CS585 Assignment 3 Written Part

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Exercise 1: Morphology

Please see attached handwritten solutions for Erosion, Dilation, Opening, and Closing.

Exercise 2: Hausdorff Distance

Part 1: Compute the Hausdorff Distance between S_1 and S_2.

We are given that $S_1 = \{A, B, C\}$, $S_2 = \{D, E, F, G\}$, as well as the coordinates for all of $\{A, B, C, D, E, F, G\}$. By definition, we have

$$H(S_1, S_2) = \max\{h(S_1, S_2), h(S_2, S_1)\},$$

so let's compute $h(S_1, S_2)$ and $h(S_2, S_1)$, respectively:

$$\begin{aligned}
h(S_1, S_2) &:= \max_{s_1 \in S_1} \left\{ \min_{s_2 \in S_2} \{d(s_1, s_2)\} \right\} \\
&= \max \left\{ \min_{s_2 \in S_2} \{d(A, s_2)\}, \min_{s_2 \in S_2} \{d(B, s_2)\}, \min_{s_2 \in S_2} \{d(C, s_2)\} \right\} \\
&= \max \Big\{ \min \{d(A, D), d(A, E), d(A, F), d(A, G)\}, \\
&\qquad \min \{d(B, D), d(B, E), d(B, F), d(B, G)\}, \\
&\qquad \min \{d(C, D), d(C, E), d(C, F), d(C, G)\} \Big\} \\
&= \max \Big\{ \min \Big\{ \sqrt{2}, \sqrt{17}, \sqrt{32}, \sqrt{17} \Big\}, \min \Big\{ \sqrt{37}, \sqrt{2}, \sqrt{5}, \sqrt{40} \Big\}, \\
&\qquad \min \Big\{ \sqrt{34}, \sqrt{19}, \sqrt{8}, \sqrt{13} \Big\} \Big\} \\
&= \max\{\sqrt{2}, \sqrt{2}, \sqrt{8}\} \\
&= \sqrt{8}.
\end{aligned}$$

$$
\begin{aligned}
h(S_2, S_1) &:= \max_{s_2 \in S_2} \left\{ \min_{s_1 \in S_1} \{d(s_2, s_1)\} \right\} \\
&= \max \left\{ \min_{s_1 \in S_1} \{d(D, s_1)\}, \min_{s_1 \in S_1} \{d(E, s_1)\}, \min_{s_1 \in S_1} \{d(F, s_1)\}, \min_{s_1 \in S_1} \{d(G, s_1)\} \right\} \\
&= \max \Big\{ \min \{d(D, A), d(D, B), d(D, C)\}, \min \{d(E, A), d(E, B), d(E, C)\}, \\
&\qquad \min \{d(F, A), d(F, B), d(F, C)\}, \min \{d(G, A), d(G, B), d(G, C)\} \Big\} \\
&= \max \Big\{ \min \left\{ \sqrt{2}, \sqrt{37}, \sqrt{34} \right\}, \min \left\{ \sqrt{17}, \sqrt{2}, \sqrt{29} \right\}, \\
&\qquad \min \left\{ \sqrt{32}, \sqrt{5}, \sqrt{8} \right\}, \min \left\{ \sqrt{17}, \sqrt{40}, \sqrt{13} \right\} \Big\} \\
&= \max\{\sqrt{2}, \sqrt{2}, \sqrt{5}, \sqrt{13}\} \\
&= \sqrt{13}.
\end{aligned}
$$

Consequently,

$$
H(S_1, S_2) = \max\{h(S_1, S_2), h(S_2, S_1)\} = \max\{\sqrt{8}, \sqrt{13}\} = \sqrt{13}.
$$

Part 2: Compute Hausdorff Distance between the triangle and the rectangle.

To compute the distance between two polygons, rather than simply two sets of discrete points, we shall invoke the "Algorithm for computing $h(A, B)$" described in the McGill link. That being said, however, because the two polygons in our case are overlapping each other rather than separate, we shall only make use of **Lemma 2** and the definition of Hausdorff Distance itself.

Let S_1 now denote the red triangle and S_2 now denote the blue rectangle. **Lemma 2** states that we may consider only the vertices of the starting figure, find the minimum distance for each such vertex, and then take the maximum of all those minima. To find the minima, we consider the length of the perpendicular line segment that starts from the current vertex of the starting figure to the "closest" edge of the target figure. Consequently, for $h(S_1, S_2)$, we have

$$
\begin{aligned}
h(S_1, S_2) &= \max \{\min\{d(A, S_2)\}, \min\{d(B, S_2)\}, \min\{d(C, S_2)\}\} \\
&= \max \{1, 1, 2\} \\
&= 2.
\end{aligned}
$$

For $h(S_2, S_1)$, we need to first compute the distances between D and $\overline{AC}$, between G and $\overline{AC}$, between E and $\overline{AB}$, and between F and $\overline{BC}$. To do so, we first need to compute the line representations of $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$, respectively. With a little bit of high-school algebra, we can obtain:

$$
\begin{aligned}
\overline{AB}: &\quad y = -\frac{2}{5}x + \frac{11}{5} \\
\overline{AC}: &\quad y = -3x - 3 \\
\overline{BC}: &\quad y = \frac{4}{3}x - 3
\end{aligned}
$$

Consequently, a perpendicular line from D to $\overline{AC}$ would be

$$
D \perp \overline{AC}: \quad y = \frac{1}{3}x + 3,
$$

which intersects $\overline{AC}$ at $z_1(-\frac{9}{5}, \frac{12}{5})$, which means $\min\{d(D, S_1)\} = d(D, z_1) = \sqrt{1.6}$. A perpendicular line from G to $\overline{AC}$ would be

$$G \perp \overline{AC}: \qquad y = \frac{1}{3}x + 3,$$

which intersects $\overline{AC}$ at $z_2(-\frac{9}{10}, -\frac{3}{10})$, which means $\min\{d(G, S_1)\} = d(G, z_2) = \sqrt{4.9}$. A perpendicular line from E to $\overline{AB}$ would be

$$E \perp \overline{AB}: \qquad y = \frac{5}{2}x - 3,$$

which intersects $\overline{AB}$ at $z_3(-\frac{52}{29}, -\frac{43}{29})$, which means $\min\{d(E, S_1)\} = d(E, z_3) \approx \sqrt{0.31}$. A perpendicular line from F to $\overline{BC}$ would be

$$F \perp \overline{BC}: \qquad y = -\frac{3}{4}x + \frac{1}{2},$$

which intersects $\overline{BC}$ at $z_4(\frac{42}{25}, -\frac{19}{25})$, which means $\min\{d(F, S_1)\} = d(F, z_4) = \sqrt{0.16}$.

Consequently, we get

$$\begin{aligned} h(S_2, S_1) &= \max\{\min\{d(D, S_1)\}, \min\{d(E, S_1)\}, \min\{d(F, S_1)\}, \min\{d(G, S_1)\}\} \\ &= \max\left\{\sqrt{1.6}, \sqrt{0.31}, \sqrt{0.16}, \sqrt{4.9}\right\} = \sqrt{4.9}. \end{aligned}$$

Therefore, overall, we have

$$H(S_1, S_2) = \max\{h(S_1, S_2), h(S_2, S_1)\} = \max\{2, \sqrt{4.9}\} = \sqrt{4.9}.$$

Exercise 3: Edge Detection

(a) Define what an edge is in an image.

An edge in an image is a change in brightness. (Verbatim definition from class.)

(b) Briefly describe three causes of edges (1 sentence each).

According to Wikipedia: (1) Edges can be affected by "focal blur" caused by a finite depth-of-field and finite point spread function; (2) edges can be affected by penumbral blur caused by shadows created by light sources of non-zero radius; (3) shading of a smooth object.

Let's decrypt what each of the above three points means: (1) At any given time, there's only one plane of focus (depth of field) and there's one certain way in which the imaging system would respond to that plane of focus (point spread function), and everything that lies outside that plane of focus appears blurred to the focal-point observer; consequently, this creates, for the observer, the impression that there are different *surfaces*, which in turn gives the observe the impression of the existence of edges between those surfaces. (2) When a shadow is cast by an object against the background, this creates a change in brightness, and thus an edge, between the background and the shadow. (3) Frequently, an object is only lit up by one or a few light sources, and that creates for the observer the impression that there's a "better lit" region and a "less lit" region of that object; consequently, this creates a change in brightness, and thus an edge, between the better-lit and the less-lit regions.

(c) Canny Edge Detector

Please see attached image.