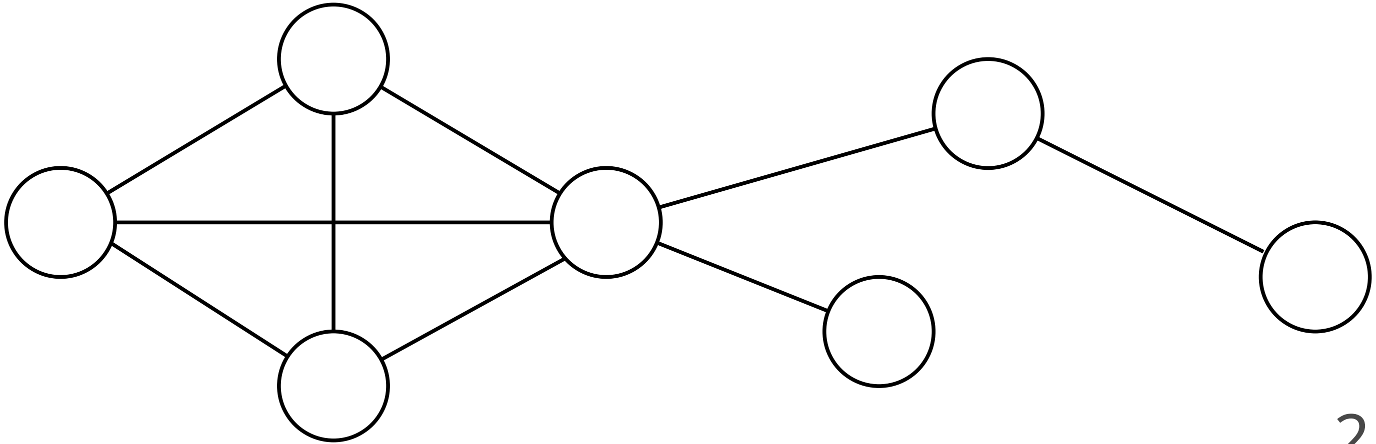
Space-stretch tradeoff in routing revisited

Tolik Zinovyev (Boston University)

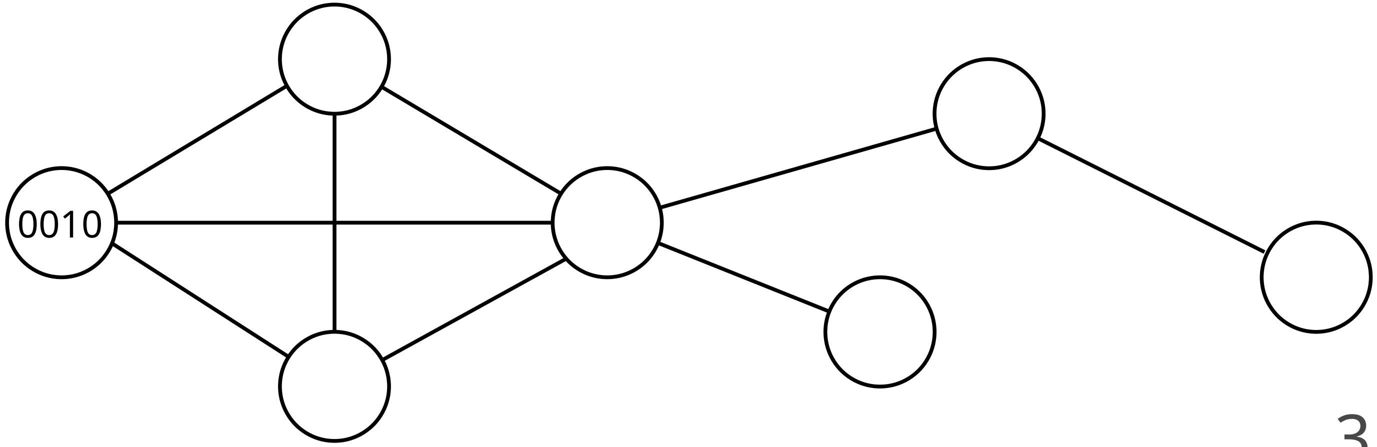
Routing in computer networks, model

- Undirected graph (network)



Routing in computer networks, model

- Undirected graph (network)
- Nodes have labels (a binary string)



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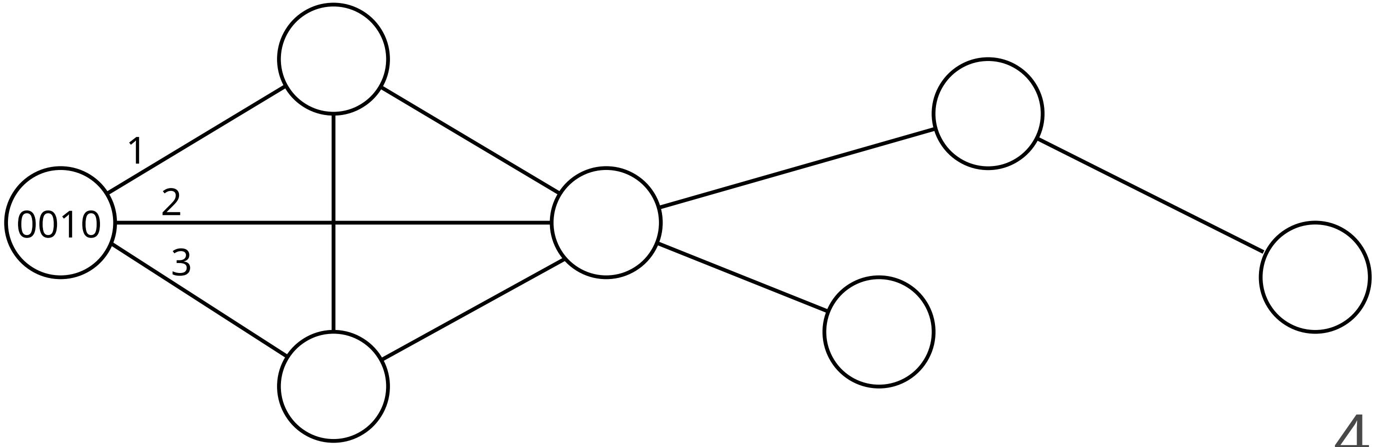
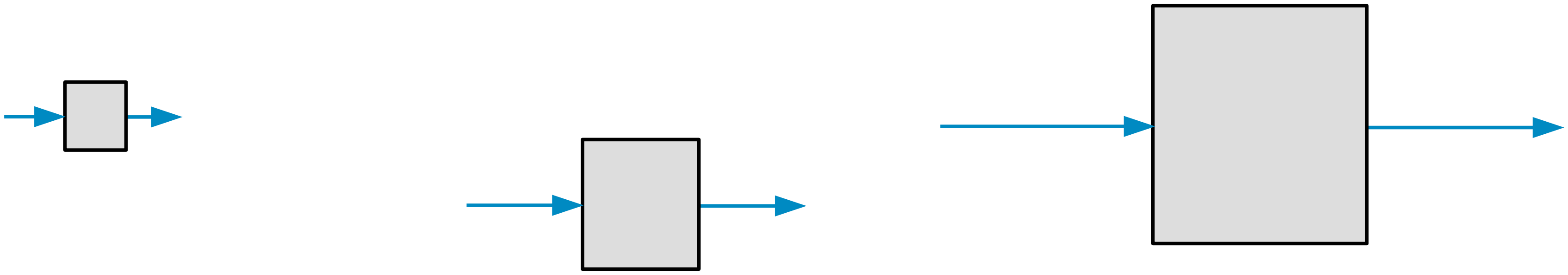


Routing in computer networks, model

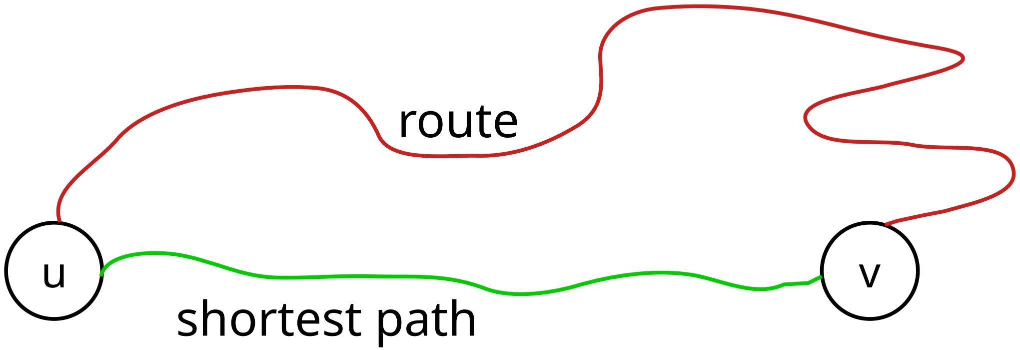
- Undirected graph (network)
- Nodes have labels (a binary string)
- Nodes have ports
- Nodes have routing programs

Routing characteristics

Space usage: program size

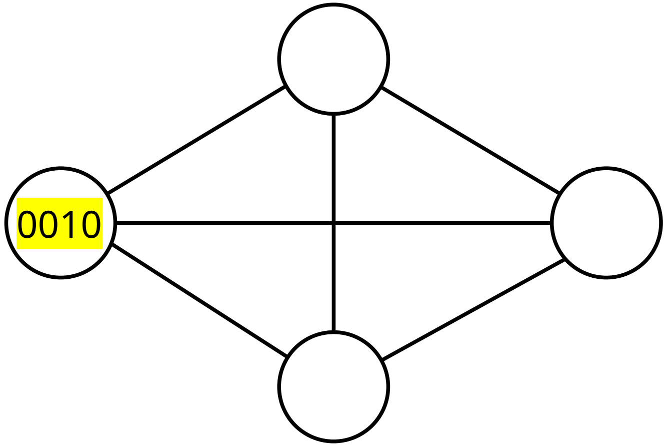


Routing stretch: route length / distance



Model, continued

- Adversarial / <u>non-adversarial</u> labels
  - Adversarial: labels given; aka name-independent model
  - Non-adversarial: designer labels; aka labeled model



Model, continued

- Adversarial / non-adversarial labels
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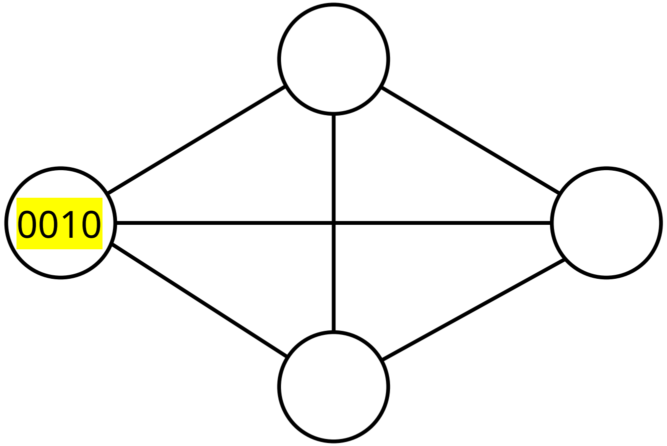

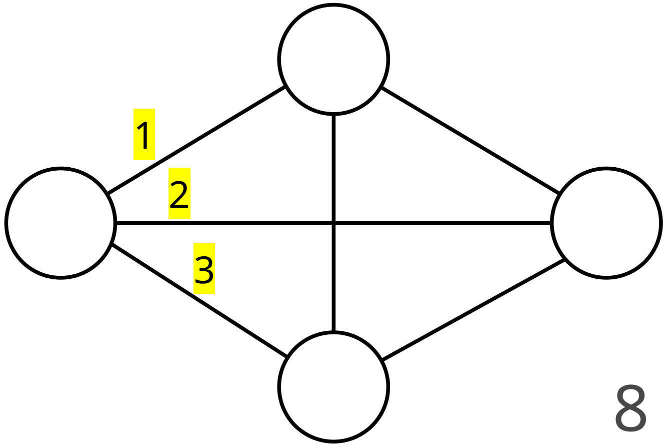

Lower bounds with non-adversarial ports

Work	Stretch	Local memory (bits)	Notes
Gavoille and Perennes (1996)	< 5/3	$\Omega(n \log n)$ on $\Omega(n)$ nodes	Node labels are [n]
Buhrman, Hoepman, Vitányi (1996)	1	$\Omega(n)$ on $\Omega(n)$ nodes	
Gavoille and Gengler (2001)	< 3	$\Omega(n)$ on some node	complex proof

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This paper	< 3	$\Omega(n)$ on cn nodes, $\forall 0<c<1$	

Lower bounds with adversarial ports

Work	Stretch	Local memory (bits)	Notes
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Thorup and Zwick (2001)	3 5 $< 2k+1$	$\Omega(n^{1/2})$ on some node $\Omega(n^{1/3})$ on some node $\Omega(n^{1/k})$ on some node	Does not work in standard model; relies on girth conjecture

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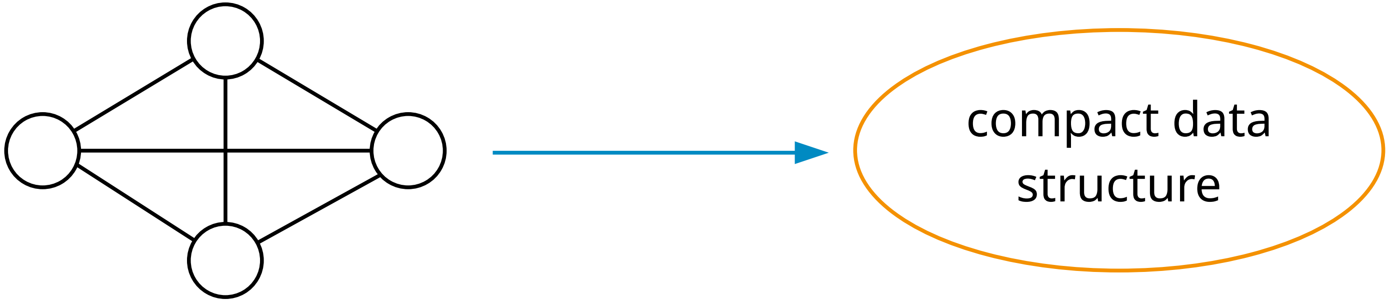
This paper	$< 2k+1$	$\Omega(n^{1/k} \log n)$ on some node	relies on girth conjecture

Previous proof does not work

- Mikkel Thorup's and Uri Zwick's proof relies on a reduction from approximate distance oracles

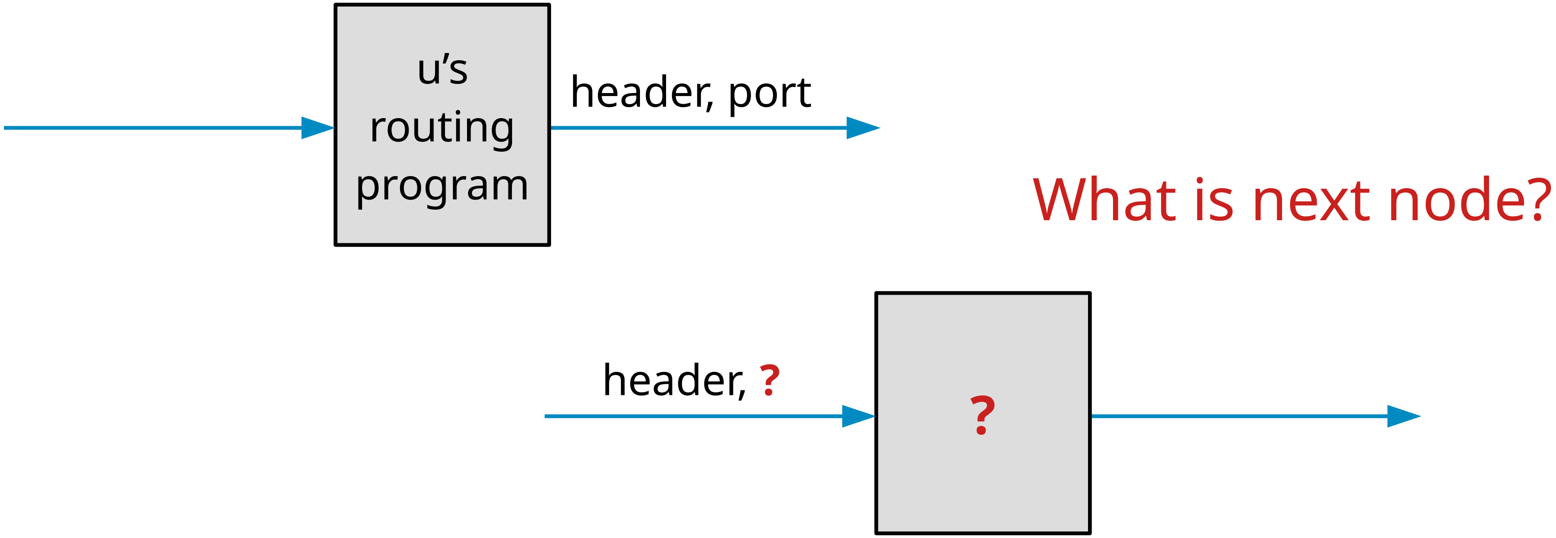


- Distance oracles with stretch <2k+1 require $\Omega(n^{1+1/k})$ bits of storage [1]

[1]: Mikkel Thorup and Uri Zwick. Approximate distance oracles. 2001

Previous proof does not work

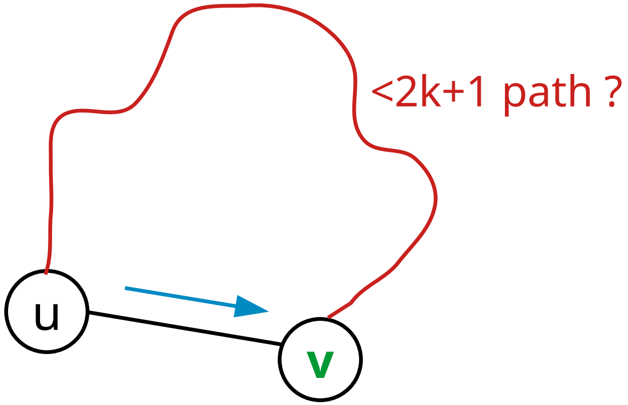
- Reduction in [2]:
  - Given a routing scheme with small size, construct small distance oracle
  - Distance oracle simulates routing and counts hops



[2]: Mikkel Thorup and Uri Zwick. Compact routing schemes. 2001

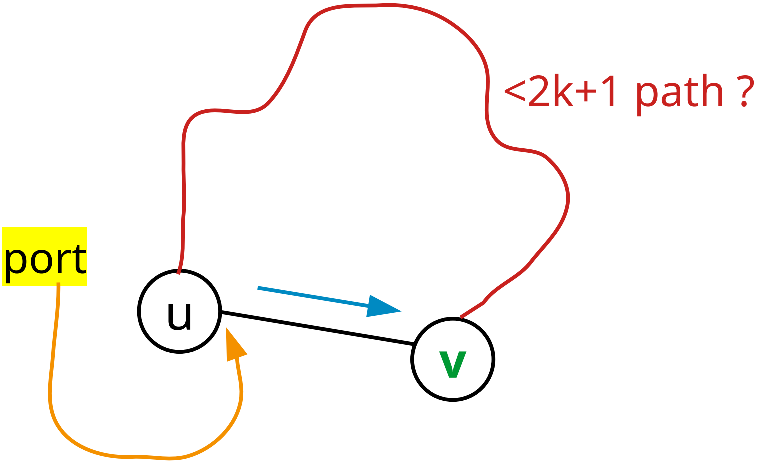
New proof

- Works in the standard model
- Borrows inspiration from Thorup's and Zwick's proof by using graphs with large girth
- Property: routing to a neighbor with stretch <2k+1 in graph with girth 2k+2 traverses edge toward neighbor



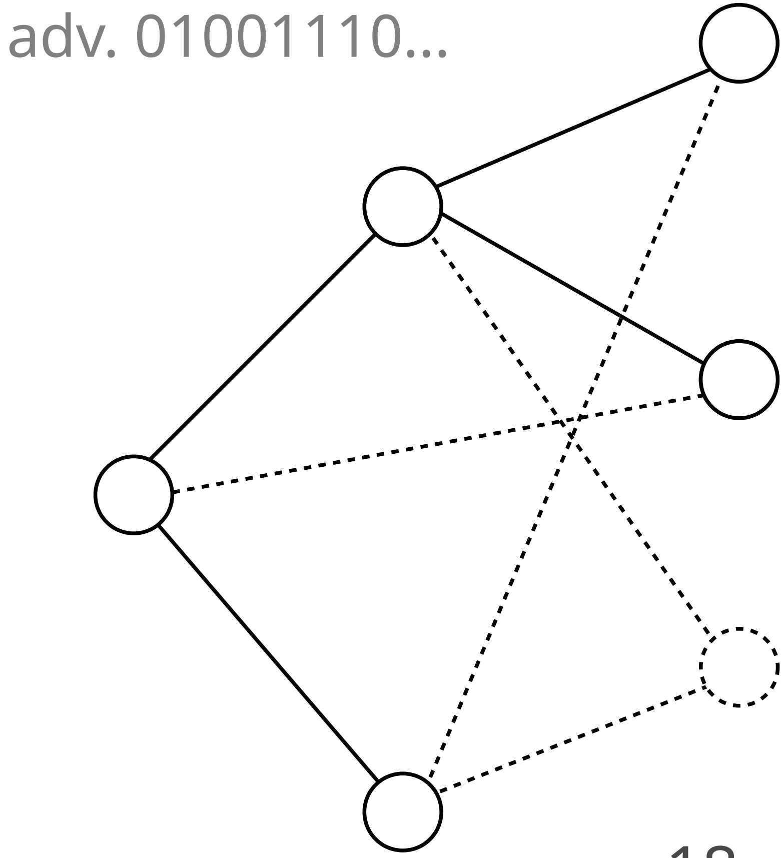
New proof

- Routing scheme must know port toward neighbor!
- Make extractor program
  - input: routing scheme, node labels, advice string
  - output: port assignment for the whole graph



Extractor program

- Start with unknown port assignment



Extractor program

- Start with unknown port assignment
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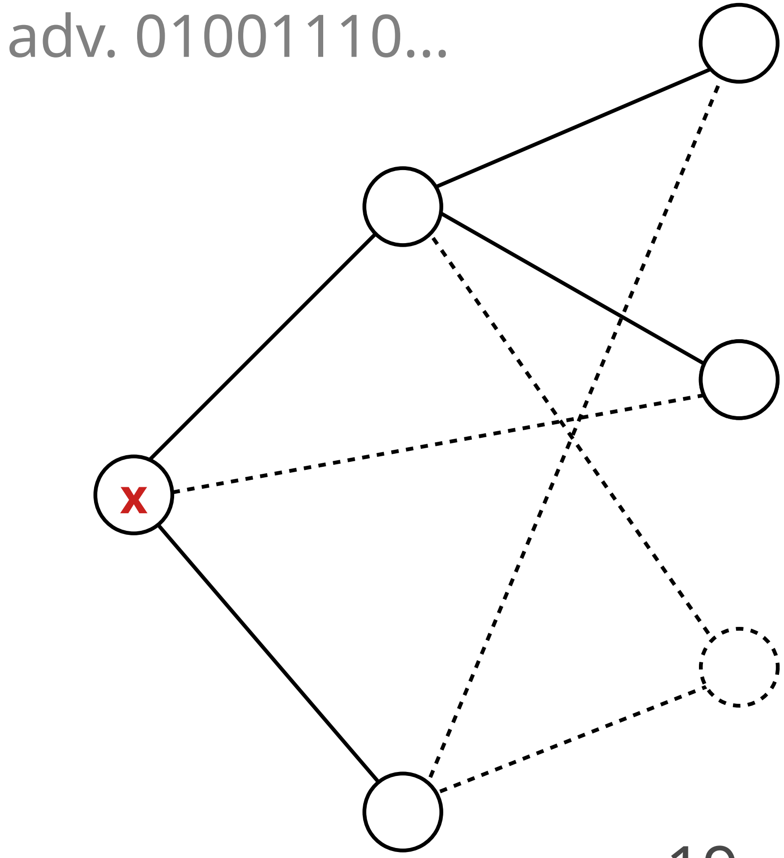

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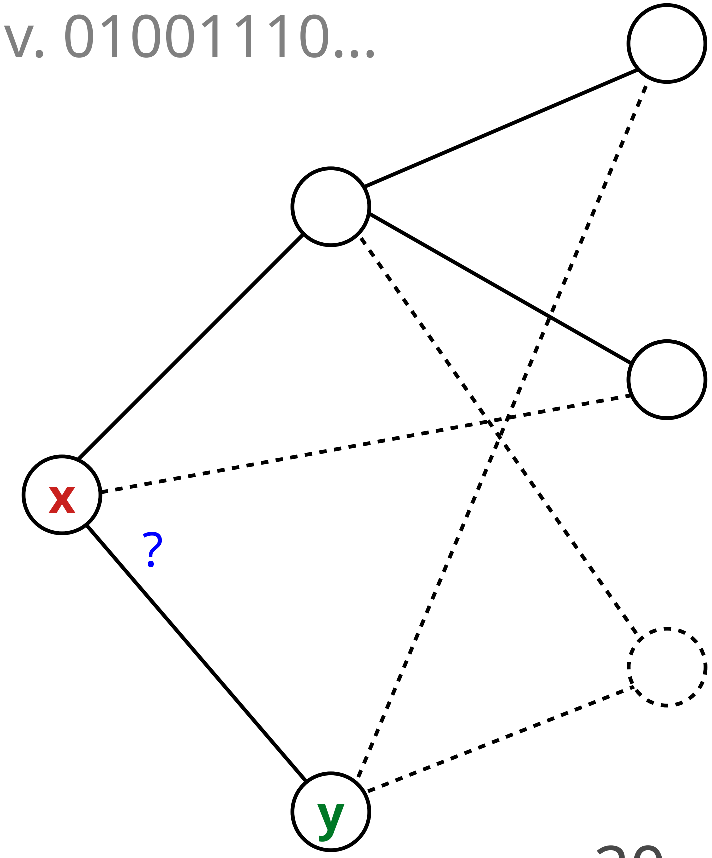
adv. 01001110...



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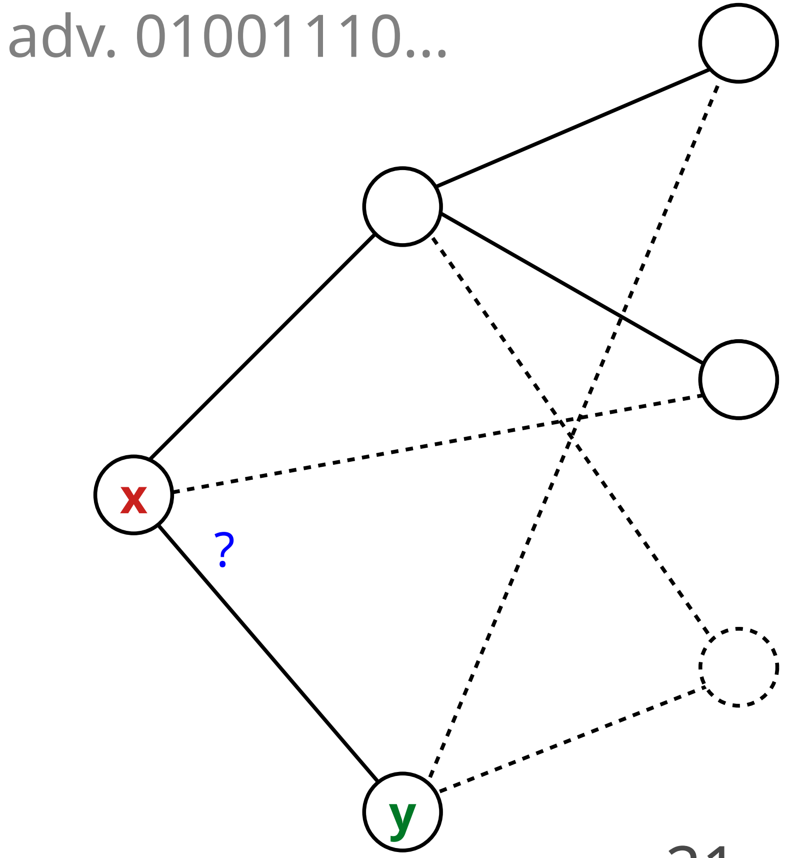

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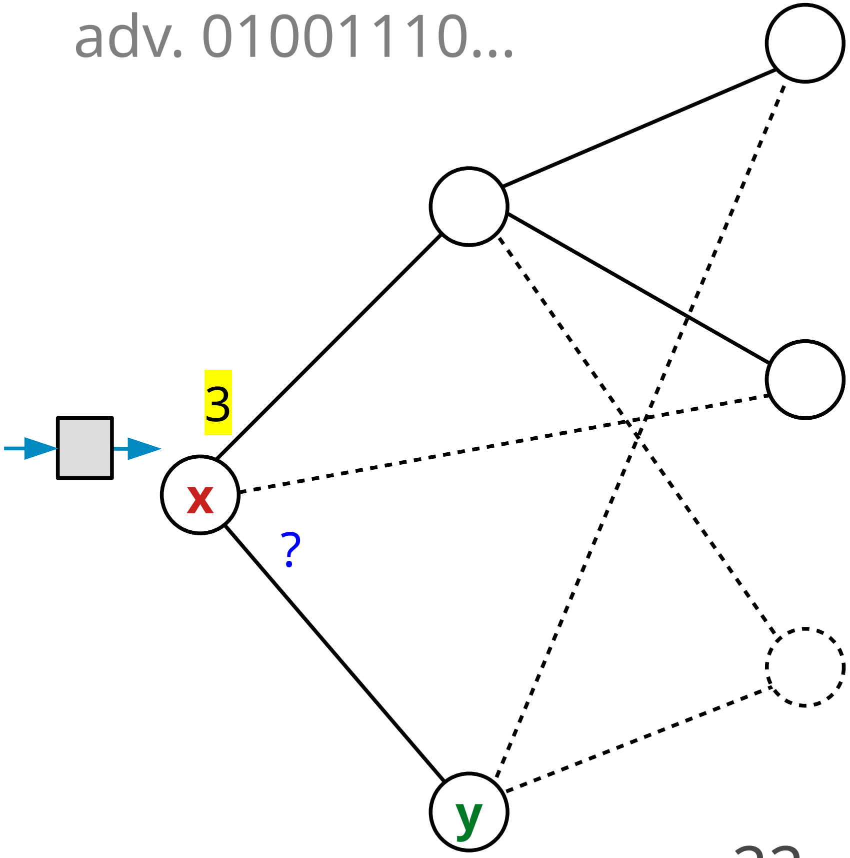

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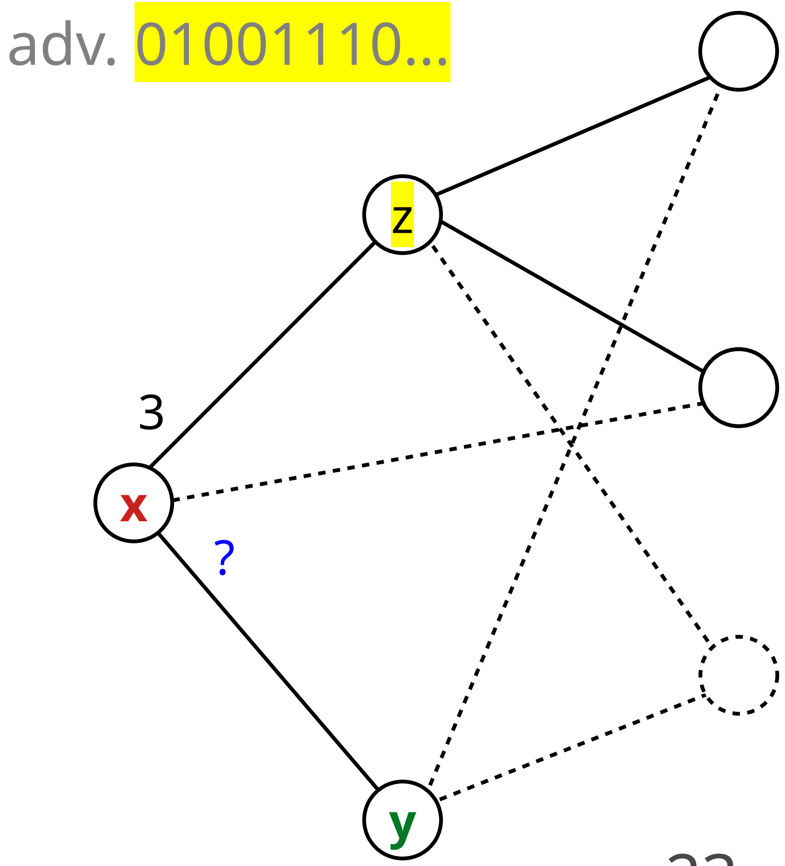

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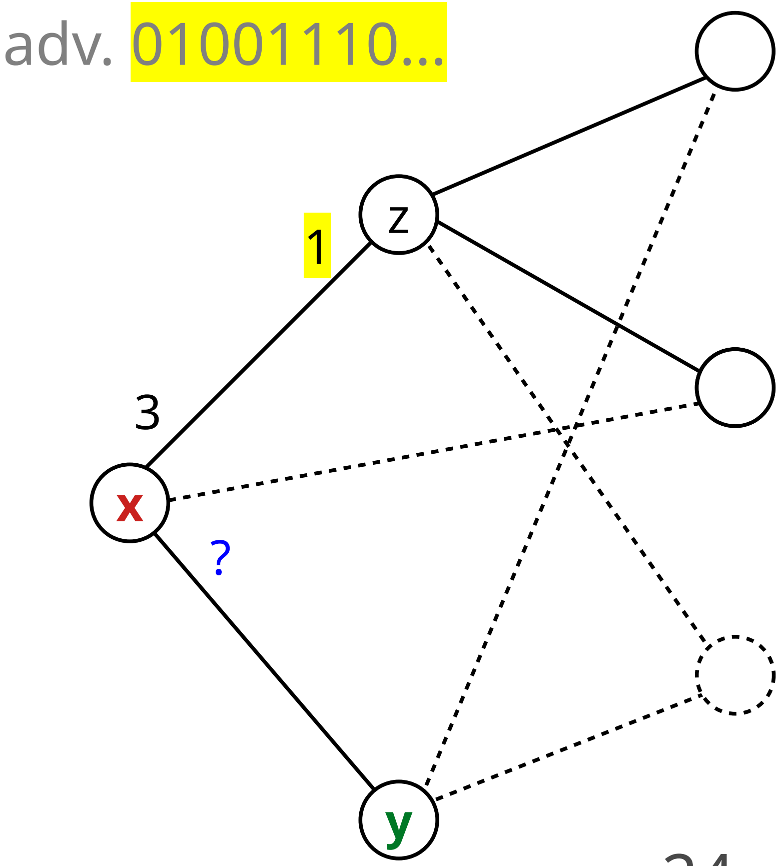

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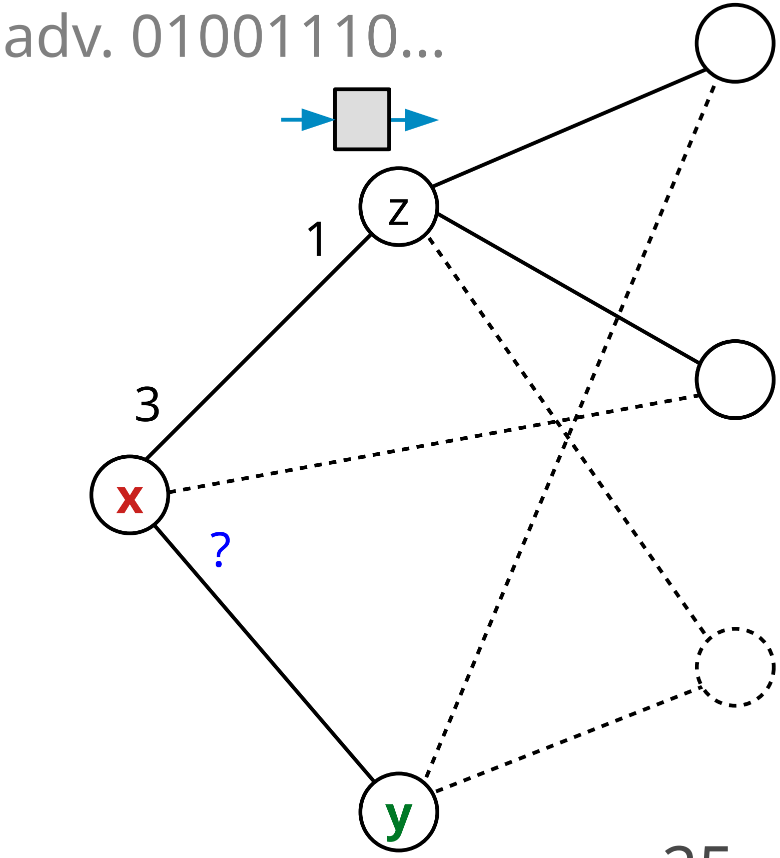

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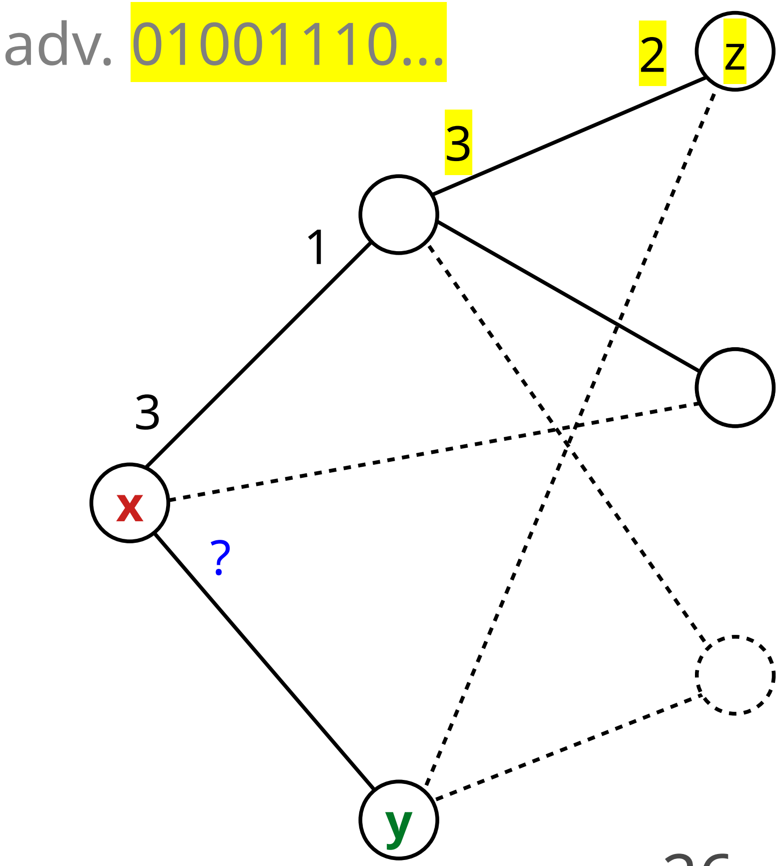

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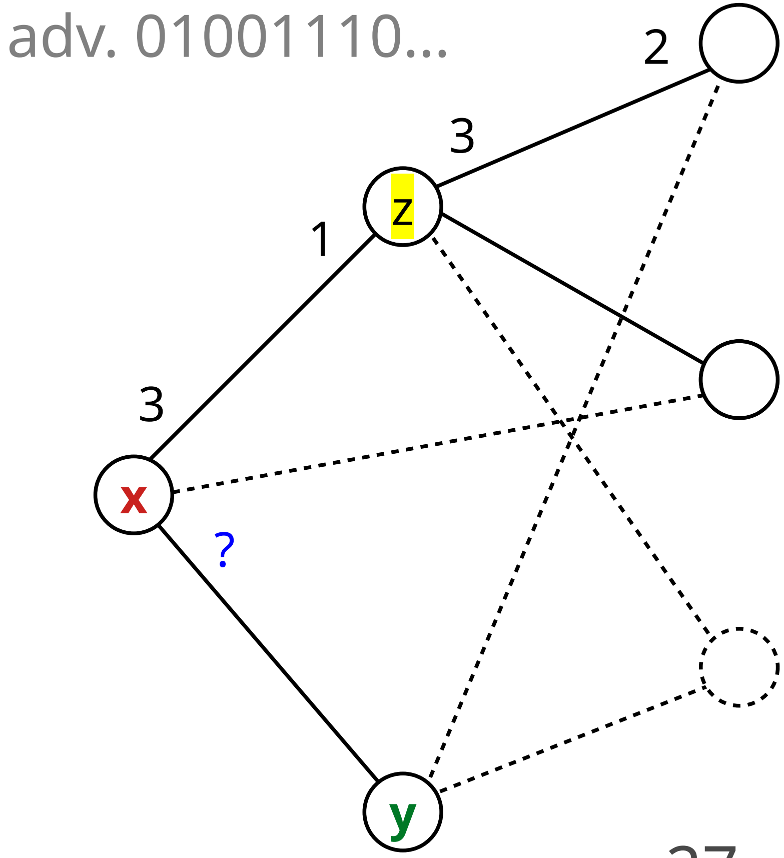

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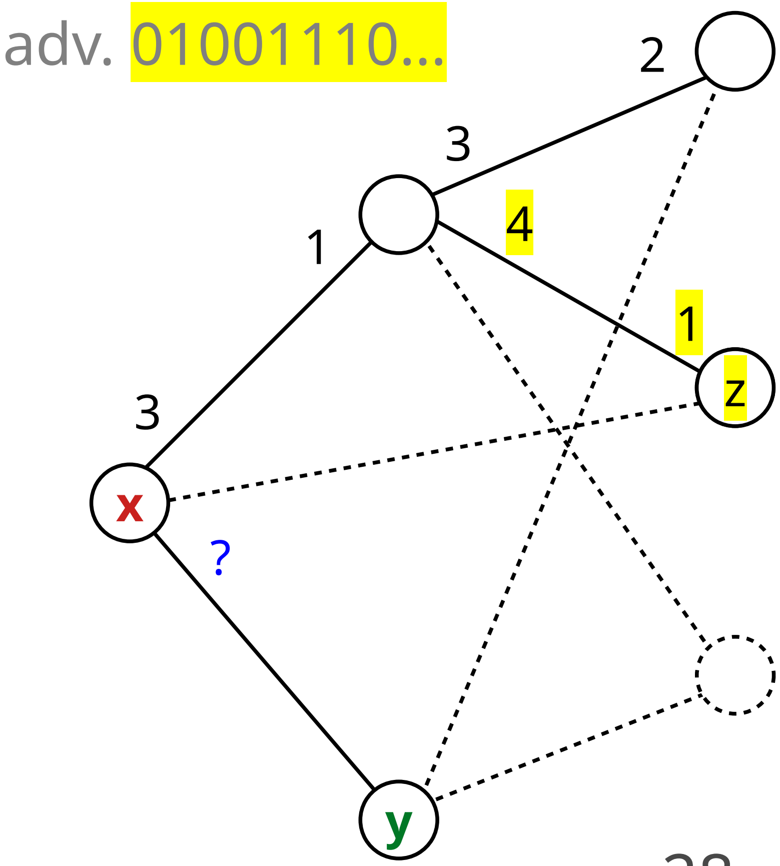

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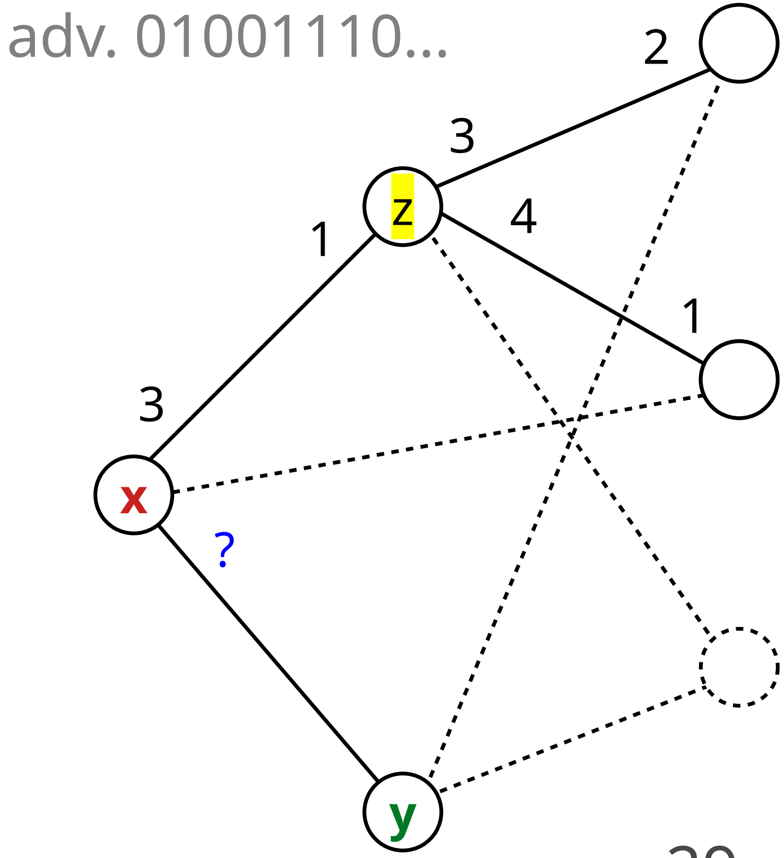

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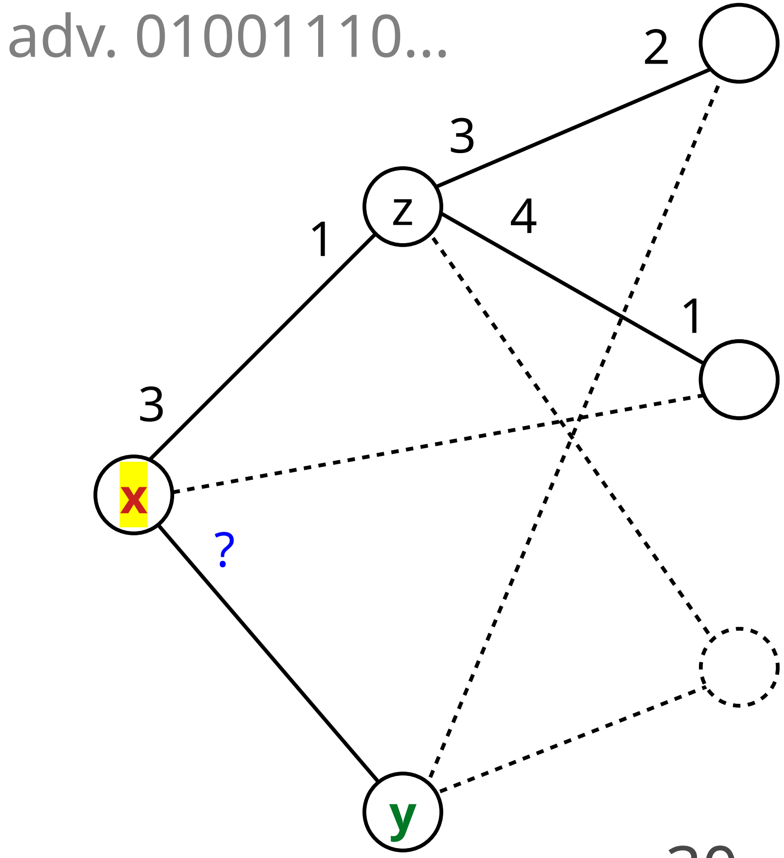

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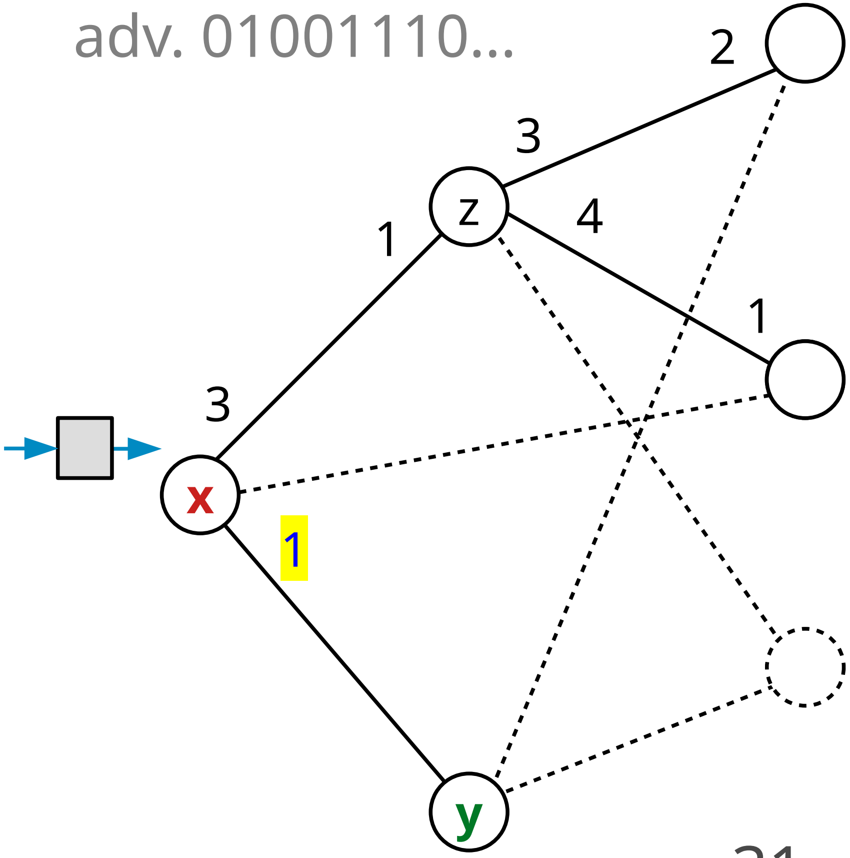

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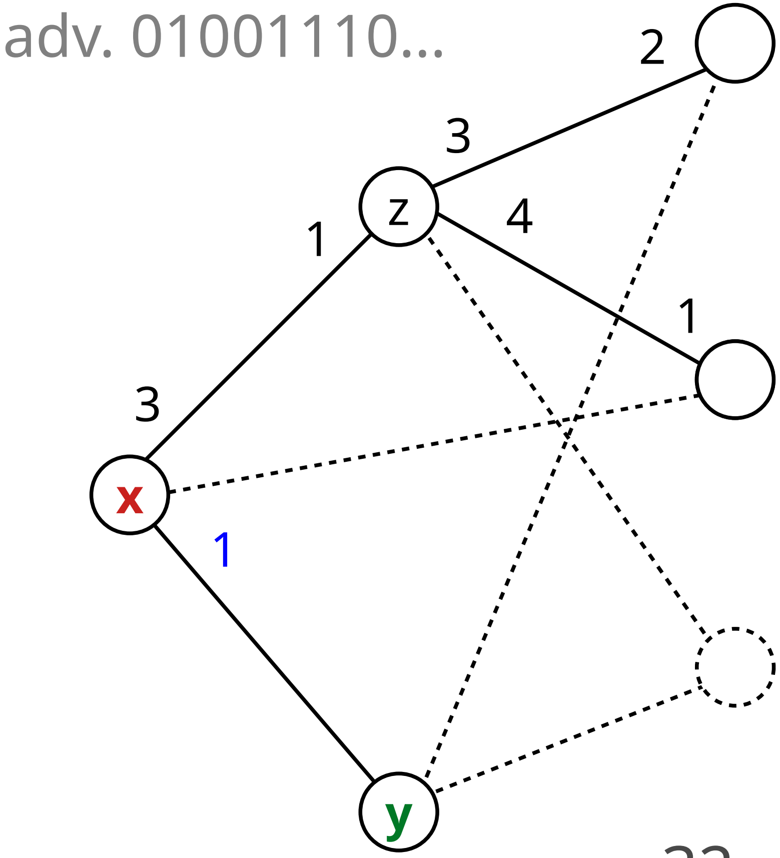
- 1 in 4k ports learned from routing scheme



Results

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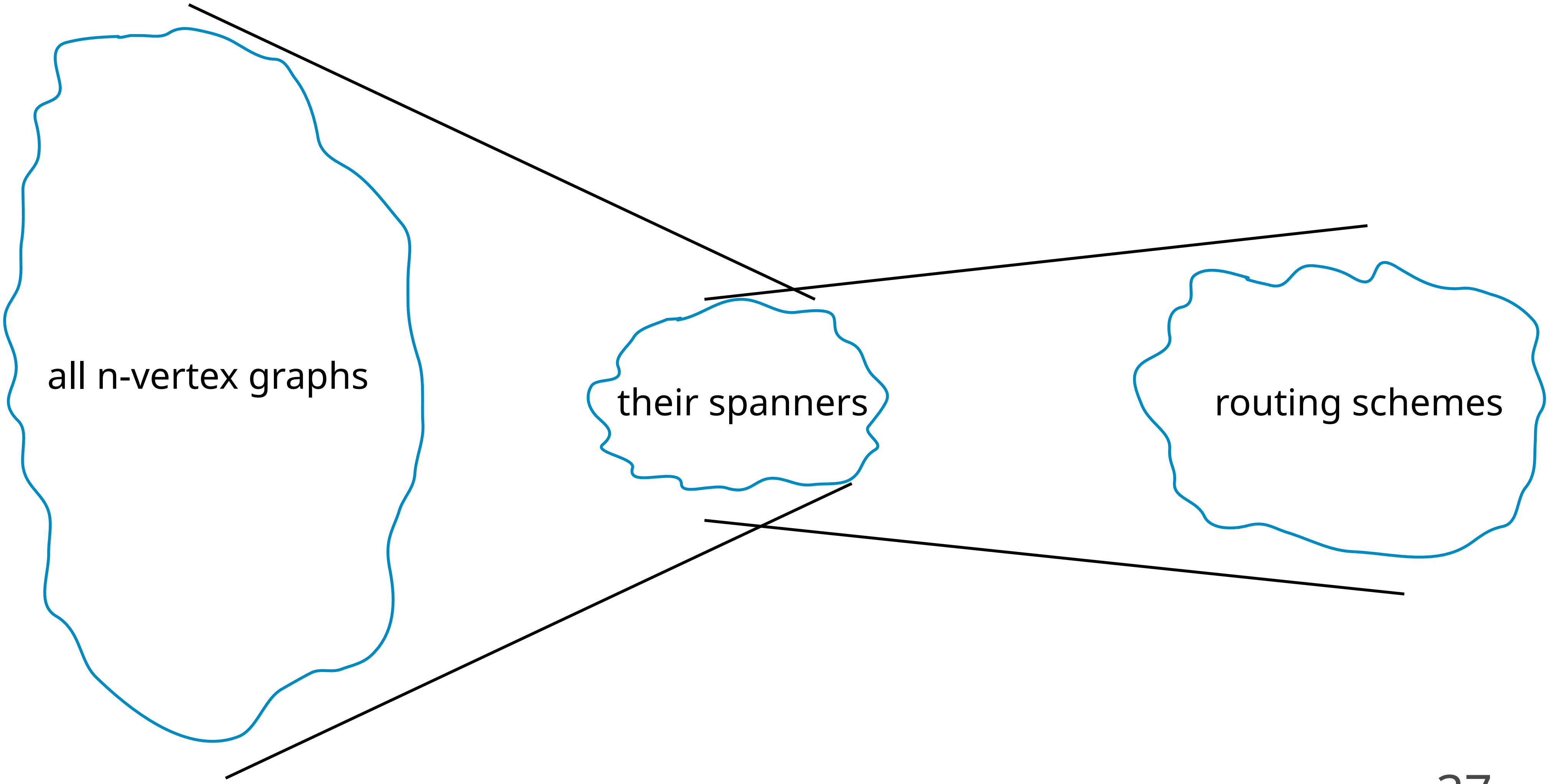
- Girth conjecture by Paul Erdős: there exists a graph with
  - n nodes
  - $\Omega(n^{1+1/k})$ edges
  - girth 2k+2
- Proven for k=1,2,3,5; weaker results for other k

Girth conjecture requirement

- All known approaches: prove that many routing schemes are necessary (including last proof)
- If $2^{\Omega(n^{(1+1/k)} \log n)}$ routing schemes are needed to satisfy all n-vertex graphs with stretch $<2k+1$, then there exists a graph with $\Omega(n^{1+1/k})$ edges and girth $2k+2$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their spanners
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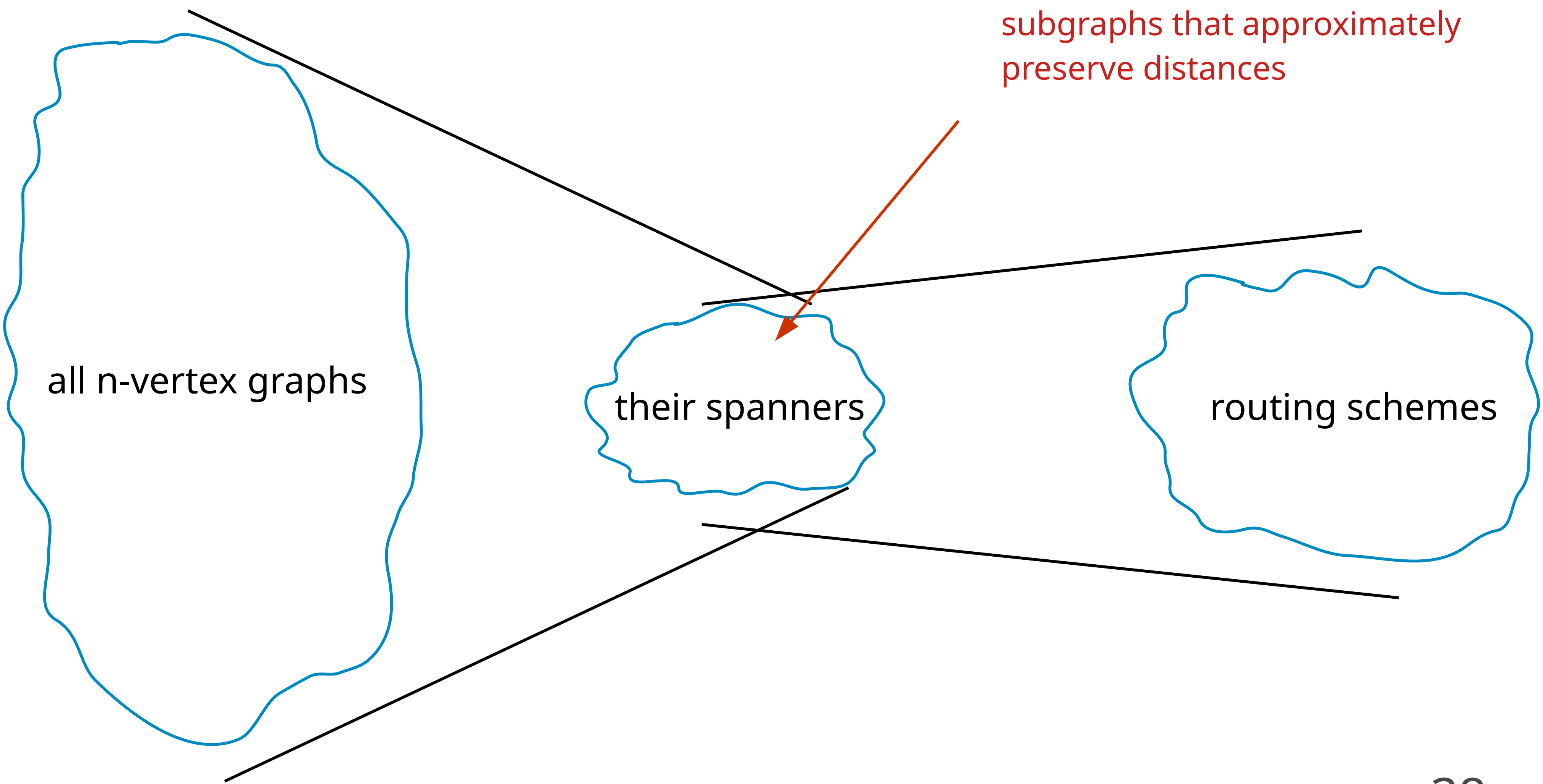
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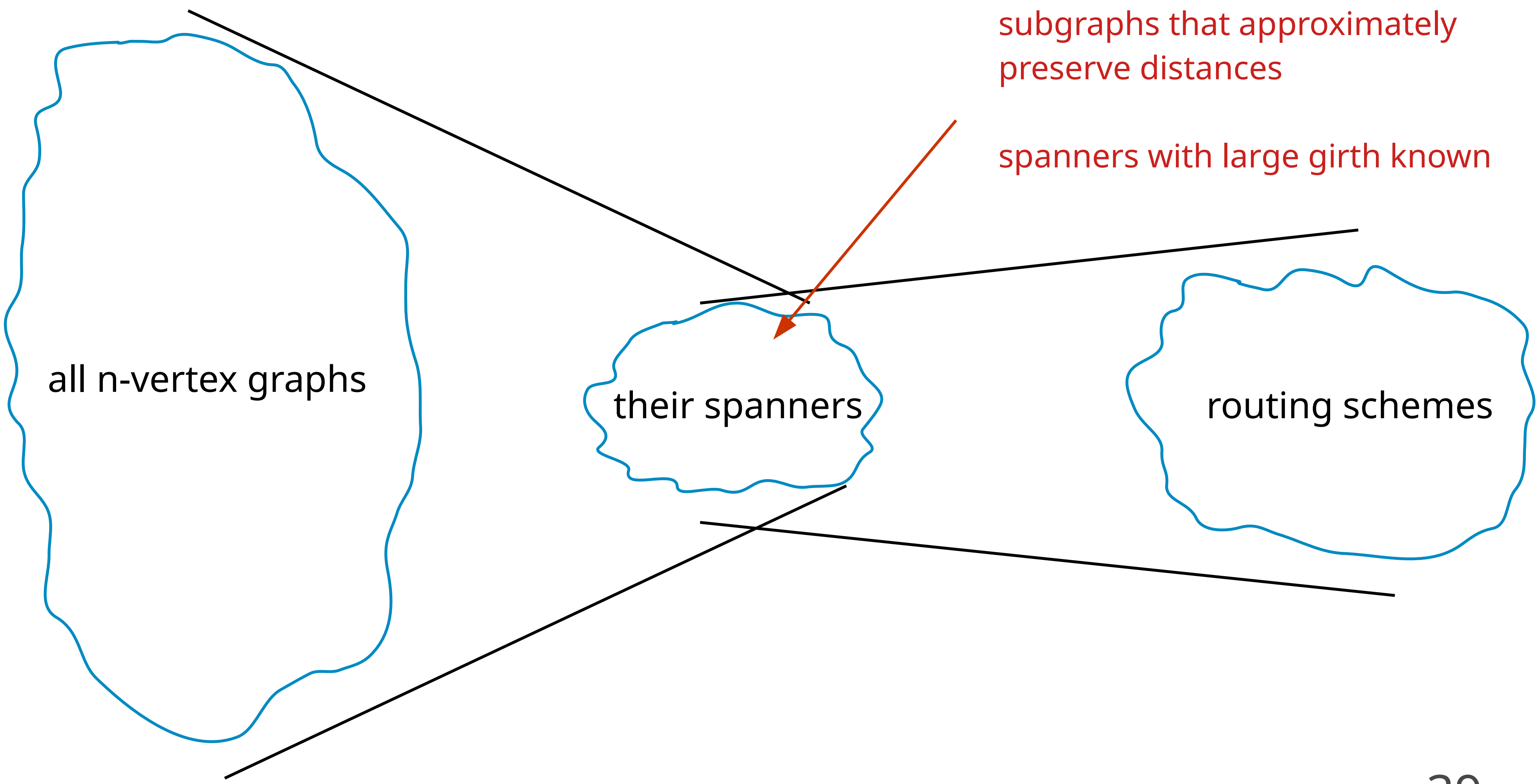
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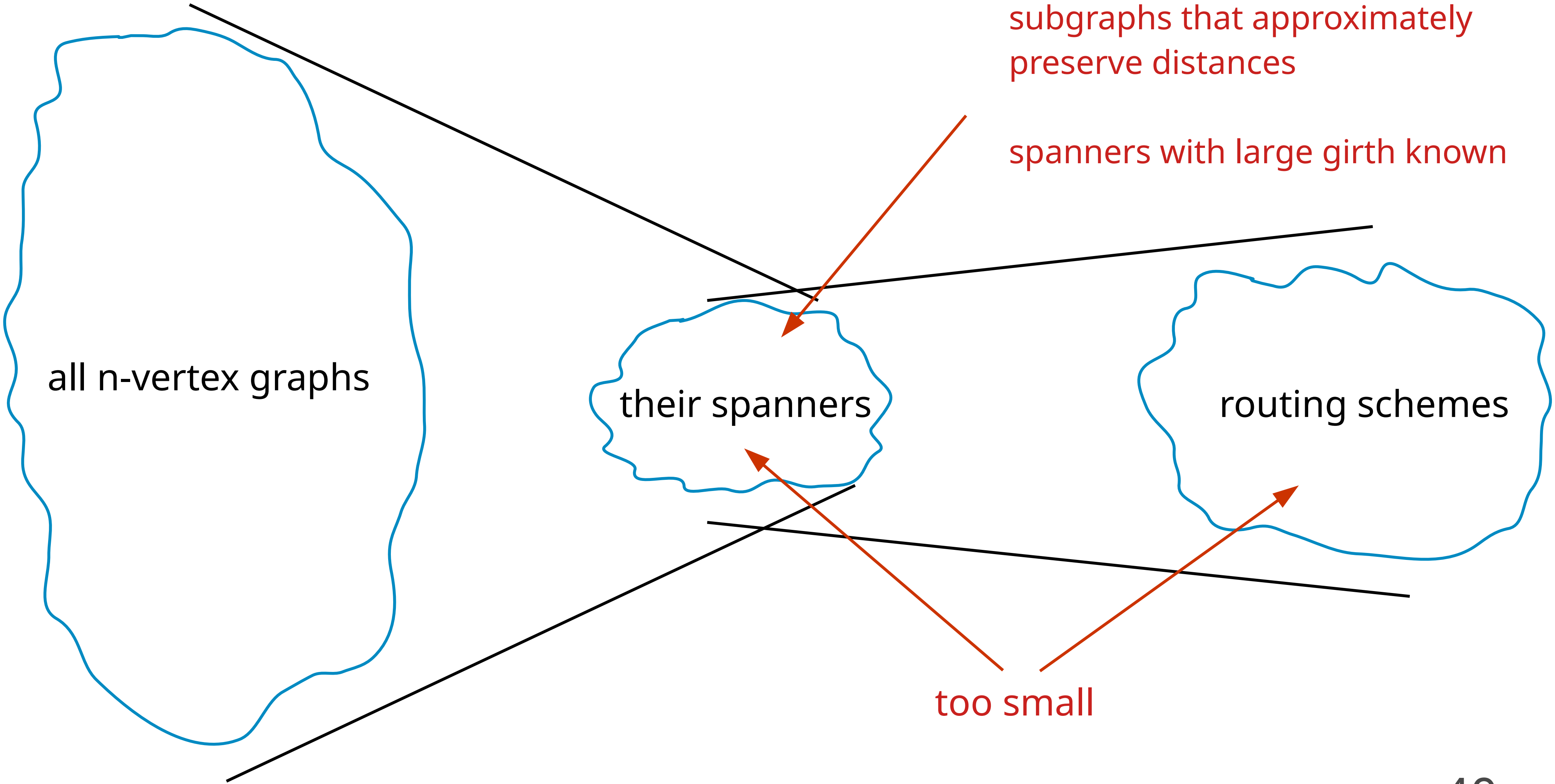
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routing schemes
too small

Open problems

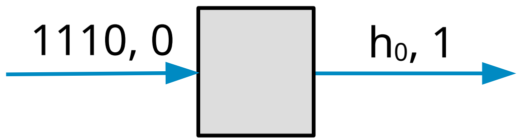
1. Can we overcome girth conjecture in labeled model?
    - (in name-independent model we can [1])

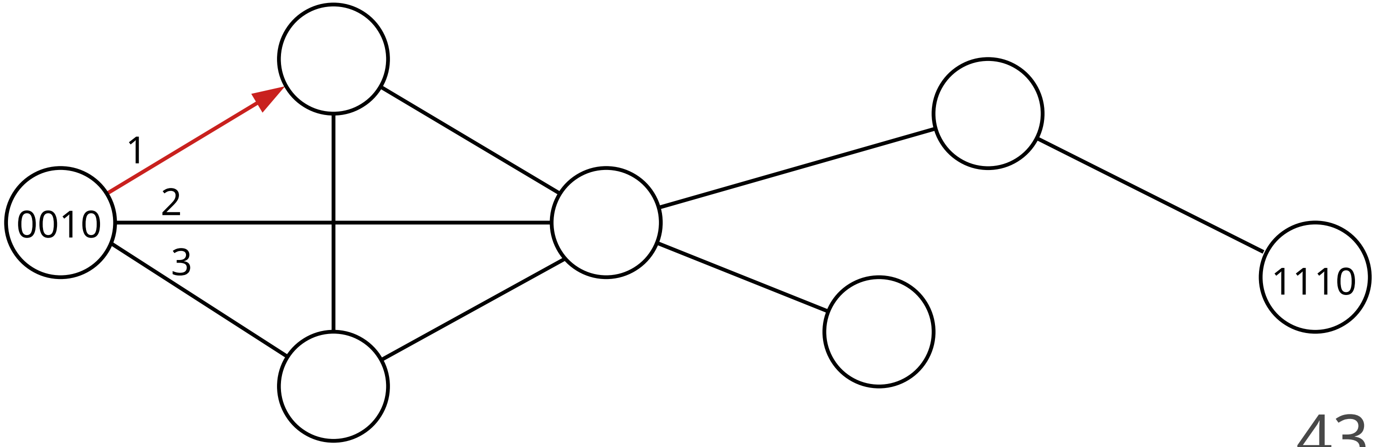
2. Unconditional (non-adversarial ports) lower bound for stretch ≥ 3?

[1]: Ittai Abraham, Cyril Gavoille, and Dahlia Malkhi. On space-stretch trade-offs: Lower bounds. 2006

Extra slides

Routing in computer networks, example





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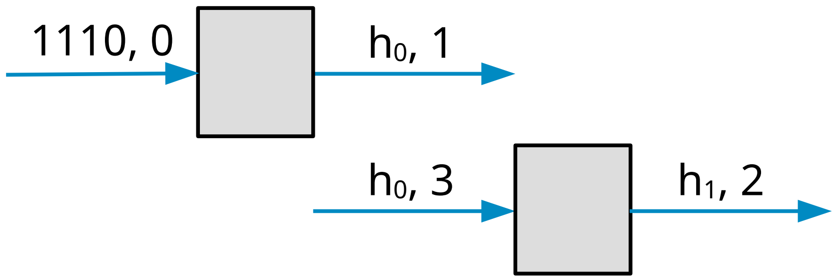

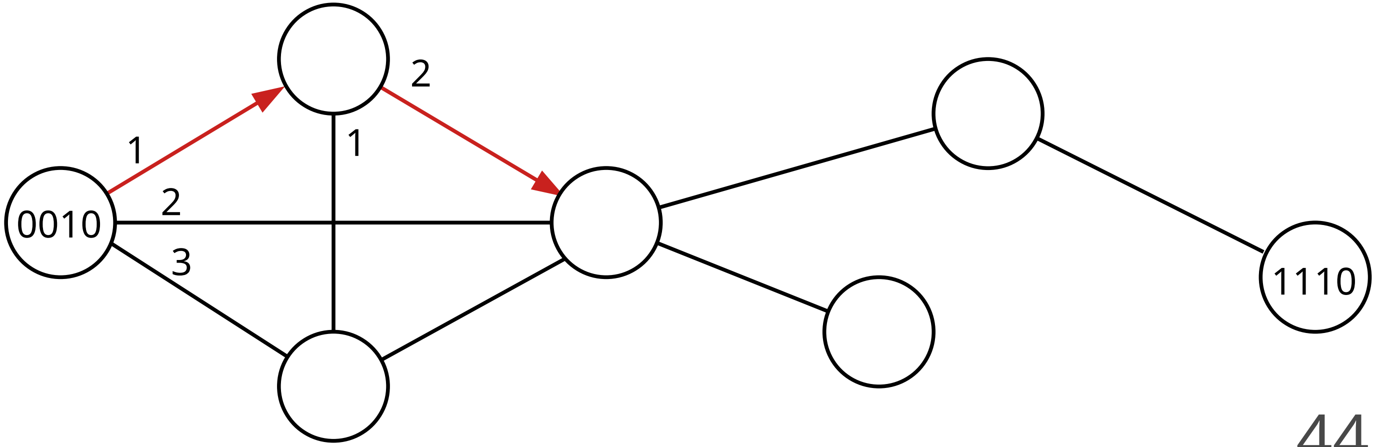

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