Micropolar APIC Method For Turbulent Fluid

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Abstract

Our project aims to simulate **Turbulent Fluid** with **Micropolar** model based on APIC transfer, creating a more animated and energetic rotational flow. Classical fluid simulations are lack of vorticity details, while micropolar provides a **microstructure** to contain more rotational information of the particles, thus provides a more thorough transfer to ensure a realistic result.

Introduction

Micropolar fluids are fluids with microstructure. Its model is a generation of the classical Navier-Stokes formulation. In addition to the linear velocity field, micropolar fluids also have a field of microrotation that represents and provides the existing and new vortices respectively. Therefore, compare to the traditional fluid simulation, it considers the rotational motion of the fluid particles, provides more turbulent details and eventually leads to a more realistic results. This idea was applied in *A Micropolar Material Model for SPH fluid* (Figure 1).

However, due to its concept, classical SPH formulations suffer from tensile instabilities and lack of consistency. Therefore, the hybrid Lagrangian/Eulerian simulation, such as APIC, will provide a more authentic result (Figure 2).

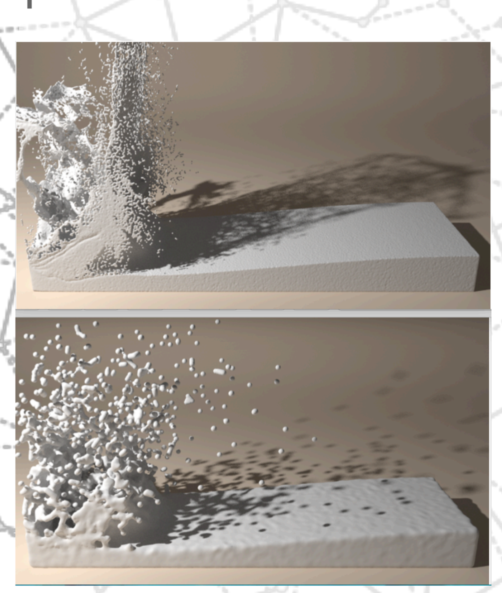

Figure 2: Comparison of APIC (Top left) and SPH (Bottom Left) within the same scene.

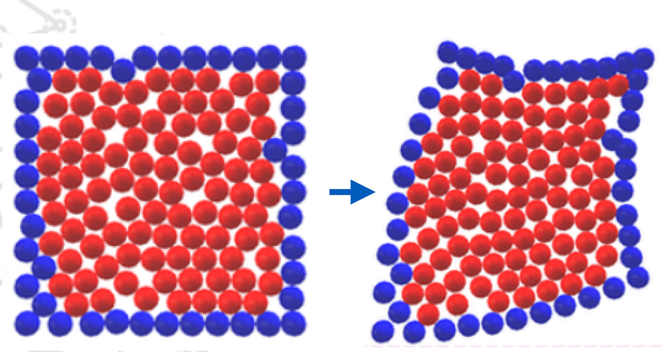

Figure 1: The deformation of micro- continuum in micropolar fluid theory.

Thus, we would like to further develop the micropolar model onto APIC fluid simulation to realize a more vivid turbulent fluid simulation.

Method

In *Micropolar Fluids: Theory and Application*, the isotropic polar fluid equation is derived by the law of conservation of linear and angular momentum. According to the derivation and summery from previous achievement, we arrive at and start with the system of equation:

$$\frac{\mathbf{v}^{n+1}-\mathbf{v}^{n}}{\Delta t} = -\frac{1}{\rho}\nabla p + \nu_t(\Delta \times \omega)$$

$$\frac{(\Theta\omega)^{n+1}-(\Theta\omega)^{n}}{\Delta t} = \nu_t(\nabla \times \mathbf{v} - 2\omega)$$

Explicit and implicit are two approaches to realize update before transfer, implicit method will be discussed in the future plan.

Explicit

Let C_p and $\mathbf{v}_p$ be particle state for velocity , and for angular velocity, G_p and ω_p are the corresponding state. It refers that for fluid, velocity vectors are stored per particle, denoted by c_{px}, c_{py}, c_{pz} , and angular velocity vectors are stored, similarly, per particle, denoted by g_{px}, g_{py}, g_{pz} . In 2D case, the direction of ω_p is towards z , i.e $\omega_p = (0, 0, \omega_p)$. We then construct G_p as c_{px} for transfer.

By solving the equation, we arrive at the two explicit recursive equations for updating velocity and angular velocity before APIC transfer:

$$\mathbf{v}_p^* = \mathbf{v}_p + \Delta t \nu_t(\Delta \times \omega_\mathbf{p})$$

$$\omega_p^* = \omega_p + \Delta t \frac{\nu_t}{\Theta}(\Delta \times \mathbf{v}_p - 2\omega_p)$$

We then transfer to the grid. The following diagram shows a brief procedure of update and transfer, figure 3 also shows the information transportation on MAC-Grid :

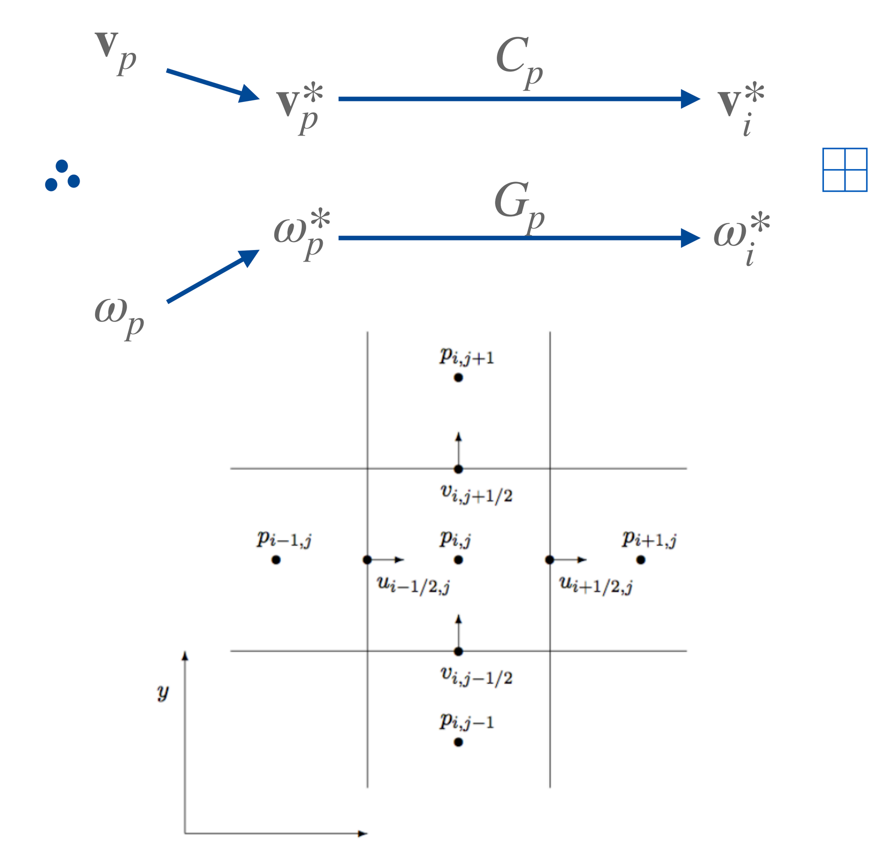

Figure 3: Transfer on MAC-Grid

Current Result

Figure 3: APIC with micropolar in 2D

We have attempted to implement from APIC2D by establishing the microstructure to it, for which APIC2D is an educational project to illustrate the Affine Particle in Cell algorithm for fluid simulation in 2D. Fix $\Theta = 2$, we then set $\nu_t = 0.01,\ 0.02$ (Figure 3, middle and bottom respectively), at the same frame, we can see that there are noticeably more vortices within the model.

The following outputs (Figure 4) are from MantaFlow with micropolar structure established. However, due to the inaccurate transfer information of G, only velocity update was able to be set, and it results a unnoticeable change 0but still missing the information of angular velocity, which leads to an inaccurate calculation of curl. Therefore, we will follow the step of velocity construction to establish an accurate structure of angular velocity.

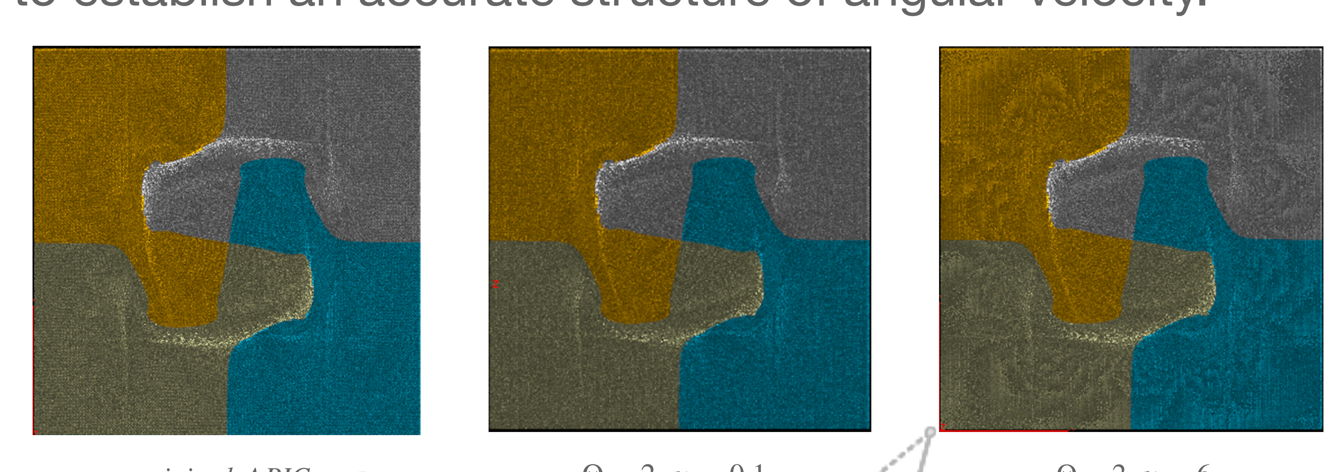

Figure 4: Vortex sheet with a sphere rotating at the center

Challenge and Implementation

There are two unknown variables in the equation mentioned previously, **kinematic transfer coefficient** ν_t and **microinertia coefficient** Θ . We, therefore, set $\Theta = 2$ according to related demonstration and mainly test on ν_t .

For explicit approach, we have finished most of implementation of 2D using **MantaFlow** (an open-source framework targeted at fluid simulation). For the 3D scale, adjustment will be made in the numerical algorithm.

For particle information, aside from velocity, we add angular velocity to it, storing in G_p , and transfer it similar as velocity at the x axis.

Future Plan

As ν_t increases, explicit method tends to be unstable, we then consider **implicit** method for update. The following is a brief plan for the next steps:

- Complete 3D implementation for explicit update method
- Provide a thorough theoretical verification and implementation guidelines for implicit update approach based on the notes
- Complete the results

Acknowledgement

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