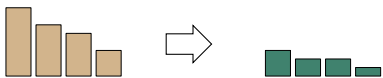

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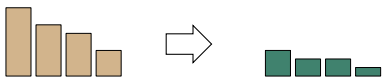

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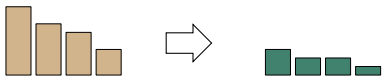

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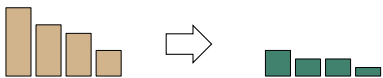

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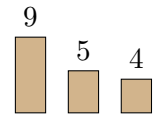

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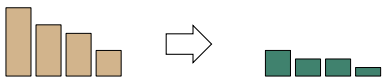

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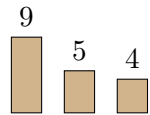

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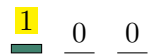

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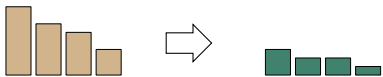

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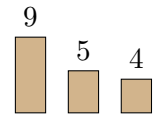

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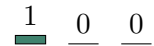

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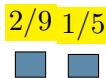

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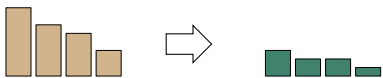

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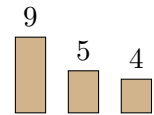

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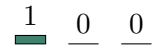

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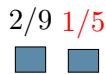

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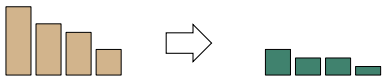

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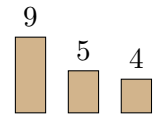

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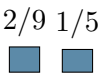

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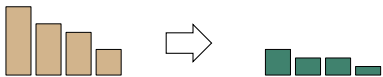

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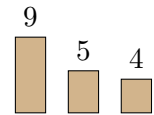

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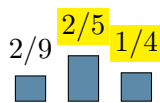

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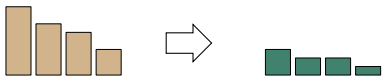

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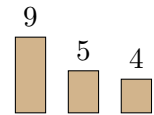

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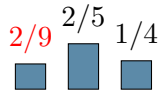

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$$\vec{t} = \lfloor s\vec{w} + c \rfloor$$

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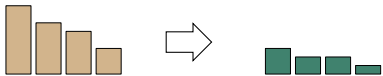

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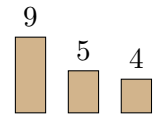

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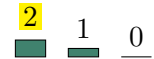

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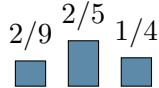

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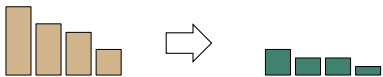

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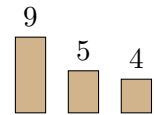

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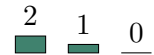

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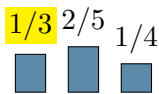

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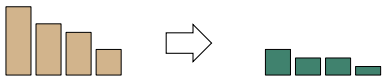

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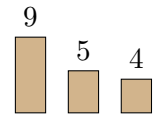

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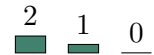

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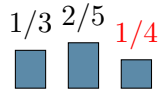

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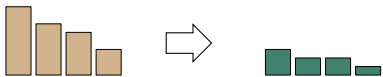

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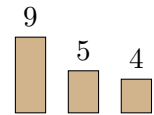

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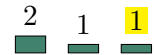

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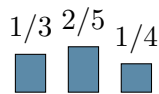

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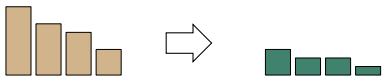

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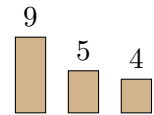

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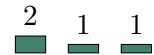

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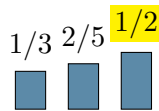

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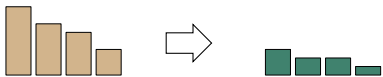

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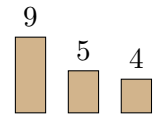

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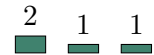

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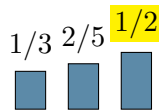

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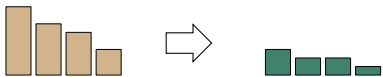

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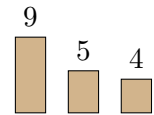

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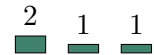

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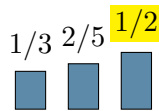

Improving Swiper: *super Swiper* (cont.)

$j \leftarrow 1$;
while $\vec{t_j}$ is not valid **do**
  $j \leftarrow j + 1$;
return $\vec{t_j}$;

- Challenge 1: compute $\vec{t_j}$ – time $O(R \log R)$
  - $O(n + R \log R)$ if $\vec{w}$ is unordered
- Challenge 2: test $\vec{t_j}$ – ?

Testing $\vec{t_j}$

- Reuse dynamic programming computations for testing ticket assignments
- generalized data structure DP:

method	mutable?	parameters	output
.apply(w, t)	mutable	party's weight w_i, # tickets t_i	none
.get()	immutable	none	integer

- guarantee: want to test $\vec{t}$
  - call DP.apply$\left(w_i, (\vec{t})_i\right)$ once for all $i \in [n]$ with $(\vec{t})_i \neq 0$ in any order
  - then DP.get() returns max # tickets the adversary gets in $\vec{t}$
- example: want to test $\vec{t} = (4, 3, 3, 0)$ for $\vec{w} = (90, 60, 50, 10)$

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  `dp` $=$ $\boxed{\text{– – – –}}$

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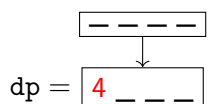

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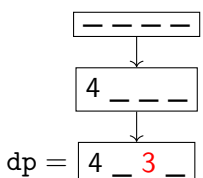

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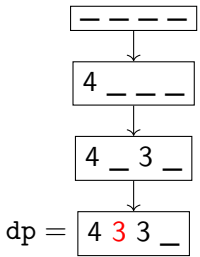

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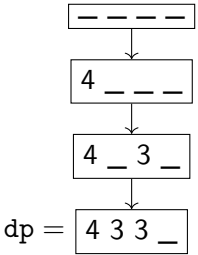

dp.apply$(90, 4)$

dp.apply$(50, 3)$

dp.apply$(60, 3)$

dp.get()

Testing $\vec{t_j}$ (cont.)

- To implement DP:

  for $k \in [n]$ **do**

  for $t \in \{0, 1, ..., T\}$ **do**

  apply (compute min weight the adversary has to corrupt from $[k]$ to get t tickets total:

  $$\mathrm{dp}[t] = \min_{S} \sum_{i \in S} w_i \text{ subject to } S \subseteq [k] \text{ and } \sum_{i \in S} t_i = t$$

  get* (**return** $\neg\exists t$ s.t. $t \geq \beta \sum_{i=1}^{n} t_i \wedge \mathrm{dp}[t] \leq \alpha \sum_{i=1}^{n} w_i$;

- time complexity
  - if $\sum_{i=1}^{n} (\vec{t})_i = T$, then DP.apply$\left(w_i, (\vec{t})_i\right)$ takes time $O(T)$
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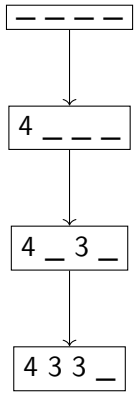

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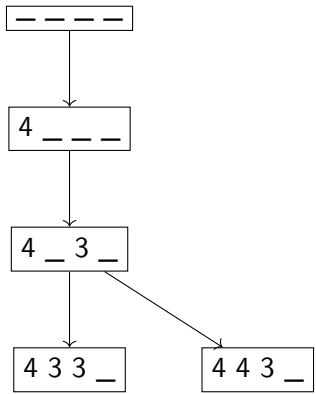

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More interesting example: want to test $(2,1,1,1)$, $(2,2,1,1)$, $(2,2,2,1)$, $(2,2,2,2)$

$4 \cdot (\log 4 + 1) = 12$ applies. Generalize: can test Swiper's $\vec{t_1}, \vec{t_2}, ..., \overrightarrow{t_{O(R)}}$ with $O(R \log R)$ applies; each apply takes time $O(R)$.

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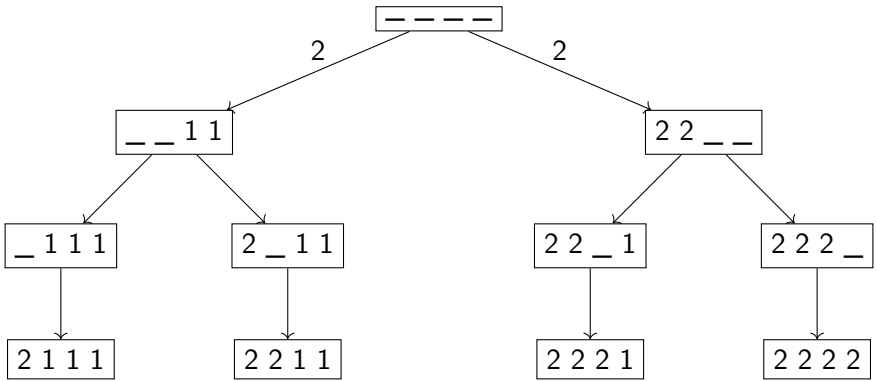

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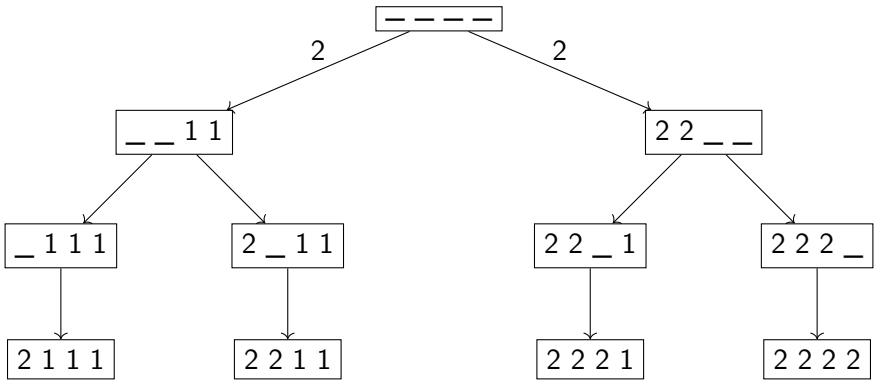

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Improving Swiper: *super Swiper* (cont.)

$j \leftarrow 1$;
while $\overrightarrow{t_j}$ is not valid **do**
  $j \leftarrow j + 1$;
return $\overrightarrow{t_j}$;

- Challenge 1: compute $\overrightarrow{t_j}$ – time $O(n + R \log R)$
- Challenge 2: test $\overrightarrow{t_j}$ – time $O(R^2 \log R)$

Total: time $O(n + R^2 \log R)$

Improving Swiper: *super Swiper* (cont.)

- linear search outputs fewer tickets
- linear search faster than binary search

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super Swiper	232	745	22384	201	383	1171	17581
	81.5µs	485µs	129ms	88.0µs	215µs	1.54ms	140ms

Table: Evaluation on the Algorand stake distribution

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- ☑ Problem statement
- ☑ Swiper overview
- ☑ Contribution 1: *super Swiper* – linear search in time $O\left(n + R^2 \log R\right)$
- ☐ **Contribution 2: lower bounds**
- ☐
- ☐
- ☐

Lower bounds

- *super Swiper* works great in practice, what about in theory?
- want to know worst case approximation factor
  - how big is output compared to optimal solution?
- our result: *super Swiper*, Swiper [TF23] + others [BHS23, FMT24, DPTX25] all have approximation factor $\Omega(n)$
  - constructed an example where output is $\Omega(n)$ but $\mathrm{OPT} = O(1)$

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- ☐ **Contribution 3: algorithms with better guarantees**
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Algorithms with better theoretical guarantees

- Would like algorithms with good approximation factor; made some progress
- useful fact: assuming $w_1 \geq w_2 \geq ... \geq w_n$,
  $\exists$ an optimal solution with $t_1 \geq t_2 \geq ... \geq t_n$
  - take an optimal solution $(t_1, ..., t_i, ...t_j, ..., t_n)$ with $t_i < t_j$
  - swap t_i and t_j
- bruteforcing sorted ticket assignment gives exact solution (approximation factor 1), takes time

$$O\left(n + R^{3/2} e^{C\sqrt{R}}\right)$$

  - $R \leq O(n)$ – size of optimal solution
  - $C = \pi\sqrt{6}/3 \approx 2.57$
  - practical when $R < 100$
- corollary: polytime algorithm with approximation factor $O\left(n/\log^2 n\right)$

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Algorithm based on linear programming

- Weight reduction can be formulated as an integer linear program

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n} t_i \\ \text{subject to} \quad & \sum_{i \in A} t_i < \beta \sum_{i=1}^{n} t_i \quad \forall A \subseteq [n] \text{ s.t. } \sum_{i \in A} w_i \leq \alpha \sum_{i=1}^{n} w_i \\ & t_i \in \mathbb{Z}_{\geq 0} \qquad\qquad \forall i \in [n] \end{aligned}$$

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Algorithm based on linear programming (cont.)

- Let $\mathrm{OPT}_{\alpha,\beta}$ denote the smallest number of tickets
- solution of size $\mathrm{OPT}_{\alpha,(1-\delta)\beta}$ in polynomial time for any constant $\delta > 0$
  - example: security $\alpha = \frac{1}{5} \to \beta = \frac{1}{3}$, size optimal for $\alpha = \frac{1}{5} \to (1-\delta)\beta = \frac{3}{10}$
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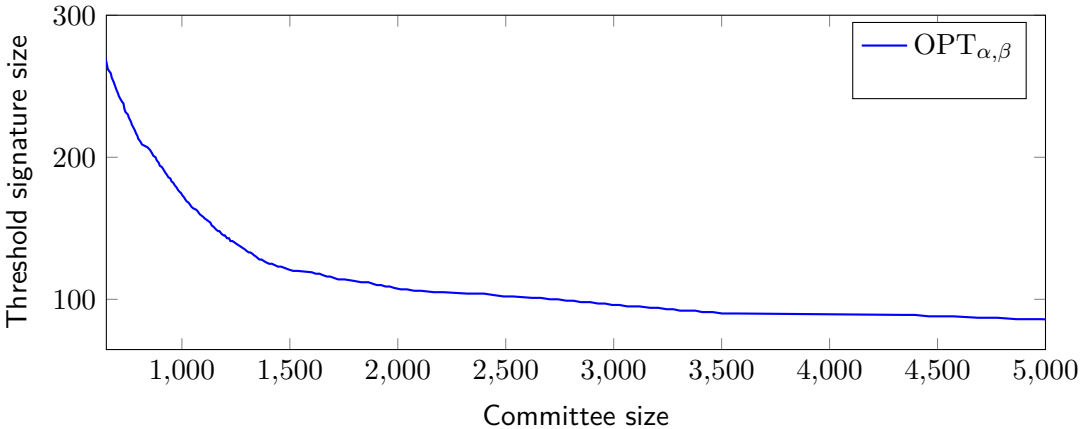
- ex.: Approximate Lower Bound Arguments [CKRZ24] yields decentralized threshold signatures



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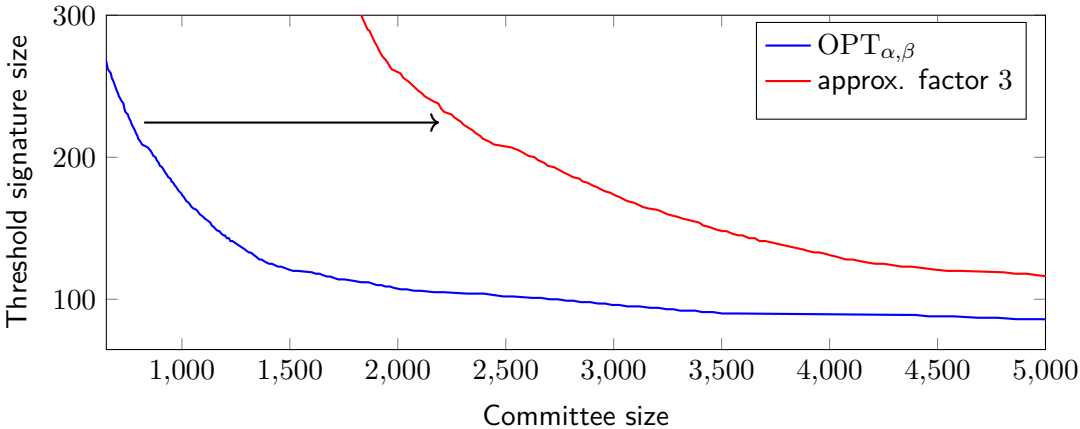

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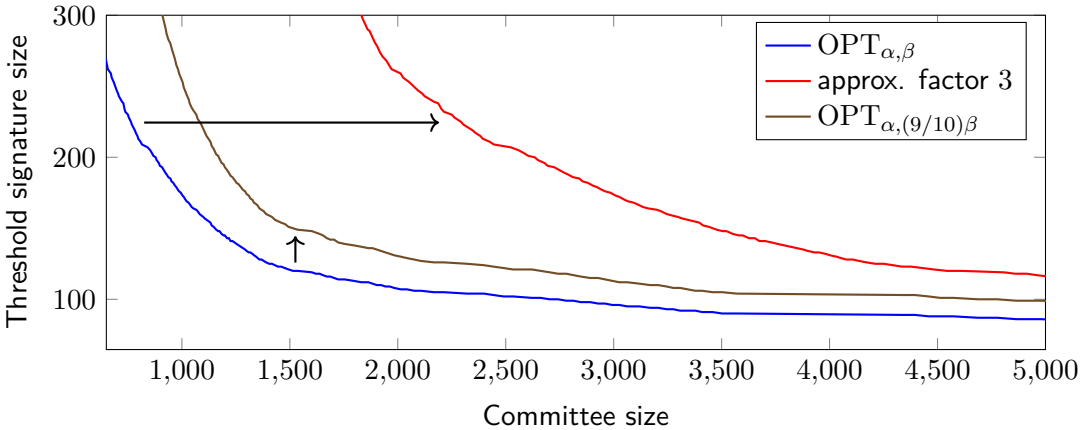

Algorithm based on linear programming (cont.)

- LP algorithm is impractical
- time complexity $\Omega\left(n^{10}\right)$ for reasonable parameters

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- sub-exponential time exact algorithm can be practical!
- *super Swiper* is almost optimal in practice

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Table: Evaluation on the Aptos stake distribution

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  - exact algorithm in time $O(n) + 2^{O(\sqrt{R})}$ – sometimes practical
- more theoretical work to be done
  - NP-hard?
  - better approximation factor in polytime
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- thanks go to:
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Weight reduction problem:
given $w_1, ..., w_n \in \mathbb{Z}_{\geq 0}$, find $t_1, ..., t_n \in \mathbb{Z}_{\geq 0}$ such that
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Additional slides

Super Swiper

- test $\vec{t_1}, \vec{t_2}, \vec{t_3}, \ldots$ in batches
- batch $k \geq 0$ tests $\vec{t_j}$ for $j \in \left(j_k, j_{k+1}\right]$
- $j_0 = 0$, $j_{k+1} = j_k + 2^k$ (batches grow)
- batch k takes time $O\left(k \cdot 2^{2k}\right)$
- total time dominated by last batch: $O(R^2 \log R)$

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Super Swiper(cont.)

Batch algorithm

- goal: test $\overrightarrow{t_j}$ for $j \in (l, r]$, $l = 2^k - 1$, $r = 2^{k+1} - 1$, in time $O(k \cdot 2^{2k})$
- create array `deltas`: indices to increment (in $\overrightarrow{t_l}$) that generate $\overrightarrow{t_{l+1}}, \overrightarrow{t_{l+2}}, ..., \overrightarrow{t_r}$
  - $|\texttt{deltas}| = 2^k$
- create `dp_head` : DP and call $\texttt{dp\_head.apply}(w_i, (\overrightarrow{t_l})_i)$ for $i \notin \texttt{deltas}$
  - takes time $O(2^{2k})$
- observation: could test $\overrightarrow{t_{l+1}}, ..., \overrightarrow{t_r}$ and save $\approx 50\%$ of CPU cycles
  - for each $\overrightarrow{t_j}$ only apply indices $i \in \texttt{deltas}$
- invoke $\textsc{ProcessBatchRecursive}(\overrightarrow{t_l}, \texttt{deltas}, \texttt{dp\_head})$
  - takes time $O(k \cdot 2^{2k})$

Super Swiper(cont.)

Batch algorithm

- goal: test $\vec{t_j}$ for $j \in (l, r]$, $l = 2^k - 1$, $r = 2^{k+1} - 1$, in time $O(k \cdot 2^{2k})$
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Super Swiper(cont.)

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Super Swiper(cont.)

Divide and conquer

Base case. If $|\texttt{deltas}| = 1$:

- call $\texttt{dp\_head.apply}\left(w_i, (\overrightarrow{t_{j+1}})_i\right)$ where $i \in \texttt{deltas}$
- check $\texttt{dp\_head.get}()$

ProcessBatchRecursive($\overrightarrow{t_j}$, deltas, dp_head)

- Input:
  - $\overrightarrow{t_j}$: a ticket assignment
  - deltas: an array of indices that generate $\overrightarrow{t_{j+1}}, \overrightarrow{t_{j+2}}, ..., \overrightarrow{t_{j+|\texttt{deltas}|}}$
  - dp_head: DP data structure where $\left(w_i, \left(\overrightarrow{t_j}\right)_i\right)$ are applied exactly for $i \notin \texttt{deltas}$
- Output:
  - the smallest valid ticket assignment $\overrightarrow{t_{j'}}$ where $j < j' \leq j + |\texttt{deltas}|$, or $\perp$ if no such solution exists

Super Swiper(cont.)

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- Input:
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- Output:
  - the smallest valid ticket assignment $\overrightarrow{t_{j'}}$ where $j < j' \leq j + |\texttt{deltas}|$, or $\perp$ if no such solution exists

Super Swiper(cont.)

Recursive case. If $|\texttt{deltas}| > 1$:

- split deltas into two halves
  - deltas_l, deltas_r
- to test left half:
  - dp_l $\leftarrow$ dp_head
  - call dp_l.apply$\left(w_i, \left(\overrightarrow{t_j}\right)_i\right)$ for $i \in$ deltas_r $\setminus$ deltas_l
  - call ProcessBatchRecursive$(\overrightarrow{t_j}$, deltas_l, dp_l$)$
- to test right half:
  - $m \leftarrow j + |\texttt{deltas\_l}|$
  - dp_r $\leftarrow$ dp_head
  - call dp_r.apply$\left(w_i, \left(\overrightarrow{t_m}\right)_i\right)$ for $i \in$ deltas_l $\setminus$ deltas_r
  - call ProcessBatchRecursive$(\overrightarrow{t_m}$, deltas_r, dp_r$)$

ProcessBatchRecursive$(\overrightarrow{t_j}$, deltas, dp_head$)$

- Input:
  - $\overrightarrow{t_j}$: a ticket assignment
  - deltas: an array of indices that generate $\overrightarrow{t_{j+1}}, \overrightarrow{t_{j+2}}, ..., \overrightarrow{t_{j+|\texttt{deltas}|}}$
  - dp_head: DP data structure where $\left(w_i, \left(\overrightarrow{t_j}\right)_i\right)$ are applied exactly for $i \notin$ deltas
- Output:
  - the smallest valid ticket assignment $\overrightarrow{t_{j'}}$ where $j < j' \leq j + |\texttt{deltas}|$, or $\perp$ if no such solution exists

Super Swiper(cont.)

Recursive case. If $|\texttt{deltas}| > 1$:

- split `deltas` into two halves
  - `deltas_l`, `deltas_r`
- to test left half:
  - `dp_l` $\leftarrow$ `dp_head`
  - call `dp_l.apply`$\left(w_i, \left(\vec{t_j}\right)_i\right)$ for $i \in$ `deltas_r` $\setminus$ `deltas_l`
  - call ProcessBatchRecursive$(\vec{t_j}, \texttt{deltas_l}, \texttt{dp_l})$
- to test right half:
  - $m \leftarrow j + |\texttt{deltas\_l}|$
  - `dp_r` $\leftarrow$ `dp_head`
  - call `dp_r.apply`$\left(w_i, \left(\vec{t_m}\right)_i\right)$ for $i \in$ `deltas_l` $\setminus$ `deltas_r`
  - call ProcessBatchRecursive$(\vec{t_m}, \texttt{deltas_r}, \texttt{dp_r})$

ProcessBatchRecursive($\vec{t_j}$, `deltas`, `dp_head`)

- Input:
  - $\vec{t_j}$: a ticket assignment
  - `deltas`: an array of indices that generate $\overrightarrow{t_{j+1}}, \overrightarrow{t_{j+2}}, ..., \overrightarrow{t_{j+|\texttt{deltas}|}}$
  - `dp_head`: DP data structure where $\left(w_i, \left(\vec{t_j}\right)_i\right)$ are applied exactly for $i \notin$ `deltas`
- Output:
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Super Swiper(cont.)

Recursive case. If $|\texttt{deltas}| > 1$:

- split `deltas` into two halves
  - `deltas_l`, `deltas_r`
- to test left half:
  - $\texttt{dp\_l} \leftarrow \texttt{dp\_head}$
  - call $\texttt{dp\_l.apply}\left(w_i, \left(\vec{t_j}\right)_i\right)$ for $i \in \texttt{deltas\_r} \setminus \texttt{deltas\_l}$
  - call ProcessBatchRecursive($\vec{t_j}$, `deltas_l`, `dp_l`)
- to test right half:
  - $m \leftarrow j + |\texttt{deltas\_l}|$
  - $\texttt{dp\_r} \leftarrow \texttt{dp\_head}$
  - call $\texttt{dp\_r.apply}\left(w_i, \left(\vec{t_m}\right)_i\right)$ for $i \in \texttt{deltas\_l} \setminus \texttt{deltas\_r}$
  - call ProcessBatchRecursive($\vec{t_m}$, `deltas_r`, `dp_r`)

ProcessBatchRecursive($\vec{t_j}$, `deltas`, `dp_head`)

- Input:
  - $\vec{t_j}$: a ticket assignment
  - `deltas`: an array of indices that generate $\vec{t_{j+1}}, \vec{t_{j+2}}, \ldots, \vec{t_{j+|\texttt{deltas}|}}$
  - `dp_head`: DP data structure where $\left(w_i, \left(\vec{t_j}\right)_i\right)$ are applied exactly for $i \notin \texttt{deltas}$
- Output:
  - the smallest valid ticket assignment $\vec{t_{j'}}$ where $j < j' \leq j + |\texttt{deltas}|$, or $\perp$ if no such solution exists

Super Swiper(cont.)

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ProcessBatchRecursive($\vec{t_j}$, `deltas`, `dp_head`)

- Input:
  - $\vec{t_j}$: a ticket assignment
  - `deltas`: an array of indices that generate $\vec{t_{j+1}}, \vec{t_{j+2}}, \ldots, \vec{t_{j+|\texttt{deltas}|}}$
  - `dp_head`: DP data structure where $\left(w_i, \left(\vec{t_j}\right)_i\right)$ are applied exactly for $i \notin \texttt{deltas}$
- Output:
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Super Swiper(cont.)

Back to batch algorithm

- goal: test $\vec{t_j}$ for $j \in (l, r]$, $l = 2^k - 1$, $r = 2^{k+1} - 1$

Running time analysis:

- DP.apply always called on $\vec{t}$ with $\text{size}(\vec{t}) < 2^{k+1}$; takes time $O(2^k)$
- $m = |\texttt{deltas}|$: ProcessBatchRecursive$(\cdot, \texttt{deltas}, \cdot)$ calls DP.apply $\leq m$ times (at current stack level)
- $T(m)$ – total # calls to DP.apply
  - $T(1) = 1$
  - $T(m) \leq 2T(m/2) + m$
- solving: $T(2^k) = O(k \cdot 2^k)$; total time $O(k \cdot 2^{2k})$

ProcessBatchRecursive($\vec{t_j}$, deltas, dp_head)

- Input:
  - $\vec{t_j}$: a ticket assignment
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Super Swiper(cont.)

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